Multiple Input Production Economics

EXAMPLES

AAE 320

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Equal Margin Principle

Equal Margin Principle: expressed mathematically in two ways

1) $\frac{MP_x}{r_x} = \frac{MP_y}{r_y}$ Ratio of MP/r must be equal for all inputs

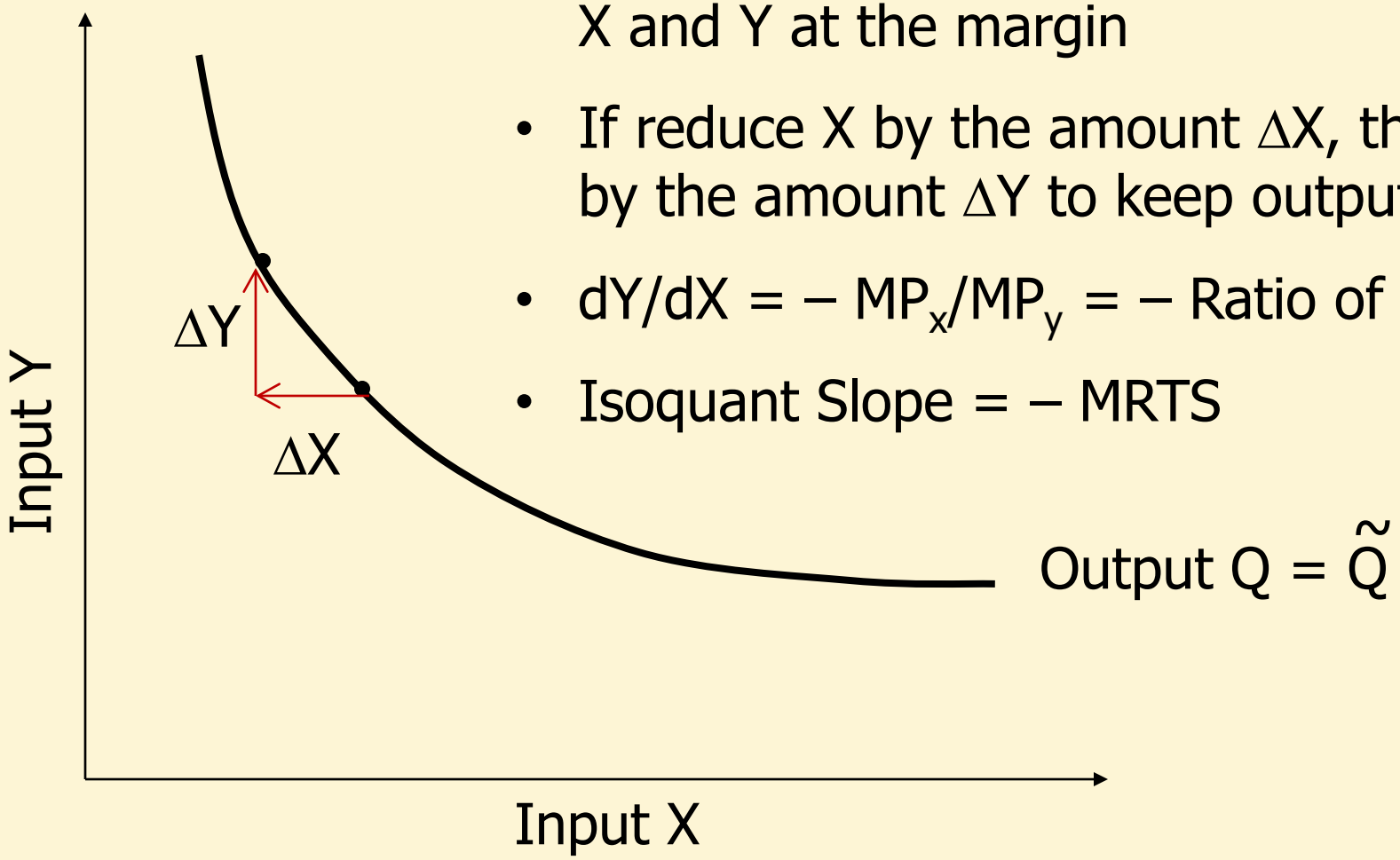
2) $\frac{MP_x}{MP_y} = \frac{r_x}{r_y}$ Ratio of MP's must equal input price ratio

Marginal Rate of Technical Substitution

- The ratio of marginal products (MP_x/MP_y) is the substitution rate between inputs in the production process
    - Slope of the tradeoff curve
- MP_x/MP_y is called the Marginal Rate of Technical Substitution (MRTS): the input substitution rate at the margin
- If you cut X by one unit, how much must you increase Y to keep output the same
- If you increase Y by one unit, how much can you cut X and still keep the same output
- Optimality condition $MP_x/MP_y = r_x/r_y$ means set substitution rates equal

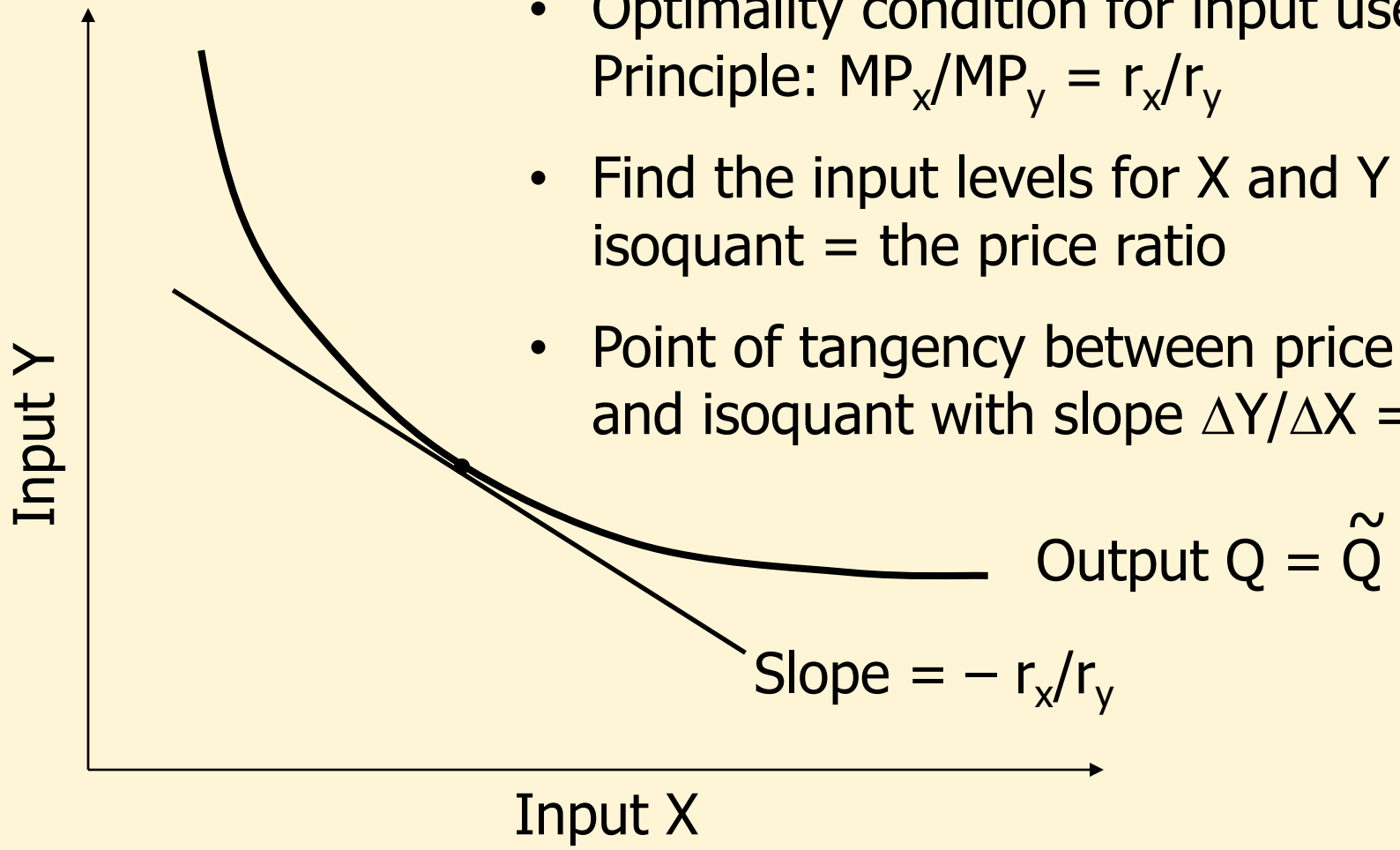
Isoquants and MP_x/MP_y

- Isoquant Slope = $\Delta Y/\Delta X$ = Substitution rate between X and Y at the margin
- If reduce X by the amount ΔX, then must increase Y by the amount ΔY to keep output fixed
- $dY/dX = -MP_x/MP_y$ = – Ratio of MP's = – MRTS
- Isoquant Slope = – MRTS



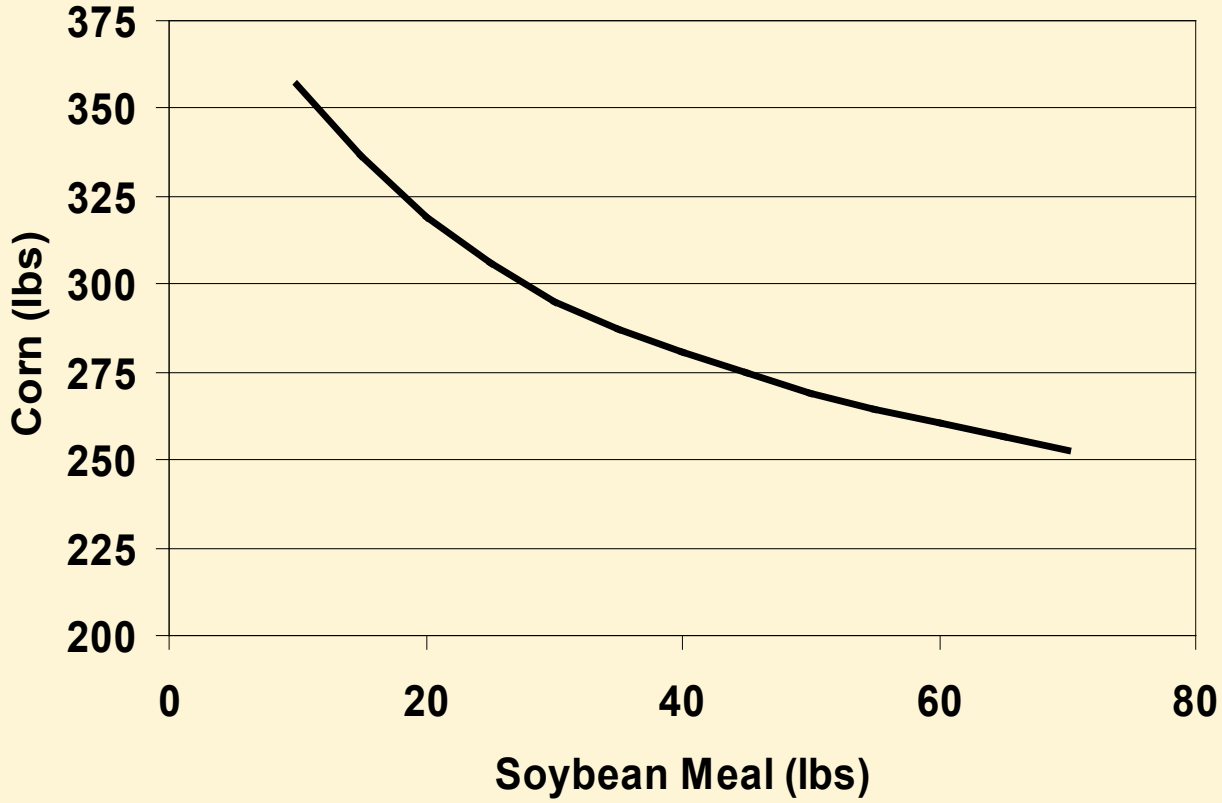
Isoquants and $\frac{MP_x}{MP_y} = \frac{r_x}{r_y}$

- Optimality condition for input use from the Equal Margin Principle: $MP_x/MP_y = r_x/r_y$
- Find the input levels for X and Y that set slope of the isoquant = the price ratio
- Point of tangency between price line with slope $– r_x/r_y$ and isoquant with slope $\Delta Y/\Delta X = – MP_x/MP_y$



Soybean Meal (lbs)	Corn (lbs)
10	376.8
15	356.3
20	339.0
25	326.0
30	315.0
35	307.5
40	300.6
45	294.6
50	289.2
55	284.4
60	280.2
65	276.4
70	272.9

Soybean meal and corn needed for 125 lb feeder pigs to gain 125 lbs



Which feed ration do you use?

Soy Meal (lbs)	Corn (lbs)	MRTS	P ratio
10	376.8		2.20
15	356.3	4.10	2.20
20	339.0	3.46	2.20
25	326.0	2.60	2.20
30	315.0	2.20	2.20
35	307.5	1.50	2.20
40	300.6	1.38	2.20
45	294.6	1.20	2.20
50	289.2	1.08	2.20
55	284.4	0.96	2.20
60	280.2	0.84	2.20
65	276.4	0.76	2.20
70	272.9	0.70	2.20

- Economically Optimal input use is where MRTS = input price ratio, $-\Delta Y/\Delta X = r_x/r_y$
- (Same as $MP_x/MP_y = r_x/r_y$)
- Soy Meal Price = \$0.088/lb, Corn Price = \$0.04/lb, Ratio = 0.088/0.04 = 2.20
- Keep straight which is X and which is Y!!!

$1.20 = -(294.6 - 300.6)/(45 - 40)$

$0.96 = -(284.4 - 289.2)/(55 - 50)$

Soy Meal (lbs)	Corn (lbs)	MRTS	P ratio
10	376.8		2.20
15	356.3	4.10	2.20
20	339.0	3.46	2.20
25	326.0	2.60	2.20
30	315.0	2.20	2.20
35	307.5	1.50	2.20
40	300.6	1.38	2.20
45	294.6	1.20	2.20
50	289.2	1.08	2.20
55	284.4	0.96	2.20
60	280.2	0.84	2.20
65	276.4	0.76	2.20
70	272.9	0.70	2.20

- Suppose Soy Meal Price = \$0.10/lb, Corn Price = \$0.05/lb, Ratio = 0.10/0.05 = 2.0
- Interpolation

	B_{btwn} Unknown		A_{btwn} Known
	Soy	Corn	MRTS
Row 0	30	315.0	2.20
Between			2.00
Row 1	40	300.6	1.50

$$B_{btwn} = B_0 + (A_{btwn} - A_0)\frac{B_1 - B_0}{A_1 - A_0}$$

$$B_{btwn} = 30 + (2.0 - 2.2)\frac{40-30}{1.5-2.2} = 32.9$$

$$B_{btwn} = 315 + (2.0 - 2.2)\frac{300.6-315}{1.5-2.2} = 310.9$$

Example 1

Grain (lbs)	Hay (lbs)	MRTS	price ratio
825	1350		
900	1130	2.93	
975	935		
1050	770	2.20	
1125	625	1.93	
1200	525		
1275	445	1.07	

The table is rations of grain and hay that put 300 lbs of gain on 900 lb steers.

1) Fill in the missing MRTS
2) If grain is $0.06/lb and hay is $0.03/lb, what is the economically optimal feed ration?

Example 1: Answer

Price Ratio: \$0.06/\$0.03 = 2.0

= – (935 – 1130)/(975 – 900)

Grain (lbs)	Hay (lbs)	MRTS	price ratio
825	1350		2.0
900	1130	2.93	2.0
975	935	2.60	2.0
1050	770	2.20	2.0
1125	625	1.93	2.0
1200	525	1.33	2.0
1275	445	1.07	2.0

B_{btwn} Unknown		A_{btwn} Known
Grain	Hay	MRTS
1050	770	2.20
		2.00
1125	625	1.93

$$\text{Grain } B_{btwn} = 1050 + (2.0 - 2.2)\frac{1125-1050}{1.93-2.2} =$$

$$\text{Hay } B_{btwn} = 770 + (2.0 - 2.2)\frac{625-770}{1.93-2.2} =$$

= – (525 – 625)/(1200 – 1125)

Example 1: Answer

Price Ratio: \$0.06/\$0.03 = 2.0

= – (935 – 1130)/(975 – 900)

Grain (lbs)	Hay (lbs)	MRTS	price ratio
825	1350		2.0
900	1130	2.93	2.0
975	935	2.60	2.0
1050	770	2.20	2.0
1125	625	1.93	2.0
1200	525	1.33	2.0
1275	445	1.07	2.0

= – (525 – 625)/(1200 – 1125)

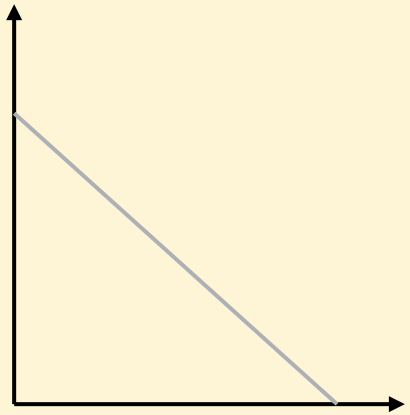
B_{btwn} Unknown		A_{btwn} Known
Grain	Hay	MRTS
1050	770	2.20
1106	663	2.00
1125	625	1.93

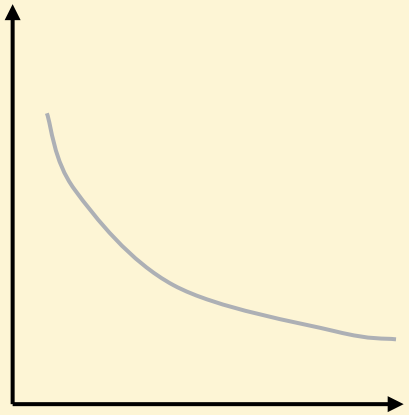
$$\text{Grain } B_{btwn} = 1050 + (2.0 - 2.2)\frac{1125-1050}{1.93-2.2} = 1106$$

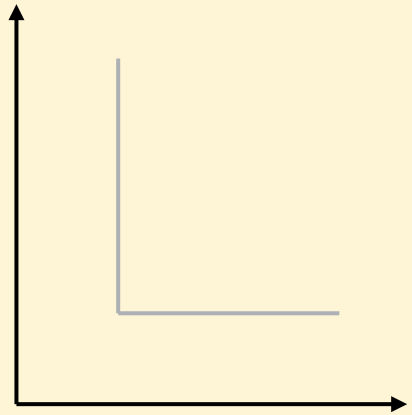
$$\text{Hay } B_{btwn} = 770 + (2.0 - 2.2)\frac{625-770}{1.93-2.2} = 663$$

Types of Input Substitution

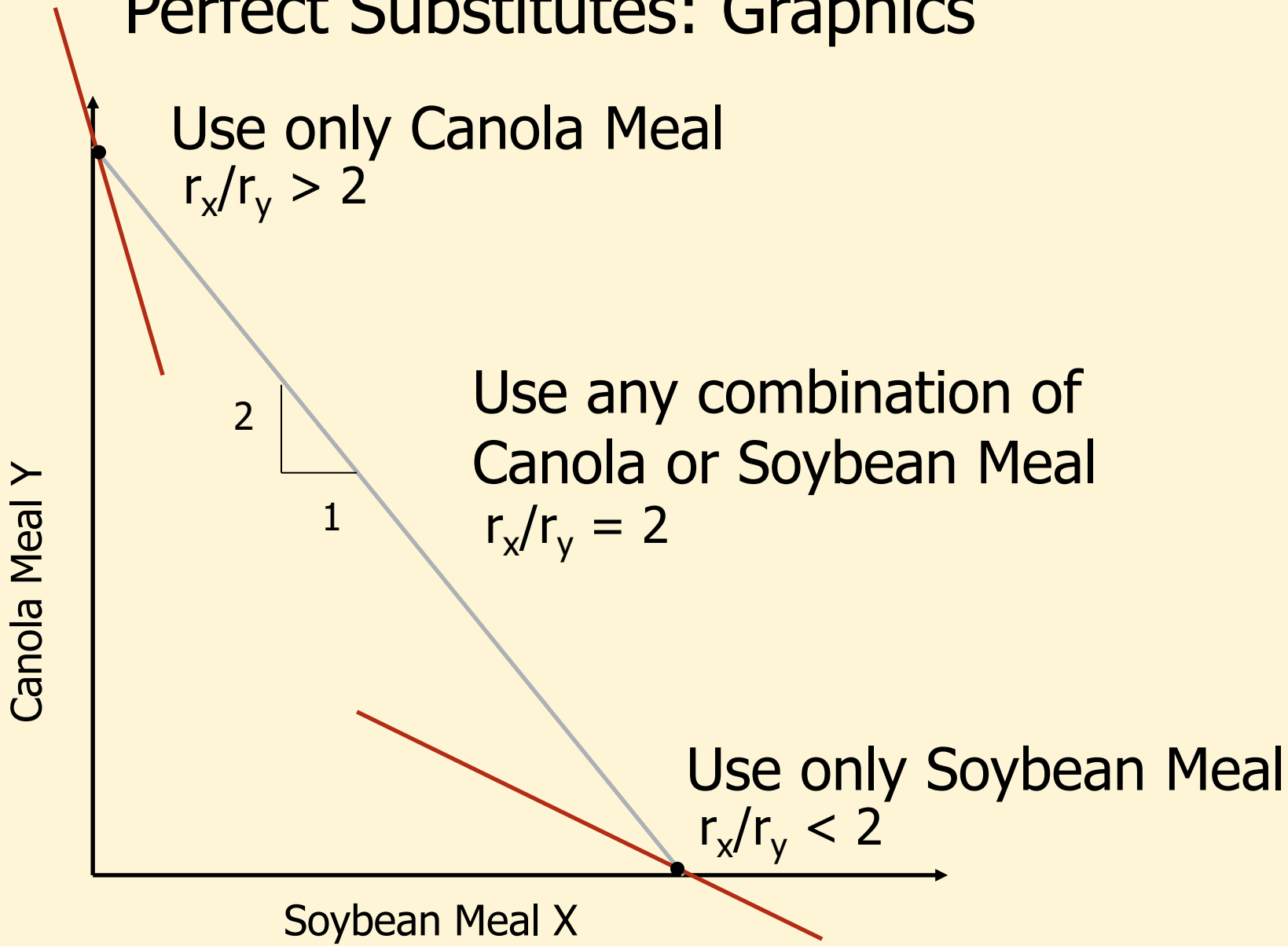
- Perfect substitutes: soybean meal & canola meal, corn & sorghum, wheat and barley
- Imperfect substitutes: corn & soybean meal
- Non-substitutes/perfect complements: tractors and drivers, wire and fence posts



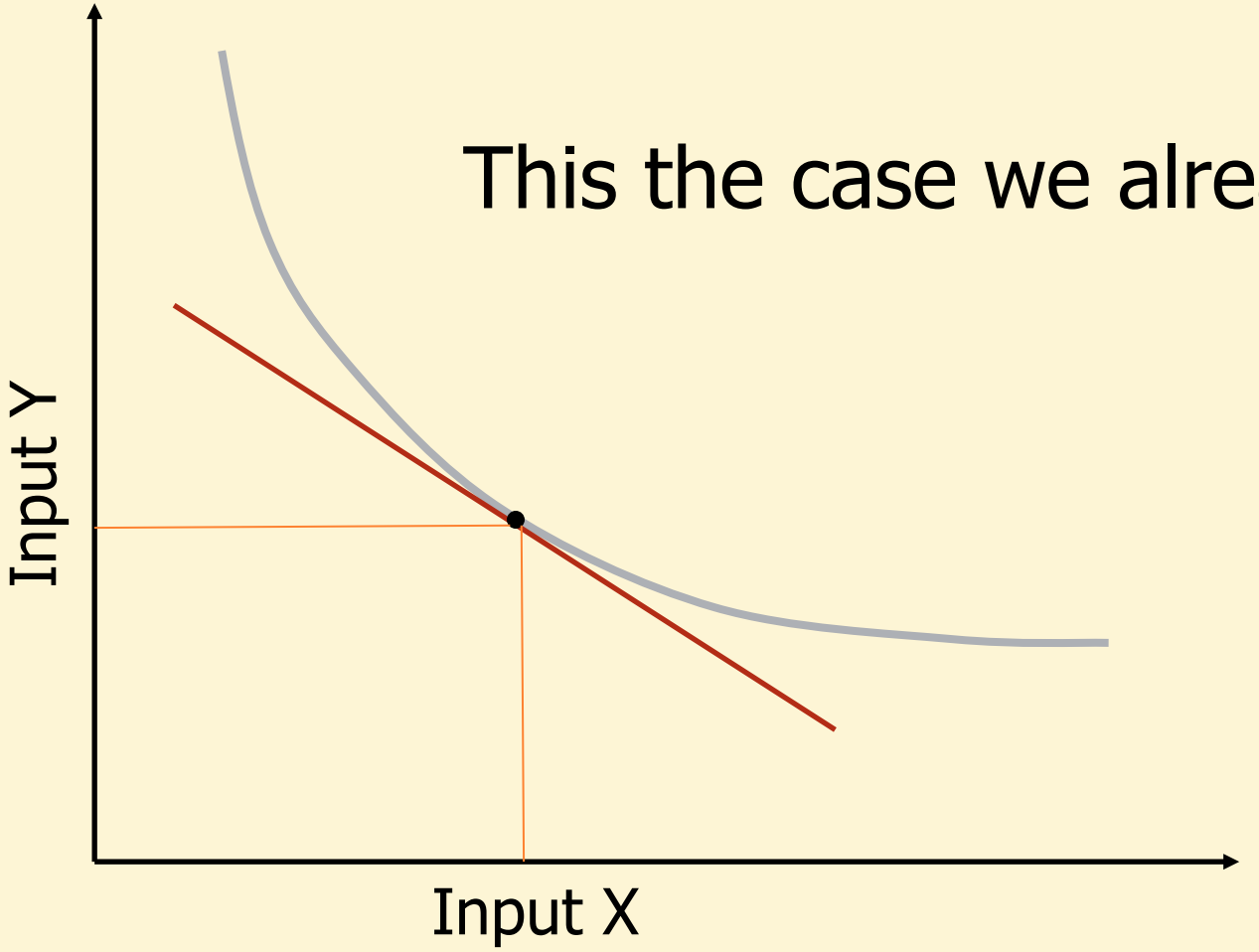




Perfect Substitutes: Graphics



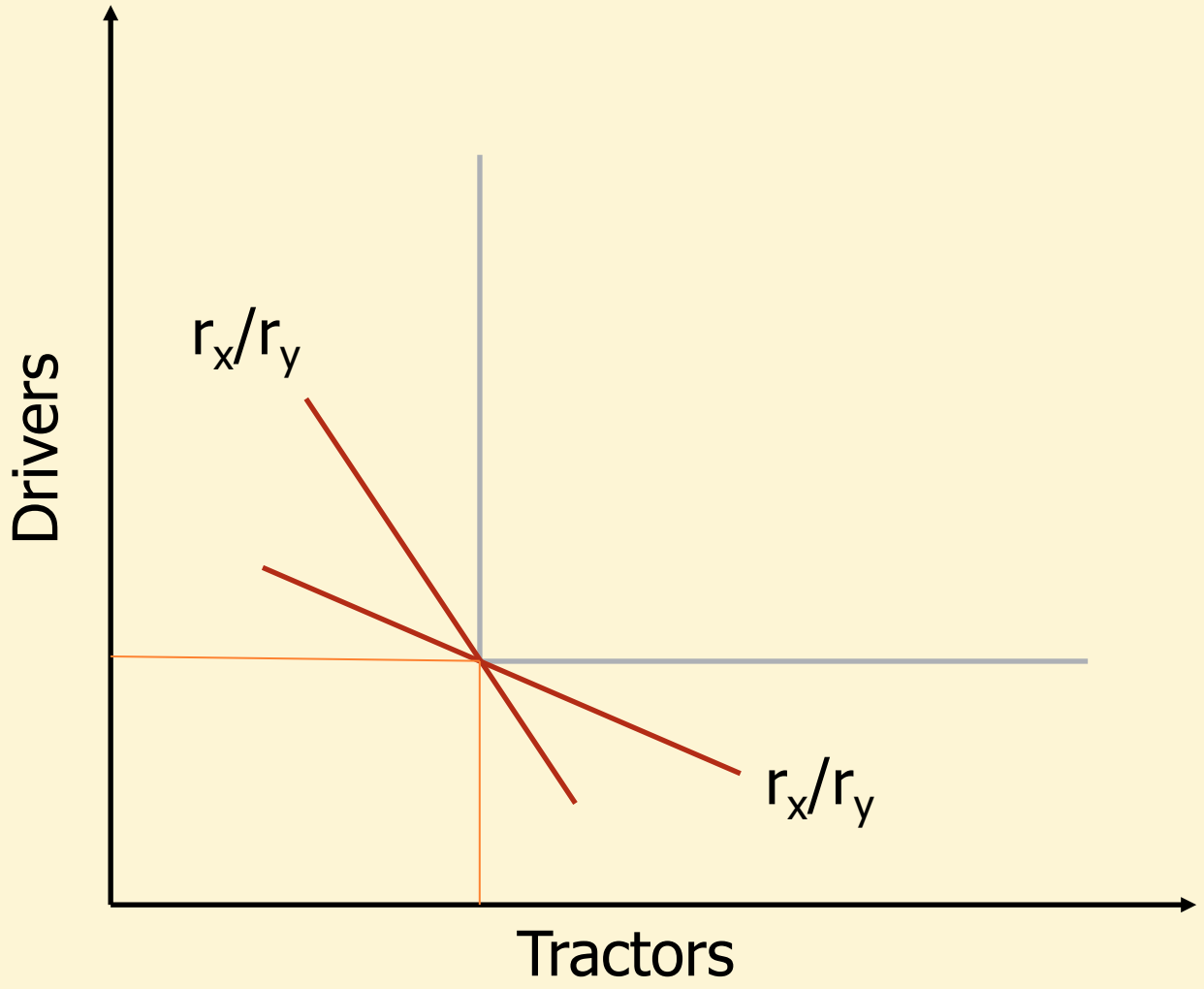
Economics of Imperfect Substitutes



This the case we already did!

Perfect Complements: Graphics

Price ratio does not matter



Example 2

- Corn yield is $Y = 10 + 12N – 0.2N^2 + 15W – 0.3W^2 – 0.1NW$,
- Y = corn yield (bu/ac), N = nitrogen (lbs/ac), W = water (acre inches)
- Corn price = \$3/bu, N price = \$0.5/lb, W price = \$12/acre inch
- What is the profit maximizing amount of nitrogen (N) and water (W) to use per acre to grow corn (Y)?
- Set Up:
- FOCs:
- SOCs:
- Yield & Profit at optimal N and W:

Example 2 Answer

- Set Up: π =
- FOC_N:
- FOC_W:

Example 2 Answer

- Set Up: $\pi = 3(10 + 12N - 0.2N^2 + 15W - 0.3W^2 - 0.1NW) - 0.5N - 12W$
- FOC_N: $3(12 - 0.4N - 0.1W) - 0.5 = 0$
- FOC_W: $3(15 - 0.6W - 0.1N) - 12 = 0$
- Solve FOC_N for N:
  -
- Substitute into FOC_W:
  -
  -
- N =

Example 2 Answer

- Set Up: $\pi = 3(10 + 12N – 0.2N^2 + 15W – 0.3W^2 – 0.1NW) – 0.5N – 12W$
- FOC_N: $3(12 – 0.4N – 0.1W) – 0.5 = 0$
- FOC_W: $3(15 – 0.6W – 0.1N) – 12 = 0$
- Solve FOC_N for N: $12 – 0.4N – 0.1W = 0.5/3 = 0.167$
  - $12 – 0.167 – 0.1W = 0.4N \qquad N = 29.58 – 0.25W$
- Substitute into FOC_W: $15 – 0.6N – 0.1N = 12/3 = 4$
  - $15 – 4 – 0.6W – 0.1(29.58 – 0.25W) = 0$
  - $11 – 0.6W – 2.958 + 0.025W = 0 \quad 8.042 = 0.575W$ **W = 13.99 = 14.0**
- $N = 29.58 – 0.25W = 29.58 – 0.25(14.0) =$ **26.1 = N**

Example 2 Answer Continued

- SOCs:
  - π_{NN} =
  - π_{WW} =
  - π_{NW} =
  - Then
- Yield =
- Yield =
- Profit =

Example 2 Answer Continued

- SOCs:
  - $\pi_{NN} = 3(-0.4) = -1.2 < 0$ passes
  - $\pi_{WW} = 3(-0.6) = -1.8 < 0$ passes
  - $\pi_{NW} = 3(-0.1) = -0.03$
  - Then $(-1.2)(-1.8) - (-0.03)^2 = 2.16 - 0.09 = 2.07 > 0$ passes
- Yield = $10 + 12(26.1) - 0.2(26.1)^2 + 15(14) - 0.3(14)^2 - 0.1(26.1)(14)$
- Yield = 301.6 bu/ac
- Profit = $3(301.6) - 0.5(26.1) - 12(14) = \$723.75$/ac (minus other costs)