Advertising in Differentiated Markets

By

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Abstract: This paper considers the role of promotional expenditures by firms in a differentiated product oligopoly model. Advertising is informative and has two types of effects on demand, accommodating effects, which increase total demand in the product class, and predatory effects, which serve only to redistribute consumers among brands in the product class. Depending on the relative importance of each type of effect, advertising can make the demand facing individual brands more elastic or less elastic. This has important implications for whether or not the market-determined levels of advertising are excessive from the welfare perspective. We compare oligopoly outcomes under multi-brand promotion by a monopolist, as well as under both quantity-setting and price-setting forms of oligopoly to the social optimum and identify the linkage between the nature of oligopoly interaction and the predominance of certain forms of advertising effects.

Key words: Oligopoly; product differentiation; informative advertising

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1. Introduction

Promotional competition is important in differentiated product markets. In 2001, total per capita advertising in measured media was $388 in the United States and per capita advertising in measured and unmeasured media (including direct mail and local spot radio and cable) was estimated to be $820 in the U.S. (Advertising Age). Advertising is an important industry in the U.S., in and of itself, and represented a 2.3 percent share of GNP in 2001. Major advertisers include Proctor & Gamble, General Motors, and Unilever, who spent more than $3 billion each on advertising in 2001. In the product categories of restaurants, pharmaceuticals, and soft drinks, moreover, advertising as a percent of sales is traditionally very high. In 2001, the four leading national advertisers in the restaurant category (McDonalds, Wendy’s International, Yum Brands, and Diageo) maintained advertising budgets between 12.8 percent and 22.1 percent of U.S. sales, and, in the soft drink industry, Coca Cola and Pepsi Co. spent 11.9 and 12.1 percent of U.S. sales revenue on advertising. Advertising expenses are large and increasing as a percent of sales in consumer goods industries, such as detergents, cigarettes, and beer, where products are highly substitutable and competition among brands is intense.

The role of advertising in differentiated markets is not well understood. While it has long been recognized that advertising can have two types of effects on demand, either increasing general demand for the product or altering the distribution of consumers between brands within the product category, the relative importance of these two effects remains a subject of considerable debate. In the empirical literature, the relative importance of these two types of effects seems to depend on the particular industries studied. Kelton and Kelton (1982) find that advertising in the brewing industry primarily alters market share between brands and Sass and Saurman (1995) show that advertising bans in the malt beverage industry significantly impact concentration, whereas Roberts and Samuelson (1988) find that cigarette advertising increases
the level of total market demand, but does not influence the distribution of market shares. In the theoretical literature, these forms of advertising effects have been largely handled separately. In the informative advertising literature following Butters (1977) and Grossman and Shapiro (1984), the central role of advertising is to inform consumers of the existence of products. Informative advertising improves the matching between consumers and brands, which makes advertising a predatory activity from the standpoint of firms, as facilitating better matches between existing consumers and existing brands serves only to redistribute market share. In models of persuasive advertising following Stigler and Becker (1977) and Dixit and Norman (1978), the role of advertising is to improve consumer’s perceptions of the attributes in traded commodities, which makes brands within a product category appear more differentiated to consumers, increasing total demand.

This paper develops a model of informative advertising that encompasses both types of effects. The basic insight of the paper is that informative advertising has the potential to have effects both on individual market shares among brands and on total market demand for the product category that comprises the brands. Advertising that informs consumers of the existence of a brand clearly stimulate new consumption in a product category that otherwise would not take place. However, the focus to date in models of informative advertising has been on models that fix total demand for the product category to examine market equilibrium in models based on Chamberlin (1933) that employ the “large-group” assumption of Dixit and Stiglitz (1977). This eliminates entirely the effect of advertising on expanding total demand in the product category.

By combining both effects of advertising in the model, the model sheds some light on the relationship between advertising and demand elasticities. A higher level of informative advertising tends to make demand more elastic (see, e.g., Grossman and Shapiro (1984)), whereas a higher level of persuasive advertising may increase perceived product differentiation.

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1 For a survey of the empirical literature, see Scherer and Ross (1990).
2 Exceptions include Shapiro (1980), who considers a monopoly model, and Grossman and Shapiro (1984), who simulate the effects of advertising in an oligopoly equilibrium with small numbers of firms. Nonetheless, all analytical results in the latter paper are derived under the large group assumption.
and make demand less elastic. Here, purely informative advertising is capable of producing either outcome, and the type of outcome that emerges in equilibrium depends fundamentally on the relative strength of these two influences.

The relative magnitude of each type of effect also has important welfare implications. When advertising results in a shift in product demand, the familiar welfare result is that advertising is excessive (see, e.g., Dixit and Norman (1978)). In informative advertising models, where advertising has purely distributive effects, informative advertising levels tend to be too low under monopoly (Shapiro (1980)), but excessive under monopolistic competition and large numbers oligopoly models (Grossman and Shapiro (1984)). Firms advertise too little under monopoly, because a monopolist considers only the implication of advertising for profits, and not the benefits consumers receive from improved information. Firms advertise too much under large numbers oligopoly, because the positive effect of informative advertising on consumers is swamped by the negative effect on rival brands when only market share is at stake. The aim of this paper is to examine the connections between informative advertising and welfare in a duopoly model that permits a role for advertising both in increasing total product demand and in redistributing market share.

The paper considers advertising in differentiated product markets. To date, very little research has considered advertising in differentiated product markets, and has focused instead on homogeneous products, both in models of informative advertising (Butters (1977), Stegman (1991), Gary-Bobo and Michel (1991), and Stahl (1994)) and in models of persuasive advertising (Stigler and Becker (1977), Nichols (1985)). This is surprising given that a tremendous volume of advertising occurs in differentiated markets.

The paper also compares the outcome of informative advertising under both price-setting and quantity-setting forms of oligopoly. Familiar differences exist between price- and quantity-setting models in other contexts (e.g., levels of capital investment); however, the implications for advertising remain largely unexplored. Gary-Bobo and Michel (1991) consider informative advertising under quantity competition, but their model is quite unlike ours in the sense that they
consider neither heterogeneous product markets nor compare outcomes under price and quantity forms of competition. This seems to be an important link. It is a stylized fact that advertising-intensity is positively-related to industry-average profitability (Schmalensee (1986)), and this result has generally been corroborated in the theoretical literature (see, e.g., Dixit and Norman (1978), Grossman and Shapiro (1984), Gary-Bobo and Michel (1991)), but the linkages to market structure and product differentiation remain somewhat less clear. For a given set of market conditions, it is well known that equilibrium prices and profits are always higher under quantity competition than under price competition, yet it is not at all understood whether this implies systematic differences in advertising levels.

The model considers promotional activities in differentiated product markets where oligopoly firms face competition from competing brands within a product category. Consumers have stable preferences both for consumption within the product category and for the individual characteristics that define the particular brands, but are uncertain about the characteristics contained in individual brands (and about whether brands even exist at all that offer a desired mix of characteristics), unless they receive advertising messages to inform them. Advertising thus affects demand by providing information about the location of the advertising brand in product space. The technology that links advertising to consumer demand is the “shotgun”-type process first considered by Butters (1977) and elaborated by Grossman and Shapiro (1984). Consumers have heterogeneous tastes. Each consumer has tastes for the characteristics that define the product category, but is unaware of the characteristics offered by individual brands unless she is “hit” by an advertising message. Advertising messages, should firms choose to send them, randomly strike customers, and the ads inform consumers by providing what amounts to a brand “address” in characteristic space. The technology for dispersing ads does so only randomly (hence the term “shotgun”), as advertisers are assumed to be unable to target messages towards only those consumers who find the brand most attractive.

We consider the case of two, differentiated brands that are sold in a common marketplace. Prices are posted in the marketplace, so that consumers have full information on
the relative prices of goods. However, without receiving an advertising message about each particular brand, consumers are aware of neither its characteristics nor its “address” within the central market. This stylized representation of the central market can be understood as follows. Imagine standing in a supermarket in which products randomly disappear off the shelves and the only goods that remain through this process are the particular brands an individual consumer has seen advertised. The product space of the supermarket for an individual consumer ultimately depends on the particular set of ads to which she has been exposed, and, to the extent that consumers differ in their tendency to view ads, each consumer therefore perceives the product space in the market differently according to her particular set of advertising messages.

In the next section we derive demand curves for duopoly firms and describe the effects of informative advertising on demand. In Section 3 we compute the monopoly and oligopoly equilibria under conditions of both price and quantity competition. We then turn to the welfare analysis in Section 4.

2. Informative Advertising and Consumer Demand

In this section, we detail the effect of advertising on consumer demand for heterogeneous products. Advertising is the only source of information in the model and consumers rely on information received from ads to locate specific brands in the product space. Each consumer is passive in the sense that she does not search for a brand that best suits her taste, nor does she engage in any activities designed to acquire information, other than viewing ads. Thus, we implicitly assume that the cost of search is high relative to the surplus offered by brands in the product category.

The model contains two substitute brands, each of which is promoted by a firm who exerts advertising effort. We choose a familiar type of product differentiation, based on the characteristics approach associated with Lancaster (1975), and focus on the role of advertising in matching heterogeneous consumers to the products that suit their tastes. The basic demand structure is represented by a Hotelling (1927) “linear city”, in which each brand is located at the
endpoint of a unit segment that connects the characteristic space, and each consumer is identified by a point on the line segment between them that corresponds to her most preferred brand.

Suppose each consumer consumes at most one unit in the product category and receives a value of \( v \) from consuming a unit of her most preferred brand. Consumers must bear transportation costs of \( t \) per unit of distance to travel to either brand. Given a price for each firm, \( p_i, i = 0,1 \), a consumer located at a distance of \( x \in [0,1] \) enjoys surplus of:

\[
U = \begin{cases} 
  v - p_0 - tx & \text{if purchase made at firm 0} \\
  v - p_1 - t(1 - x) & \text{if purchase made at firm 1} \\
  0 & \text{otherwise}
\end{cases}
\]

Consumers are distributed uniformly on the unit interval and have a common (gross) valuation of the product, \( v \), although the net valuation of a particular brand to consumers depends on their location in characteristic space.

The role of advertising in the model is to convey information about a firm’s product to consumers. Consumers purchase a unit in the product class only if they are aware of the existence of a brand that offers positive surplus. If the consumer is aware of both brands, then she selects the brand that offers the greatest surplus; if a consumer is aware of only one brand, then she either chooses to consume the brand or to consume nothing at all in the product category. From the perspective of each firm, there are thus three types of consumers: (i) those who are unaware of any brand in the category (i.e., consumers who do not receive the advertising message of any firms); (ii) those who are aware of the brand but are unaware of the rival brand; and (iii) those who are aware of both brands.

In full information duopoly models, it is typically assumed that the market is fully covered between the two firms in the sense all consumers on the line segment make a purchase at the equilibrium prices. Indeed, full coverage of the market is a necessary condition for oligopoly equilibrium, as strategic interaction cannot otherwise take place between firms. This assumption of full coverage has been retained in the literature on informative advertising.
Our departure point from existing models of informative advertising begins by describing several ways in which a differentiated market can be fully covered. The central idea is that the market can be covered for some consumer types, yet left uncovered for others, while preserving standard notions of oligopoly equilibria. For consumers who receive no advertising messages, the market is clearly uncovered: These consumers fail to purchase regardless of location and price. For the remaining consumers who receive at least one advertising message, we define the market to be completely covered, incompletely covered, or not covered at all as follows.

**Definition 1. Complete coverage.** The market is completely covered if all consumers who receive at least one advertising message purchase a brand at prevailing prices: \( t \leq (v - p_i) \), for \( i = 0, 1 \).

**Definition 2. Incomplete coverage.** The market is incompletely covered if some consumers who receive only one advertising message do not purchase the brand at the prevailing prices, but all consumers who receive both advertising messages do purchase: \( t/2 \leq (v - p_i) < t \), for \( i = 0, 1 \).

In the case of complete coverage, the equilibrium prices are such that a consumer located at the far endpoint of the line segment finds it worthwhile to travel the entire unit distance to consume the more distant brand. In the case of incomplete coverage, a consumer located at the far endpoint of the line segment finds it prohibitively costly to travel the entire unit distance to consume the more distant brand. Nonetheless, consumers in both cases find it worthwhile to travel half the unit distance to consume their favored brand. Under either definition, the market is fully covered among consumers who receive both advertising messages, but only in the case of complete coverage is the market also fully covered among those receiving only one.

Figure 1 depicts the case of incomplete coverage. Let \( x_i \) denote the location of the consumer who is indifferent between the two brands given that she is fully informed of both brands. \( x_i \) is depicted in Figure 1, and is given by
\[ x_i = \frac{t + p_i - p_0}{2t}. \] 

All fully informed consumers with a location \( x \leq x_i \) find brand 0 to be their first choice at the posted prices. If the density of consumers is given by \( n \), then the total number of fully informed consumers served by brand 0 is \( X_i^0 = nx_i \).

Under incomplete coverage, some consumers who receive only one advertising message do not buy the advertised brand. For a consumer who is informed only about brand 0, let \( x_U \) denote the location of the consumer who is indifferent between purchasing brand 0 or purchasing nothing at all within the product category. By the definition of incomplete coverage, this consumer resides on the unit interval at all permissible prices. The indifferent consumer who receives only a brand 0 message is labeled \( x_U \) in Figure 1, and satisfies

\[ x_U = \frac{v - p_0}{t}. \]

Those consumers located between \( x_I \) and \( x_U \) would prefer brand 1 to brand 0 with complete information. The total number of partially informed consumers served by brand 0 is \( X_U^0 = nx_U \).

In the case of complete coverage, the location of the fully informed consumer who is indifferent between the two brands is again given by (1). As in the case of incomplete coverage, all fully informed consumers make a purchase and select between the two brands according to their tastes. By the definition of complete coverage, all consumers who receive only one advertising message also purchase the good. Thus, the total number of fully informed consumers served by brand 0 is \( \hat{X}_I^0 = X_I^0 = nx_I \) and the total number of partially informed consumers served by brand 0 is \( \hat{X}_U^0 = n \), the entire consumer population receiving only the advert of brand 0.

The remaining case to consider is that of no coverage at all, \( (v - p_i) < t/2 \), for \( i = 0,1 \). This is the case of local monopoly power. Among fully informed consumers, a consumer arbitrarily close to the midpoint of the segment prefers to buy nothing at all at prevailing prices than to buy her favored brand. Under local monopoly, it makes no difference to either firm if a consumer if fully informed or only partially informed, as consumers do not switch between
brands based on relative prices. The reaction functions fail to intersect. The total number of consumers served by brand 0 under local monopoly is given by $\hat{X}_0 = X^0_U = nx_u$ in (1) and (2).

Firms use advertising as a vehicle to inform consumers. Following Butters (1977), we assume that firms have no ability to target advertising towards consumers located at a particular point on the line segment. Advertising is of the “shotgun-type” and is non-discriminatory in the sense that a consumer at a given location in characteristic space has an identical likelihood of being hit by an ad as a consumer at some other location in characteristic space.

Let $\phi_i$, $i = 0,1$, denote the advertising intensity of firm $i$. Advertising intensity is measured in terms of the reach of the ad campaign, so that $\phi_i$ is interpreted as the fraction of the consumer population that is exposed, at least once, to the advertising message of firm $i$. This divides consumers at each location in the product space into four types: With probability $\phi_0\phi_1$ a consumer simultaneously receives advertising messages for each brand, with probability $\phi_0(1-\phi_1)$ a consumer receives the brand 0 message but not that for brand 1, with probability $(1-\phi_0)\phi_1$ a consumer does not receive the brand 0 message but does for brand 1, and with probability $(1-\phi_0)(1-\phi_1)$ a consumer receives advertising messages from neither brand.

The demand facing the oligopolists depends on the equilibrium outcomes for advertising and for the market prices. For the remainder of this section, we describe demand facing a representative firm under three possible scenarios for the endogenous market prices: (i) local monopoly power ($v-p<t/2$); (ii) incomplete coverage ($t/2 \leq v-p < t$); and (iii) complete coverage ($t \leq v-p$). In subsequent sections, we identify the range of parameter values consistent with oligopoly equilibrium in each case and compare the private and social outcomes.

2A. Local Monopoly Power

Consider the case in which firms with local monopoly power engage in advertising. Under local monopoly, the gross value of the product relates to the equilibrium price as $v-p < t/2$, so that a consumer located at the midpoint of the line segment does not purchase the brand, even when fully informed. Local monopolists strategically interact neither in prices, nor in advertising.
levels. Thus, demand facing the representative firm depends only on the number of consumers in the monopoly region that receive his advertising message. Although an advertising message is just as likely to be received by a consumer located at some other point in the characteristic space, the message has no impact on behavior outside the monopoly region of the firm.

Demand for the representative firm under local monopoly is

\[
\hat{X}_i^0 = n x_i = \left( n \hat{\phi} / t \right) (v - \hat{p}),
\]

where \( \hat{\phi} \) is the probability that a consumer, residing somewhere on the line segment, receives the advertising message, and \( (v - \hat{p})/t \) is the conditional probability that, given the message is received, the consumer then buys the product. That is, consumers who receive an advertising message only from brand 0 and subsequently choose to buy brand 0 are located uniformly on the interval \([0, (v - \hat{p})/t]\) and those who choose not to buy brand 0 are located on the interval \(((v - \hat{p})/t, 1]\).

Notice that the demand function (3) is linear in the monopoly advertising level. The marginal product of advertising therefore equals its average product: Letting \( \epsilon_\phi = \left( \partial X(\cdot) / \partial \phi \right)(\phi/X(\cdot)) \) denote the elasticity of demand with respect to advertising, \( \hat{\epsilon}_\phi = 1 \) for the local monopolist. Notice also that the price elasticity of demand, \( \hat{\epsilon}_p = \hat{p} / (v - \hat{p}) \), is not a function of the advertising level under local monopoly.

The essential feature of this model is that it relies solely on a form of advertising that shifts total demand in the product category. The behavior of the local monopolist is not conditioned by the goal of acquiring market share of existing product customers through inter-brand rivalry. Both prices and advertising serve to attract new customers to the product category that otherwise would not consume either brand at all.

**2B. Complete Coverage**

The next case to consider is that of complete coverage \( t \leq v - p \). Since the seminal work of Butters (1977) and Grossman and Shapiro (1984), this is the case considered by virtually all studies of informative advertising. Here, firms strategically interact at every location on the line
segment. Indeed, a firm can sell to a consumer with preferences aligned perfectly with the characteristics of the rival brand under complete coverage if his advertising message is the only one received by the consumer.

The demand for brand 0 is given by
\[ X^0(\tilde{p}, \tilde{\phi}) = \phi^0_0 \tilde{\phi}^0 \tilde{X}^0 + \phi^0_1 (1 - \tilde{\phi}_1) \tilde{X}^0, \]
and similarly for brand 1, where \( \tilde{p} = (\tilde{p}_0, \tilde{p}_1) \) is the vector of brand prices and \( \tilde{\phi} = (\tilde{\phi}_0, \tilde{\phi}_1) \) is the vector of advertising intensities. Among those consumers who receive an advertising message for brand 0, the total demand for brand 0 sums the demand of fully informed and partially informed consumers. All consumers who do not receive an advertising message for brand 0 do not buy brand 0, and those consumers who do receive the brand 0 message are divided between those who also receive the advertising message for brand 1, and who are therefore sensitive to the relative prices of brands in (1), and those who do not receive the brand 1 message, and who are accordingly not price sensitive.

Substituting the number of consumers of each type served by brand 0 into this expression, the demand for brand 0 under complete coverage is
\[ \tilde{X}^0(\tilde{p}, \tilde{\phi}) = n \phi^0_0 \tilde{\phi} X^0 \left( \phi^0_1 \left( \frac{t + \tilde{p}_1 - \tilde{p}_0}{2t} \right) + (1 - \tilde{\phi}_1) \right) \]
and similarly for brand 1. Notice that the brands in demand expression (4) are gross substitutes, \( \partial \tilde{X}^i / \partial \tilde{p}_j = n \phi^0_0 \phi^0_i / 2t \) for all \( i \neq j, i = 0,1. \)

The demand function (4) satisfies the standard and intuitive property that improved consumer information (through large advertising levels in \( \tilde{\phi} \)) increases demand elasticities under oligopoly. In the symmetric case, the price elasticity of demand facing the representative firm, \( \tilde{e}_i = \left( \partial \tilde{X}^i / \partial \tilde{p}_i \right) \tilde{p}_i / \tilde{x}_i, \) evaluated at \( \tilde{p}_1 = \tilde{p}, \tilde{X}^i = \tilde{X}, \) and \( \tilde{\phi}_i = \tilde{\phi} \) for \( i = 0,1, \) is given by
\[ \tilde{e} = \frac{\tilde{p} \tilde{\phi}}{t(2 - \tilde{\phi})} \]

Throughout the paper, we denote variables with subscripts and functions with superscripts.
which is an increasing function of $\phi$. Improved information increases the demand elasticities (and thus reduces prices). The intuition for this is straightforward. The elasticity of demand in (5) can be decomposed into a weighted share of the demand elasticities of fully informed and partially informed subgroups of consumers. For the subgroup of fully informed consumers, the elasticity of demand is given by (1) as $\varepsilon_i = p/t$, whereas, for the subgroup of partially informed consumers, demand is infinitely inelastic (up to the reservation level). Because the share of fully informed consumers in the demand function (4) increases as the equilibrium advertising levels increase, informative advertising always makes demand more elastic.

The essential feature of the complete coverage outcome is that pricing behavior is influenced solely by competition to acquire market share. All consumers who receive at least one advertising message purchase a brand within the product class, so that price competition between firms is a zero sum game for market share. This is precisely the opposite motivation as arises under local monopoly, where both increases in advertising and decreases in price always increase total product demand.

2C. Incomplete Coverage
Under incomplete coverage ($t/2 \leq v - p < t$), at least one consumer is unwilling to travel the entire length of the line segment to consume the more distant brand. The distinguishing feature between this case and the case of complete coverage is that the number of partially informed consumers who purchase the brand is now endogenous. Among fully informed consumers, a price reduction by one brand predates consumers of the rival brand, as in the case of complete coverage. This coincides with the standard oligopoly outcome in a variety of contexts under full information and price setting behavior. However, a price reduction among partially informed consumers now has a total demand shifting effect. A reduced price now increases total demand in the product category by inducing more distant consumers to purchase a product they otherwise would not buy at higher prices.
The case of incomplete coverage is conceptually important for two reasons. First, it extends the literature on informative advertising in differentiated oligopoly markets to consider environments where price reductions enhance total demand for the product class. This bridges existing models of advertising by encompassing the wide range of cases in which pricing behavior (for a given level of advertising) is neither fully predatory, nor fully accommodating from the perspective of the rival firm. Second, as we demonstrate below, when the level of sales to partially informed consumers is endogenously determined, the demand functions have well-defined inverse functions. This allows advertising behavior to be examined under both price and quantity competition in differentiated oligopoly markets.

The demand for brand 0 is given by

$$X^0(p, \phi) = \phi_0 \phi_i X^i_0 + \phi_0 (1 - \phi_i) X^i_U$$

and similarly for brand 1, where $p = (p_0, p_1)$ is the vector of brand prices and $\phi = (\phi_0, \phi_1)$ is the vector of advertising intensities. Using the definitions in (1) and (2), the demands are

$$X^0(p, \phi) = \frac{n \phi_0}{2t} \left( \phi_0 t + 2(1 - \phi_i) v - (2 - \phi_i) p_0 + \phi_i p_1 \right), \quad (6a)$$

and

$$X^1(p, \phi) = \frac{n \phi_1}{2t} \left( \phi_1 t + 2(1 - \phi_i) v + \phi_0 p_0 - (2 - \phi_0) p_1 \right), \quad (6b)$$

As in the case of complete coverage, the brands are gross substitutes,

$$\frac{\partial X^i}{\partial p_j} = n \phi_i \phi_j / 2t$$

for all $i \neq j$, $i = 0, 1$. Each demand function (6a) and (6b) is weighted average of consumers who receive one ad and those who receive two.

In the literature on informative advertising, a standard result is that improved information increases the demand elasticities. Indeed, we found this to be the case above under the assumption of complete coverage. Because the act of providing information, itself, benefits consumers and enhances market efficiency through improved matching between consumers and brands, informative advertising has been viewed as fundamental to this result. Not so. In the symmetric case, the price elasticity of demand facing firm $i$, $\varepsilon_i = \left( \frac{\partial X^i}{\partial p_i} \right) p_i / x_i$, evaluated at $p_i = p$, $X^i = X$, and $\phi_i = \phi$ for $i = 0, 1$, is given by
where the subscript refers to the demand elasticity in the incomplete coverage oligopoly equilibrium characterized by Bertrand (price) competition. This satisfies
\[
\frac{\partial \varepsilon_p}{\partial \phi} = \frac{2p(v-p-t)}{[2(v-p)(1-\phi)+\phi t]^2},
\]
which is negative by the definition of incomplete coverage.

Improved information in the incomplete coverage equilibrium reduces the demand elasticities (and thus increases prices). The intuition for this is as follows. Decomposing the elasticity of demand (7) into a weighted share of the demand elasticities of the informed and partially informed subgroups of consumers, the elasticity of demand for the subgroup of fully informed consumers is given, as before, by \( \varepsilon_i = p/t \), but the elasticity of demand for the subgroup of partially informed consumers is now given by \( \varepsilon_u = p/(v-p) \) in (3). By the definition of incomplete coverage, it follows that \( \varepsilon_i < \varepsilon_u \). As the equilibrium advertising levels increase, therefore, the demand facing each firm is comprised of a greater share of fully informed consumers, and hence becomes more inelastic.

An important property of the demands (6a) and (6b) is that the quantity demanded is influenced differentially by own and rival prices. Specifically, the own-price effect is larger in magnitude than the cross-price effect, \( |\partial X^i(\cdot)/\partial p_i| > |\partial X^i(\cdot)/\partial p_j| \). Unlike the outcome of a standard Hotelling model with full information, and also unlike the outcome of a model with complete coverage, this implies that the demands are invertible. Letting \( X = (X_0, X_1) \) denote the vector of output levels, the inverse demands are
\[
p^0(X, \phi) = \alpha_0 - \beta_0 X_0 - \gamma X_1, \tag{8a}
\]
\[
p^1(X, \phi) = \alpha_1 - \gamma X_0 - \beta_1 X_1, \tag{8b}
\]
where, \( \alpha_0 = \frac{v(2-\phi_0-\phi_1)+\phi_1 t}{2-\phi_0-\phi_1} \), \( \beta_0 = \frac{(2-\phi_0)t}{n\phi_0(2-\phi_0-\phi_1)} \), and \( \gamma = \frac{t}{n(2-\phi_0-\phi_1)} \), and where \( \alpha_1 \) and \( \beta_1 \) are similarly defined.
It is well known that when firms compete in quantities residual demand is less elastic than when firms compete in prices. That is, firm behavior under quantity competition is less competitive, for a given set of demand conditions, than firm behavior under price competition.\(^4\)

For a given level of advertising, this is true here as well. Under symmetric market conditions, the demand elasticity facing quantity-setting firm \(i\) is \(\varepsilon_i = p_i / (\beta_i x_i)\). Evaluating this elasticity at \(p_i = p,\ X^i = X,\) and \(\phi_i = \phi\) for \(i = 0,1,\) and converting into terms of the equilibrium prices gives

\[
\varepsilon_c = \left( \frac{4(1-\phi)}{(2-\phi)^2} \right) \varepsilon_h,
\]

(9)

where the expression in brackets is strictly less than unit value at all permissible advertising levels \(\phi \in [0,1]\), which implies \(\varepsilon_h > \varepsilon_c\). Thus, equilibrium prices are higher (for a given level of advertising) when firms compete in quantities than when firms compete in prices.\(^5\)

3. Monopoly Outcomes

Consider the outcome under monopoly ownership of both brands. There are three possibilities, which coincide with the three types of demand conditions outlined above. In each case, we consider a monopolist who faces symmetric production costs per brand and incurs independent advertising costs for each brand.\(^6\)

3A. Local Monopoly

Under local monopoly conditions \((v - p < t / 2)\), demand for each brand is given by (3). Monopoly profit per brand is

\[
\hat{\pi}^m(\hat{p}_m, \hat{\phi}_m) = (\hat{p}_m - c)(n\hat{\phi}_m / t)(v - \hat{p}_m) - nA(\hat{\phi}_m) - F,
\]

where \(F\) is fixed cost, which is assumed to be sunk, \(c\) is the constant unit cost of production, and \(A(\phi)\) is the private cost of sending an advertising message through a unit population density.

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\(^5\) As in the case of price competition, it is easy to verify that advertising that improves information also reduces the demand elasticities (and increases equilibrium prices) under quantity competition.

\(^6\) Multi-product firms often have separate advertising budgets for each product. For example, soft drink producers run simultaneous ad campaigns for cola, cherry, and lemon-lime flavored beverages without “cross-advertising” by promoting multiple brands in the product line at a time.
We assume throughout that $A'(\phi) > 0$ and $A''(\phi) > 0$. The latter assumption reflects the idea that it becomes increasingly expensive for advertising to reach higher fractions of the population, either because the advertising media become saturated, or because consumers differ in their tendency to view ads. A useful specialization of the advertising cost function, which we employ in various places below, is $A(\phi) = -z \ln(1 - \phi)$, where $z < 0$ is a constant. This is the form of advertising technology considered by Butters (1977) and Grossman and Shapiro (1984).

The first-order necessary conditions for the local monopoly equilibrium are

$$\hat{p}_{m}^*(\hat{\phi}_m, \hat{\phi}_m) = (n\hat{\phi}_m / t)(v - 2\hat{p}_m + c) = 0$$

and

$$\hat{\phi}_m^*(\hat{\phi}_m, \hat{\phi}_m) = (n / t)(\hat{p}_m - c)(v - \hat{p}_m) - nA'(\hat{\phi}_m) = 0.$$ 

In (10), the monopoly sets the price above marginal cost according to the reciprocal of the demand elasticity, and in (11), the marginal product of advertising, which is the parallel shift in demand, is set equal to its marginal cost.

The implicit solution to (10) and (11) is $\hat{p}_m^* = (v + c) / 2$ and $(v - c)^2 / 4t = A'(\hat{\phi}_m^*)$ in the general case. Notice that the optimal price is not a function of the advertising level. This is because advertising does not affect the extent of the market served by each brand.

Under the advertising cost specialization $A(\phi) = -z \ln(1 - \phi)$, the optimal level of advertising is given by $\hat{\phi}_m^* = 1 - 4tz / (v - c)^2$. Notice that when $t$ increases, which means consumers are more sensitive to brand characteristics, the monopolist advertises his brands less. Put differently, the result that a monopolist advertises more when the brands are relatively homogeneous is rather striking.

Substituting $\hat{p}_m^*$ into the condition for a local monopoly, the local monopoly outcome occurs for combinations of consumer reservation values, production costs, and transportation costs that satisfy $v - c < t$. With linear demand, a local monopolist fails to cover half the market.

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7 This form of advertising cost can be interpreted as selecting to run ads in a number of magazines with identical and independent readerships. For details, see Grossman and Shapiro (1984).
with each brand at monopoly prices only when a single brand priced at marginal cost cannot cover the entire market.

Substituting the optimal price for a local monopolist into the profit expression gives profit per brand as a function of the advertising level

\[ \hat{\pi}^m(\hat{\phi}_m^*) = \frac{n\hat{\phi}_m^*(v-c)^2}{4t} - nA(\hat{\phi}_m^*) - F. \]  

(12)

For the advertising cost specialization \( A(\phi) = -z \ln(1-\phi) \), this reduces to

\[ \hat{\pi}^m(\hat{\phi}_m^*) = nz \left[ \frac{(v-c)^2}{4tz} + \ln \left( \frac{4tz}{(v-c)^2} \right) - 1 \right] - F. \]

3B. Complete Coverage

Under complete coverage \((t \leq v - p)\), the monopolist sells each brand to all consumers who receive an advertising message for that brand. The monopolist, by definition, sets his price sufficiently low that even the most distant consumer from the brand is induced to purchase, which implies that the equilibrium prices must be set below the reservation value of a consumer at a unit distance away from the brand. On the other hand, decreasing prices any further from this position has no consequence on the total quantity sold, so that any lower price than this cannot be optimal. Hence, the optimal monopoly price under complete coverage is \( \hat{p}_m^* = v - t \).

The monopolist also has the choice of whether to promote both brands or to promote only one. That is, the monopolist may choose to advertise only a single brand and make no attempt at all to sell the second brand. Doing so would not harm the monopolist’s sales—we have assumed complete coverage by each brand—but doing so would save on advertising costs. Advertising two brands is wasteful from the perspective of a monopolist with complete coverage, because some consumers receive two advertising messages that would purchase anyway having received only one. Specifically, a consumer is hit by advertisements for both brands with probability \( \hat{\phi}^2 \) when only one ad is sufficient to induce a purchase.
To see that the monopolist only chooses to advertise one brand under complete coverage, consider the monopoly profit level (per brand) in each case. If only a single brand is promoted, monopoly profit is

\[ \pi^m(\phi_m) = n\phi_m(v - c - t) - nA(\phi_m) - F, \]  

(13)

where \((v - c - t)\) is the (constant) markup of price over marginal cost, and \(\phi_m\) is the probability that the representative consumer purchases the brand, which is simply equal to the fraction of the population exposed to the (single) ad.

Letting \(0 \leq \phi_m \leq 1\) denote the optimal level of advertising in the symmetric case when both brands are promoted, monopoly profit per brand is

\[ \pi^m(\phi_m) = \phi_m n(v - c - t)(1 - \phi_m / 2) - nA(\phi_m) - F. \]  

(14)

For later reference, the first order condition associated with promotion of both brands in (14) is

\[ (v - c - t)(1 - \phi_m)(1 - \phi_m) = A'(\phi_m). \]  

(14a)

Notice that total cost per brand is identical in (13) and (14) for a given level of advertising, \(\phi_m = \phi_m\), but that total revenue differs. This is because the probability that a consumer who receives an ad will subsequently buy the advertised brand depends both on whether the consumer, when fully informed, prefers the brand to the alternative brand offered by the monopolist, and on whether the consumer receives advertising messages from one or from both brands. Consumers receive advertising messages from both brands with probability \(\phi_m^2\) and a fully informed consumer (on average) buys the advertised brand half the time and buys the alternative brand the remaining half the time. Consumers receive an advertising message from the representative brand and no other advertising messages with probability \(\phi_m(1 - \phi_m)\), and these partially informed consumers buy the advertised brand all the time. Thus, for a representative brand advertised by the monopolist, the probability of making a successful sale to a consumer is \(\phi_m(1 - \phi_m / 2)\).

At this point, it is straightforward to verify that a monopolist always promotes only a single brand when conditions support complete coverage. To see this, suppose the monopolist selected a level of advertising to maximize profits in (14), and let \(\phi_m^*\) denote this level. By
inspection of (13), monopoly profit would be greater at \( \phi_m^* \) if the monopolist chose, instead, to promote only the single brand. Formally, \( \pi^m(\phi_m^*) - \pi^m(\phi_m^*) = (n/2)\phi_m^2 (v - c - t) > 0 \). The monopolist could do still better, of course, under single brand promotion at some other level of advertising, \( \phi_m^* \). It follows by revealed preference that a monopolist always chooses to promote only one brand under complete coverage.

The first order condition associated with (13) is

\[
\tilde{\pi}^m (\phi_m) = n(v - c - t) - n A'(\phi_m) = 0. \tag{15}
\]

The optimal level of advertising, \( \phi_m^* \), is identified by (15). For the advertising cost specialization \( A(\phi) = -z \ln(1 - \phi) \), the equilibrium advertising level is \( \phi_m^* = 1 - \frac{z}{v - c - t} \) and equilibrium profit is

\[
\tilde{\pi}^m (\phi_m^*) = n \left[ v - c - t - z \left( 1 - \ln \left( \frac{z}{v - c - t} \right) \right) \right] - F.
\]

Under complete coverage, the monopolist only produces one brand. It remains to identify parameter values that support the monopoly equilibrium (15) under complete coverage. We return to this issue after discussing the remaining case of incomplete coverage in the following section.

3C. Incomplete Coverage

Under incomplete coverage \( (t/2 \leq v - p < t) \), the monopolist considers three types of solutions. First, the monopolist may choose to promote only a single brand, even though doing so incompletely covers the market. Second, the monopolist may choose to promote both brands, but price them each at \( p = v - t/2 \) to preserve a local monopoly market for each brand with independent demands.\(^8\) Third, the monopolist may choose to promote both brands and price them at \( p < v - t/2 \). All fully informed consumers receive positive surplus from consuming their preferred brand at such prices, so that the demand for each brand is given by (6a) and (6b).\(^9\)

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\(^8\) The local monopoly regions would actually intersect at the midpoint of the line segment, but a fully informed consumer at this point receives zero net surplus from either brand so that the demands are independent.

\(^9\) Equivalently, one could use the inverse demands (8a) and (8b).
If the monopolist promotes only one of the two brands, the equilibrium profit level for the promoted brand is identical to that under local monopoly conditions in (12). Substituting the optimal price for the single brand, \( \hat{p}^*_m = (v + c)/2 \), into the conditions for incomplete coverage, the net value of the product, \( v - c \), that supports this outcome is in the range \( t \leq v - c < 2t \). For values of \( v - c \) below this range, a local monopoly is optimal, but both brands are promoted as in Section 3A, whereas for values of \( v - c \) above this range, coverage is complete and the outcome is as in Section 3B.

If the monopolist promotes both brands but prices in a manner to maintain independent demands in two local monopoly markets, then \( \hat{p}'_m = v - t/2 \) in the symmetric case, and total sales per brand are \( \hat{X}(\hat{p}'_m, \hat{\phi}'_m) = n\hat{\phi}'_m / 2 \). Profit per brand is

\[
\hat{\pi}(\hat{p}'_m, \hat{\phi}'_m) = n\hat{\phi}'_m \left( \frac{2(v-c) - t}{4} \right) - nA(\hat{\phi}'_m) - F,
\]

which has the first order condition,

\[
\hat{\pi}'(\hat{p}'_m, \hat{\phi}'_m) = n \left( \frac{2(v-c) - t}{4} \right) - nA'(\hat{\phi}') = 0.
\]

Let \( \hat{\phi}'_m \) denote the solution to (17). For the advertising cost specialization \( A(\phi) = -\varphi \ln(1-\phi) \),

\[
\hat{\phi}'_m = 1 - \frac{4z}{2(v-c) - t}.
\]

Finally, the monopolist can choose to promote both brands, but price them sufficiently low that the demands are independent, as in (6a) and (6b). Pricing the brands below \( \hat{p}'_m \) reduces the profit that the monopolist earns on fully informed consumers, because all informed consumers purchase in either case, but do so now at lower prices. But pricing the brands below \( \hat{p}'_m \) also increases sales to partially informed consumers, who are now willing to travel farther to consume the lower priced brand. Given that two brands are promoted, the decision of how to price them depends on the relative magnitude of these two effects.

If the monopolist promotes both brands but prices them below demand \( \hat{p}'_m \), the demand for each brand is given by (6a) and (6b). Monopoly profit per brand in the symmetric case, \( p_m = p_0 = p_1 \) and \( \phi_m = \phi_0 = \phi_1 \) is

\[
\pi''(p_m, \phi_m) = (p_m - c) \frac{n\phi_m}{2t} \left( 2(p_m - p_m)(1 - \phi_m) + \phi_m t \right) - nA(\phi_m) - F.
\]


The first order conditions are
\begin{equation}
\pi_p^m(p_m, \phi_m) = \frac{n\phi_m}{2t} \left(2(v - p_m)(1 - \phi_m) + \phi_m t - 2(1 - \phi_m)(p_m - c)\right) = 0
\end{equation}
and
\begin{equation}
\pi_\phi^m(p_m, \phi_m) = \frac{n(p_m - c)}{t} \left((v - p_m)(1 - 2\phi_m) + \phi_m t\right) - nA'(\phi_m) = 0.
\end{equation}
Equation (18) gives the familiar pricing condition under monopoly, \( (p - c)/p = 1/\varepsilon_m \), where
\begin{equation}
\varepsilon_m = \frac{2(1 - \phi_m)p_m}{2(v - p_m)(1 - \phi_m) + \phi_m t}
\end{equation}
is the demand elasticity. Equation (20) can be written
\begin{equation}
(p_m - c)(\phi_m + (1 - 2\phi_m)(v - p_m)/t) = A'(\phi_m),
\end{equation}
which has the following interpretation. Because \( n\phi_m \) consumers receive the advertising message for each brand, a small increase in \( \phi_m \) of \( d\phi_m = 1/n \) informs (on average) one more consumer. With probability \( \phi_m^2 \), this consumer is fully informed, in which case she buys each brand exactly half the time. Thus, the marginal probability that a fully informed consumer purchases the brand is \( \phi_m \). With probability \( \phi_m(1 - \phi_m) \), this consumer is only partially informed, in which case she buys the brand with frequency \( (v - p_m)/t \), which is the length of the segment served by the monopolist under incomplete coverage. The marginal probability that a partially informed consumer purchases the brand is \( (1 - 2\phi_m)(v - p_m)/t \). A small increase in \( \phi_m \) therefore generates \( \phi_m + (1 - 2\phi_m)(v - p_m)/t \) sales and raises profits by \( (p_m - c)(\phi_m + (1 - 2\phi_m)(v - p_m)/t) \). The monopolist equates this marginal gain to the marginal cost of informing a consumer, \( A'(\tilde{\phi}^m) \).

Unlike the outcomes under local monopoly and complete coverage, notice that the elasticity of demand with respect to advertising is no longer unit valued. For a monopolist who promotes brands with intersecting demands, this elasticity is
\begin{equation}
\varepsilon_\phi = \frac{2[(v - p)(1 - 2\phi) + \phi t]}{2(v - p)(1 - \phi) + \phi t}.
\end{equation}
Advertising has a smaller direct impact on demand, \( \varepsilon_\phi < 1 \), than in the other cases, because a higher level of advertising for one brand now has a negative effect on demand for the other. Increased advertising for one brand reallocates a portion of fully informed consumers between
brands that otherwise would have purchased the alternative brand, so that the average product of advertising now exceeds its marginal product.

The optimal prices and advertising levels per brand for the monopolist who produces two brands solve (19) and (20) simultaneously. Let \((p_m^*, \phi_m^*)\) denote this solution.

To see whether profit is greater for a monopolist who promotes one brand or promotes two brands with intersecting demands, we compare profits in each case for a given level of advertising. For a monopolist who chooses to promote both brands at \(p_m^* < \hat{p}_m'\), the equilibrium price, as a function of \(\phi_m\), is given by (19) to be

\[
p_m(\phi_m) = \frac{v + c}{2} + \frac{t\phi_m}{4(1 - \phi_m)}
\]

Profit per brand, as a function of the advertising level, is found by substituting (22) into (18), which gives

\[
\pi^m(\phi_m) = \frac{n\phi_m(1 - \phi_m)}{4t} \left( v - c + \frac{\phi_m t}{2(1 - \phi_m)} \right)^2 - nA(\phi_m) - F.
\]

Total monopoly profit is \(2\pi^m(\phi_m)\). Evaluating total profit at the optimal level of single brand promotion, \(\hat{\phi}_m^*\), and comparing this expression with (12) yields

\[
2\pi^m(\hat{\phi}_m^*) - \hat{\pi}^m = \frac{n\phi_m^*(1 - \phi_m^*)}{4t} \left[ (1 - 2\hat{\phi}_m^*)(v - c)^2 + 2t\hat{\phi}_m^*(v - c) + \frac{(t\phi_m^*)^2}{2(1 - \phi_m^*)} \right].
\]

If the term in square brackets is positive, then promoting two brands at the advertising level \(\hat{\phi}_m^*\) leads to greater profit than promoting only one. Rearranging this term, \(2\pi^m(\hat{\phi}_m^*) > \hat{\pi}^m(\hat{\phi}_m^*)\) when

\[
(1 - \hat{\phi}_m^*)(v - c)^2 - \hat{\phi}_m^*(v - c)(v - c - 2t) + \frac{(t\phi_m^*)^2}{2(1 - \hat{\phi}_m^*)} > 0.
\]

This inequality always holds, because single brand promotion is only feasible under incomplete coverage when \(v - c < 2t\). If a monopolist who promotes both brands advertises each brand at the level \(\hat{\phi}_m^*\), then profit is greater than the maximum profit level obtainable from promoting a single brand. It follows by revealed preference that \(2\pi^m(\hat{\phi}_m^*) \geq \hat{\pi}^m(\hat{\phi}_m^*) > \hat{\pi}^m(\hat{\phi}_m^*)\). Accordingly, a monopolist always chooses to promote both brands.
The remaining case to consider is a monopolist who promotes both brands, but prices them at $\hat{p}_m'$ to maintain local monopoly markets. To assess this case, we proceed similarly as above. Profits per brand (evaluated at $\hat{\phi}_m'$) compare in (16) and (23) as

$$\pi_m(\hat{\phi}_m') - \hat{\pi}_m(\hat{\phi}_m') = \frac{n \hat{\phi}_m'}{4t} \left[ (1-\hat{\phi}_m')(v-c)^2 - t(v-c)(2-\hat{\phi}_m') + \frac{t^2(2-\hat{\phi}_m')^2}{4(1-\hat{\phi}_m')} \right]$$

The critical value that determines the sign of this expression is $\frac{v-c}{t} = \frac{2-\hat{\phi}_m'}{2(1-\hat{\phi}_m')}$. For smaller values of $(v-c)/t$, $\pi_m(\hat{\phi}_m') < \hat{\pi}_m(\hat{\phi}_m')$, while the opposite result holds for larger values.

The range of parameter values that support an outcome of incomplete coverage is found by substituting (22) into the two conditions on price, $p_m \leq v - t/2$ and $p_m > v - t$. In terms of the equilibrium advertising levels, this range is defined by

$$\frac{2-\hat{\phi}_m}{2(1-\hat{\phi}_m')} < \frac{v-c}{t} \leq \frac{4-3\hat{\phi}_m}{2(1-\hat{\phi}_m')}.$$  

Notice that outcome under monopoly depends importantly on the term $(v-c)/t$. This term has two components: $v-c$ is the social benefit of consumption (gross of transportation costs) and $t$ is the transportation cost per unit of distance. The interpretation of $(v-c)/t$ is the ratio of the social benefit of consumption to its opportunity cost for the most distant consumer in product space. In There is a sense in which this measures the intrinsic value of the product space. The product space is more valuable when the reservation price of consumers greatly exceeds the production cost of firms, and for products in which this value is not greatly eroded by the transportation cost necessary to acquire it. This term depends critically on the degree of product differentiation.

When product differentiation is more important, this value decreases.

The range of outcomes is depicted in Figure 2. For a product class with a relatively small net value, $(v-c)/t < 1$, the outcome is a local monopoly with promotion of both brands at interior prices. The equilibrium prices are such that consumers on the mid-point of the segment do not consume, and the remaining consumers purchase their closest brand when fully informed and consume either their closest brand or no brand at all when partially informed. For a product
category with a relatively larger net value of \( 1 < \frac{v-c}{t} < \frac{2-\phi_m^*}{2(1-\phi_m^*)} \), where \( \phi_m^* \) is determined by (19) and (20), the outcome is a local monopoly with a corner solution for the prices at \( \hat{p}_m^* \). For a product category that satisfies \( \frac{2-\phi_m^*}{2(1-\phi_m^*)} < \frac{v-c}{t} \leq \frac{4-3\phi_m^*}{2(1-\phi_m^*)} \), the equilibrium is characterized by incomplete coverage with interdependent demands. And finally, for a product category that satisfies \( \frac{v-c}{t} > \frac{4-3\phi_m^*}{2(1-\phi_m^*)} \), the monopolist provides complete coverage with a single brand and does not promote the other.

For the advertising cost specialization \( A(\phi) = -z \ln(1-\phi) \), these boundary values are given by \( \frac{2-\phi_m^*}{2(1-\phi_m^*)} = \frac{4z + 2(v-c) - t}{8z} \) and \( \frac{4-3\phi_m^*}{2(1-\phi_m^*)} = \frac{3z + v - c - t}{2z} \).

Notice that the value of each boundary point decreases with decreases \( z \), which is the (standardized) marginal cost of advertising. This implies that pricing behavior under monopoly may change over time with the introduction of new media (e.g., the internet) that reduces the cost of exposing the consumer population to ads.

In subsequent sections, we compare the outcome under monopoly to that under oligopoly and to the social optimum. We next turn to the outcomes under oligopoly.

4. Outcomes Under Oligopoly

Under oligopoly, firms choose two strategic variables simultaneously: advertising levels, and either prices or quantities. As in the monopoly case, several outcomes can occur under oligopoly. First, values of the parameters can be such that each firm has local monopoly power \((v-c < t)\). The outcome in this case coincides with that in Section 3A. Second, the values of the parameters can be such that the demand functions of the oligopoly firms intersect. In this case, depending on the value of the model parameters, the equilibrium is either in pure strategies or in mixed strategies.
In this section, we consider non-cooperative Nash equilibrium in pure strategies under both price and quantity competition. To facilitate the comparison of outcomes, we center attention on the case of incomplete coverage \((t/2 \leq v - p < t)\).\(^\text{10}\) Throughout, we consider the problem from the perspective of a representative firm and, without loss of generality, label this firm as the producer of brand 0.

4A. Price competition

Under price competition, the profit of the representative firm who produces brand 0 is given by

\[ \pi^b(p, \phi) = (p_b - c)X^0(p, \phi) - nA(\phi_b) - F, \]

where \(p \) and \(\phi\) are the price and advertising vectors, and demand is given by (6a). The essential difference between profit under price-setting oligopoly and profit under (incomplete coverage) monopoly is that each firm now views the price and advertising level set by its rival as constant, and this fails to internalize the full effects of both pricing and advertising on joint profitability.

The first-order conditions, evaluated in the symmetric case, \(p_b = p_o = p_1\) and \(\phi_b = \phi_o = \phi_1\) are

\[ \pi^b_p(p, \phi) = \frac{n\phi_b}{2t} [2(v - p_b)(1 - \phi_b) + \phi_b t - (p_b - c)(2 - \phi_b)] = 0, \] (24)

and

\[ \pi^b_\phi(p, \phi) = \frac{n}{2t} (p_b - c)(2(v - p_b)(1 - \phi_b) + \phi_b t) - nA'(\phi_b) = 0. \] (25)

In (24), each firm marks up price over marginal cost according to the reciprocal of the demand elasticity, \((p_b - c) / p_b = 1 / \varepsilon_b\), where \(\varepsilon_b\) is defined in (7). Notice that \(\varepsilon_b > \varepsilon_m\) in (21) for a fixed advertising level. For an equivalent level of advertising, oligopoly prices are lower than those under monopoly. In (25), the representative firm equates the marginal product of advertising to its marginal cost. Because demand facing the representative firm in (6a) is linear in his own advertising level, the marginal product of advertising equals its average product. It follows that,

\(^{10}\) For the case of complete coverage, the demands are not invertible.
for equivalent prices, the advertising levels under oligopoly are higher than those which occur under monopoly (i.e., $\varepsilon_\phi < 1$ under monopoly).

The oligopoly equilibrium under price-setting behavior is given by the simultaneous solution to (24) and (25). It is possible to eliminate $p_b$ from these two equations. In terms of the equilibrium advertising level, the optimal price that satisfies (24) is

$$p(\phi_b) = \frac{2v(1 - \phi_b) + c(2 - \phi_b) + t\phi_b}{4 - 3\phi_b}.$$  \hspace{1cm} (26)

Substituting (26) into (25), the effect of advertising on oligopoly profit is

$$\frac{(2 - \phi_b)}{2t(4 - 3\phi_b)^2} \left( 2(v - c)(1 - \phi_b) + \phi_b t \right)^2 = A'(\phi_b)$$  \hspace{1cm} (27)

In equilibrium, symmetric price-setting firms set $\phi_b^*$ so that $\pi^b(\phi_b^*) = 0$ in (27). Substitution of $\phi_b^*$ into (26) recovers the oligopoly price, $p_b^* = p_b(\phi_b^*)$. Let $B = \left( p_b^*, \phi_b^* \right)$ denote this solution.

The range of parameter values that support an oligopoly outcome under incomplete coverage is found by substituting (26) into the two pricing conditions, $p_b \leq v - t / 2$ and $p_b > v - t$. In terms of the equilibrium advertising levels, this range is defined by

$$\frac{4 - \phi_b^*}{2(2 - \phi_b^*)} \leq \frac{v - c}{t} \leq 2.$$  \hspace{1cm} (26)

In the remaining case of complete coverage, profit of the representative brand is

$$\pi^0(p, \phi) = (p_0 - c)\bar{X}^0(p, \phi) - nA(\phi_b) - F.$$  \hspace{1cm} (27)

In the symmetric case, the first order condition for price implies

$$\tilde{\phi}_b(p_b - c) = t(2 - \tilde{\phi}_b)$$ \hspace{1cm} (24a)

and the first order condition for advertising implies

$$(p_b - c)(2 - \tilde{\phi}_b) = 2A'(\tilde{\phi}_b)$$ \hspace{1cm} (25a)

Under complete coverage, the range of parameter values that support an oligopoly outcome is found by substituting (26) into the pricing condition $p_b \leq v - t$. In terms of the equilibrium advertising levels, this range is defined by

$$\frac{2}{\phi_b^*} \leq \frac{v - c}{t}.$$  \hspace{1cm} (26)
The range of outcomes under price competition is depicted in Figure 3. For the range of values \((v - c)/t < 1\), the outcome is a local monopoly. For values between one and \(1 < \frac{4 - \phi^*_b}{2(2 - \phi^*_b)}\), and between 2 and \(2/\phi^*_b\), the oligopoly equilibrium is in mixed strategies. Behavior in the regions of mixed strategy equilibria may be characterized by sporadic advertising campaigns or by random sales events, none of which is considered in the present focus on pure strategy equilibria.

B. Quantity competition

Under quantity competition, the profit of the representative firm who produces brand 0 is

\[
\pi^c(X, \phi) = p^0(X, \phi)X_0 - cX_0 - nA(\phi_0),
\]

where inverse demand is given by (8a).

The first order conditions in the symmetric case, \(X_c = X_0 = X_1\) and \(\phi_c = \phi_0 = \phi_1\), are

\[
\pi^c_c(X, \phi) = v - c + \frac{\phi_c t}{2(1 - \phi_c)} - \frac{(4 - \phi_c) t X_c}{2n\phi_c(1 - \phi_c)} = 0,
\]

and

\[
\pi^c_\phi(X, \phi) = \frac{tX_c \left(n\phi^3_c + 2(2 - 3\phi_c)X_c\right)}{4n\phi^2_c(1 - \phi_c)^2} - nA'(\phi_c) = 0.
\]

It is helpful to convert these expressions into terms of market prices by substituting in for the demand function (6) under symmetry, \(X(p, \phi) = (\phi n/2t) \left(2(v - p)(1 - \phi) + \phi t\right)\). This gives

\[
\pi^c_c(p, \phi) = p_c - c - \frac{(2 - \phi_c)(2(v - p_c)(1 - \phi_c) + \phi_c t)}{4(1 - \phi_c)} = 0 \tag{28}
\]

and

\[
\pi^c_\phi(p, \phi) = \frac{n(p_c - c)((2 - 3\phi_c)(v - p_c) + \phi_c t)}{t(2 - \phi_c)} - nA'(\phi_c) = 0 \tag{29}
\]

Each firm marks up price over marginal cost in (28) according to the reciprocal of the demand elasticity, \((p_c - c)/p_c = 1/\varepsilon_c\), as in the case of both price-setting oligopoly and monopoly. For a given level of advertising, \(\varepsilon_b > \varepsilon_c > \varepsilon_m\), and it follows that the price of the representative brand under quantity competition is lower than that which emerges under monopoly in (22), but higher than that which emerges under price competition in (24). The interpretation of equation (29) is
that the marginal benefit of an additional unit of advertising, which is the value of the increased quantity evaluated at constant prices, should be set equal to its marginal cost.

Unlike the case of price competition, notice that the marginal product of advertising does not equal its average product under quantity competition. This is because own brand advertising enters equation (6a) linearly, but own brand advertising affects both the intercept and the slope in equation (8a). Oligopoly firms perceive the effect of advertising differently when viewing the price of the rival brand to be fixed than when perceiving his quantity to be fixed. Comparing (25) and (29) for equal prices, the advertising levels are higher under price competition than they are under quantity competition when \( t \leq 2(v - p) \), and this always holds by the definition of incomplete coverage. For given prices, firms advertise more under Bertrand than under Cournot.

In terms of the equilibrium advertising level, the optimal price that satisfies (28) is

\[
p_c(\phi_c) = \frac{2v(1-\phi_c)(2-\phi_c) + 4(1-\phi_c)c - \phi_c(2-\phi_c)t}{2(1-\phi_c)(4-\phi_c)}
\]  

(30)

Substituting (30) into (29), the optimal level of advertising is

\[
\frac{(2-3\phi)(2(v-c)(1-\phi)+\phi t)}{2t(1-\phi)^2(4-\phi)^2} + \frac{\phi^2(2(v-c)(1-\phi)+\phi t)}{4(1-\phi)^2(4-\phi)} = A'(\phi)
\]  

(31)

In equilibrium, symmetric firms set \( \phi_c^* \) so that \( \pi_c^*(\phi_c^*) = 0 \) in this expression. Substitution of \( \phi_c^* \) into (30) would then recover the equilibrium output level, \( p_c^* = p_c(\phi_c^*) \). Let \( C = (p_c^*,\phi_c^*) \) denote this solution.

The range of parameter values that support a quantity-setting oligopoly outcome under incomplete coverage is shown in Figure 3. These values satisfy

\[
\frac{4-3\phi_c^*}{4(1-\phi_c^*)} < \frac{v-c}{t} \leq \frac{8(1-\phi_c^*) + (\phi_c^*)^2}{4(1-\phi_c^*)}.
\]

For equivalent advertising levels, a larger range of parameter values supports incomplete coverage under quantity competition than under price competition. This is because prices are higher under quantity competition than under price competition (advertising given), which precludes consumer purchases at more distant locations, and thereby extends the range of outcomes that support the incomplete coverage equilibrium.
5. Welfare Analysis

In this section, we first detail the socially optimal level of advertising, then compare the privately and socially optimal levels under monopoly, and both forms of oligopoly. In all cases, we consider the optimal advertising level of a social planner who takes market structure and prices as given. This is the standard approach. Implicitly, we are assuming that the planner can control each externality in the oligopoly market with independent instruments on production and advertising.

From a normative policy perspective, it is well known that an output subsidy can be used to “correct” the pricing distortion under oligopoly.\(^{11}\) The focus here is on policies that correct the advertising distortion under oligopoly with a subsidy (or a tax) on advertising. In each case, we ignore second-best policy issues that arise when attempting to correct multiple distortions with a single instrument. This is done by suppressing the influence of advertising policy on the equilibrium prices and by assuming that government revenue to support the optimal policy is generated from taxes on an inelastic factor in the economy.

It is somewhat unusual to ask the question of whether advertising is socially inefficient or excessive in explicit terms of whether or not an advertising subsidy would improve welfare. Nonetheless, these questions are equivalent. Indeed, from a social cost perspective, it is useful to note that a portion of advertising expenses is often passed on to consumers; for example, advertisements represent much of the printing cost of a Sunday paper, and this is passed along to consumers through a higher delivery price.\(^{12}\)

Let the social cost of advertising to be \(A_s(\phi)\). The social cost relates to private costs as \(A(\phi) = A_s(\phi) + \tau \phi\), where \(\tau\) is a unit tax (subsidy) now introduced on advertising. For each

\(^{11}\) It is straightforward to show that the social optimum always involves price equal to marginal cost for each brand. To see this, consider the group of consumers who receive only an advertising message from brand 0 and suppose the price of brand zero is set above marginal cost, \(p_0 > c\). There would then exist a consumer in this group at some location, \(z\), who would not consume brand 0 \((v - p_0 - zt < 0)\), even though total social surplus would increase by doing so \((v - c - zt < 0)\).

\(^{12}\) Becker and Murphy (1993) consider an interesting model in which consumers pay (at least implicitly) a market price for advertising, which enters demand as a complement to advertised goods.
outcome of the model, we identify whether the optimal policy for each market structure involves a tax, $\tau_j > 0$ for $j = m, b, c$, or a subsidy, $\tau_j < 0$ for $j = m, b, c$.

5A. Local Monopoly

The socially optimal allocation depends on industry profits and the surplus consumers receive from purchasing a brand, which is influenced by total transportation costs in the market. Under local monopoly conditions, a consumer receives an advertising message from the representative brand with probability $\hat{\phi}$. At a price of $\hat{p}$, some consumers purchase the brand and some do not. The conditional probability that a consumer buys the product after receiving the advertising message is $(v - \hat{p})/t$. The average consumer who purchases the brand travels $(v - \hat{p})/2t$ to consume the brand and incurs travel cost of $T = (v - \hat{p})/2$, so that consumer surplus per brand for the average consumer is $v - \hat{p} - T = (v - \hat{p})/2$.

Aggregate welfare is given by

$$W = 2n\hat{\phi}(v - \hat{p})\left(\frac{v - \hat{p}}{2} + \hat{p} - c\right) - 2nA_s(\hat{\phi}) - 2F,$$

where the first term in $W$ represents benefits net of variable production costs and transportation costs. The fraction of the population that consumes the product is $2n\hat{\phi}((v - \hat{p})/t)$ and, for each unit of consumption, the return to consumers (on average) is $(v - \hat{p})/2$ and the return to producers is $\hat{p} - c$. The remaining two terms are social advertising costs and fixed costs, respectively.

The socially optimal $\hat{\phi}$ is defined by the first order condition for the maximization of $W$,

$$\frac{(v - \hat{p})^2}{2t} + \frac{(\hat{p} - c)(v - \hat{p})}{t} = A_s(\hat{\phi}).$$ (32)

A small increase in $\hat{\phi}$ that informs one more consumer induces a purchase with frequency $(v - \hat{p})/t$, and this contributes $(v - \hat{p})/2$ in surplus to consumers and $\hat{p} - c$ to producers. Equation (32) sets this marginal gain equal to the marginal social cost of advertising.
At the social optimum, $\tau$ is set to equate the socially and privately optimal levels of advertising. Noting that $A'(\hat{\phi}) = A'_i(\hat{\phi}) + \hat{\tau}$, the optimal tax on advertising is recovered by substitution of (11) into (32) to get

$$\hat{\tau}^* = -\frac{(v - \hat{p})^2}{2t} < 0.$$  \hspace{1cm} (33)

Under local monopoly, firms advertise too little from the social perspective. A monopolist fully internalizes the effect of advertising on industry profit, but fails to account for the marginal contribution of advertising to the welfare of consumers. Because the social benefits of advertising for each consumer $(v - c - tx)$ exceed the private benefits $(p - c)$, a monopolist always underprovides purely informative advertising, as proven by Shapiro (1980).

If an independent instrument exists to control price, then the optimal subsidy under local monopoly in (33) is evaluated at $\hat{p}^* = c_s$, where $c_s$ is the social marginal cost of production. Firms, in this case, would receive subsidies on both advertising $\hat{\tau}^* = -(v - c_s)^2 / 2t$ and on output, $\hat{s}^* = \hat{p}^* - c_s$.

5B. Complete Coverage

Under complete coverage, both brands are always promoted at the social optimum. To see this, recall that the reason a monopolist prefers to promote only one brand under complete coverage is that the monopolist captures no additional rents from fully informing consumers, but incurs additional advertising costs. With probability $\bar{\phi}^2$, a consumer receives an advertising message for both brands, when only one ad is sufficient to induce a purchase, and this makes advertising both brands wasteful from the perspective of the monopolist. From the perspective of the social planner, the outcome is just the opposite: Fully informing consumers conserves transportation costs. It is straightforward to show that this makes advertising both brands socially beneficial.

Under complete coverage, all consumers that receive at least one advertising message purchase the product. With probability $\bar{\phi}^2$, a consumer receives both advertising messages. The average consumer who receives both advertising messages incurs transportation cost of $t/4$ to
obtain her desired brand, so that consumer net benefit for the fraction of the population hit by both ads is \( v - \bar{p} - (t/4) \). With probability \( 2\phi(1-\tilde{\phi}) \), a consumer receives exactly one advertising message. The advertising message is equally likely to be received from the nearest brand as from the farthest brand, so that a partially informed consumer (on average) incurs transportation cost of \( t/2 \) to obtain her desired brand, and receives net benefit of \( v - \bar{p} - t/2 \).

Aggregate welfare is given by
\[
\tilde{W} = n\phi \left( \phi(v - \bar{p} - t/4) + 2(1-\phi)(v - \bar{p} - t/2) + (2-\phi)(\bar{p} - c) \right) - 2nA_\phi(\phi) - 2F,
\]
where the first term in \( \tilde{W} \) represents benefits net of variable production costs and transportation costs. As in the case of local monopoly, this term is divided into consumer surplus and producer surplus components.

The socially optimal \( \phi \) is defined by the first order condition for the maximization of \( \tilde{W} \),
\[
\tilde{\phi} (v - \bar{p} - t/4) + (1 - 2\phi)(v - \bar{p} - t/2) + (\bar{p} - c)(1 - \tilde{\phi}) = A_\phi'(\phi).
\]  
(34)

A small increase in \( \phi \) that informs one more consumer does one of two things under complete coverage. With probability \( \phi \), the consumer becomes fully informed, which creates value in the economy of \( v - \bar{p} - (t/4) \), whereas, with probability \( (1 - \phi) \), the consumer becomes partially informed. The new consumption by the partially informed consumer contributes to industry profits by \( \bar{p} - c \). The new consumption also reduces the likelihood that advertising by the rival brand will inform a new consumer, so that, in the symmetric equilibrium, the probability of generating a new consumer with a simultaneous small increase in \( \phi \) for each brand is \( (1 - 2\phi) \). This consumption creates value in the economy of \( v - \bar{p} - t/2 \), and the sum of these marginal gains is set equal to the marginal social cost of advertising in (34).

We first compare the outcome in (34) to that under monopoly provision. To do so, note that the selection of \( \tilde{r}_m \) by the regulator aligns social and private incentives for advertising and therefore induces the monopolist to promote both brands. Substituting \( A'(\phi) = A_s'(\phi) + \tilde{r}_m \) and (14a) into (34) yields
\[
\tilde{r}_m^* = -\left( \tilde{\phi} (v - \bar{p} - t/4) + (1 - 2\phi)(v - \bar{p} - t/2) \right) < 0.
\]  
(35)
Notice that the subsidy in (35), as in the case of local monopoly, internalizes the marginal contribution of advertising to consumer welfare. A monopolist with complete coverage does not sufficiently advertise brands from the social perspective.

If an independent instrument exists to control price, then the optimal subsidy under local monopoly in (35) can be evaluated at \( p^* = c \).\(^{13}\) A monopoly firm would receive subsidies on both advertising (35) and output, \( \tilde{s}^* = \tilde{p}_m^* - c \).

Under oligopoly provision, the outcome differs from that under monopoly in the sense that individual firms do not account for the effect of advertising on reducing the profit of rival firms. Under price-setting oligopoly, firms advertise according to the average product of advertising on demand, and this exceeds the marginal product. Relative to the social level, there are thus two offsetting incentives under oligopoly. The effect of advertising on consumers is not internalized, which reduces advertising levels, and the effect of advertising on industry profits in (34) is not fully internalized, which increases advertising levels.

Substituting \( A'(\phi) = A_e'(\phi) + \tau_b \) and (25a) into (34) gives the implicit equation for the optimal advertising policy under price-setting oligopoly,

\[
\tilde{\phi} (v - \tilde{p} - t / 4 + (1 - 2\tilde{\phi})(v - \tilde{p} - t / 2) - \phi_b (\tilde{p} - c) / 2 + \tau_b^* = 0
\]

(36)

The sum of the first two terms is the effect of advertising on consumer surplus. Under monopoly provision, the subsidy on advertising is set to exactly offset this effect in (35). Under oligopoly, the optimal subsidy on advertising is less than that under monopoly (and perhaps it is even a tax), because the third term is negative. Under price-setting oligopoly price, firms advertise excessively relative to the level that maximizes joint industry profit. For a unit density of consumers, the marginal product of advertising on industry profits is \( (\tilde{p} - c)(1 - \phi_b) \) and its average product is \( (\tilde{p} - c)(2 - \phi_b) / 2 \). The third term in (36) reflects this difference.

\(^{13}\) Under complete coverage, it should be noted that the socially optimal price is not unique. Among consumers who receive advertising messages, demand is perfectly inelastic, so that any equilibrium price below this level serves only to distribute rents between producers and consumers.
Whether the optimal policy involves an advertising subsidy or an advertising tax under price-setting oligopoly depends on the sum of these two effects. To see which effect dominates, substitute \( \tilde{p} - c = (2 - \bar{\phi})t / \bar{\phi} \) from (24a) in (36) and evaluate this expression at \( \tilde{p}^* = c. \)

Rearranging terms gives

\[
\tilde{\epsilon}_b^* = -\frac{(1 - \bar{\phi}_b^*)^2}{2 - \bar{\phi}_b^*} 2(v - c_s - t) + \frac{(1 - \bar{\phi}_b^*)^2 t}{2 - \bar{\phi}_b^*} \left( \frac{\bar{\phi}_b^*}{2(1 - \bar{\phi}_b^*)} \right)^2 - 1,
\]

Notice that the first term is negative by the definition of complete coverage at the social price \( v - c_s \geq t. \) The second term is negative when \( \bar{\phi}_b^* < 2/3 \) and positive otherwise. Hence, the sum of these terms depends on the socially optimal level of advertising, on the social value of consumption (gross of transportation costs), and on the degree of product differentiation. When the term \( v - c_s - t \) is arbitrarily small, advertising is excessive under price-setting oligopoly whenever the socially optimal advertising level exceeds \( \bar{\phi}_b^* > 2/3. \) This is a noteworthy result in light of the fact that advertising costs change over time with the introduction of new media (e.g., the internet), which may permit advertisers to send messages to consumers at lower cost.

5C. Incomplete Coverage

Under incomplete coverage, all fully informed consumers purchase the product. With probability \( \bar{\phi} \), a consumer receives both advertising messages and a fully informed consumer receives (on average) net surplus of \( v - p - (t / 4). \) With probability \( 2\phi(1 - \phi) \), a consumer receives an advertising message from only one of the two brands. For this fraction of the population, these consumers are further divided into those who buy the advertised brand and those who do not, as each brand has incomplete coverage of the market segment. The conditional probability of a purchase, given a single advertising message is the fraction of the population who buy the brand at a unit price of \( p \), which is \( (v - p) / t. \) Among those consumers who chose to buy the brand, the average consumer is located at \( (v - p)2t \) the unit distance from the brand. Thus, consumer benefits net of variable production costs and transportation costs for the fraction of the population that receive a single ad is \( (v - p)/2. \)
Aggregate welfare is given by
\[
W = n\phi \left[ \phi \left( v - p - \frac{t}{4} \right) + \frac{(1 - \phi)(v - p)^2}{t} \right] + n\phi(p - c) \left[ \frac{2}{t}(v - p)(1 - \phi) + \phi t \right] - 2nA_s(\phi) - 2F,
\]
The first term in \(W\) represents consumer benefits for the fraction of the population that consumes the good. The interpretation is straightforward. With probability \((1 - \phi)^2\), a consumer does not receive an advertisement for either brand and neither consumption nor transportation cost takes place. With probability \(\phi^2\), a consumer receives both ads and travels to the nearest firm, which is on average \(\frac{1}{4}\) the unit distance. And, with probability \(2\phi(1 - \phi)((v - p)/t)\) a consumer receives only a single ad and finds it worthwhile to travel there and consume it, which is on average \((v - p)/2t\) the unit distance. The second term is industry profit net of advertising costs.

The first-order necessary for \(\phi^*\) is
\[
\phi \left( v - p - \frac{t}{4} \right) + (1 - 2\phi) \left( \frac{(v - p)^2}{2t} \right) + \frac{(p - c)}{t} \left( (1 - 2\phi)(v - p) + \phi t \right) = A_s'(\phi).
\]

The first two terms in (37) capture the marginal change in consumer surplus from an additional unit of advertising and the third term is the marginal change in industry profit. As in the previous two cases, the latter effect is completely internalized by the monopolist, so that the optimal policy under monopoly is an advertising subsidy.

Under price-setting oligopoly, substituting \(A'(\phi) = A_s'(\phi) + \tau_b\) and (25) into (37) gives the implicit equation for the optimal advertising tax as
\[
\phi \left( v - p - \frac{t}{4} \right) + (1 - 2\phi) \left( \frac{(v - p)^2}{2t} \right) - \frac{\phi(p - c)}{t} \left( v - p - \frac{t}{2} \right) + \tau_b = 0
\]

The sum of the first two terms is the effect of advertising on consumer surplus. The third term reflects the difference between the marginal product and the average product of advertising on industry profits.

It is possible that firms advertise excessively from the social perspective under price-setting oligopoly. As in the case of complete coverage, this outcome depends on the gross value of the product \((v - c)\), on the degree of product differentiation \((t)\), and on the marginal cost of
advertising. The range of parameters for which advertising is socially excessive under price-setting oligopoly can be determined by numerical simulation.

Proceeding similarly as above, substituting \( A'(\phi) = A'_s(\phi) + \tau_c \) and (29) into (37) gives the implicit equation for the optimal advertising tax under quantity competition,

\[
\phi \left( v - p - \frac{t}{4} \right) + (1 - 2\phi) \left( \frac{(v-p)^2}{2t} \right) - 2\phi(1-\phi)(p-c) \left( v - p - \frac{t}{2} \right) + \tau_c = 0
\]  

(39)

As before, the sum of the first two terms is the effect of advertising on consumer surplus, while the third term reflects the difference between the marginal product and the average product of advertising on industry profits. Comparing (38) and (39),

\[
\tau^*_b - \tau^*_c = \frac{\phi^*(p-c)(2(v-p)-t)}{2(2-\phi^*)} > 0
\]

where the inequality holds by the definition of incomplete coverage. The optimal advertising tax under price setting oligopoly always exceeds that under quantity setting oligopoly.

Under quantity competition, firms advertise less than under price competition for given prices. Hence, there may exist a range of parameter values for which price setting firms over-advertise from the social perspective, but quantity-setting firms under-advertise.

6. Concluding Remarks

We have considered a model of purely informative advertising with differentiated products. Advertising in the model informs consumers about the characteristics of brands and improves the matching of consumers and products. Nevertheless, we show that informative advertising does not necessarily make the demand for individual brands more elastic. Depending on the degree to which the market prices enable consumers to consume brands at distant locations, advertising can make demand more elastic, less elastic, or create parallel shifts.

Relative to the social optimum, advertising levels under oligopoly can be either inefficiently low or excessive. For equivalent prices, firms advertise more under price-setting oligopoly than under quantity-setting oligopoly. In either case, whether advertising is
inefficiently high or inefficiently low depends on the net value of the product (the difference between the reservation price of consumers and the production cost of firms), the degree of product differentiation, and the marginal cost of sending ads. When sending ads is less costly, oligopoly firms tend to excessively advertise products relative to the level which maximizes aggregate welfare in the economy.
References


FIGURE 1. Surplus offered by firms 0 and 1 under incomplete coverage.
FIGURE 2. The range of outcomes under monopoly for different net product values, \((v - c)/t\).

<table>
<thead>
<tr>
<th>Local monopoly (both brands)</th>
<th>Incomplete coverage (both brands)</th>
<th>Local monopoly (single brand)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 (\frac{2 - \phi_m^<em>}{2(1 - \phi_m^</em>)})</td>
<td>(\frac{4 - 3\phi_m^<em>}{2(1 - \phi_m^</em>)})</td>
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</table>

FIGURE 3. The range of outcomes under oligopoly for different net product values \((v - c)/t\).

<table>
<thead>
<tr>
<th>Local Monopoly</th>
<th>Incomplete Coverage</th>
<th>Complete Coverage</th>
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<tbody>
<tr>
<td>Mixed strategies</td>
<td>Mixed strategies</td>
<td>1 C B</td>
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