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Market Concentration and the Analysis of Vertical Market Structures

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Abstract: This paper investigates the pricing of differentiated products in a vertical sector, with a focus on the role of industry concentration. We propose concentration indices that extend the classical Hirschman-Herfindahl Index (HHI) to include vertical structures that manage differentiated products. We also identify how substitution/complementarity among products affects pricing. We compare our proposed indices with those proposed by Gans (2007). Additionally, we illustrate the utility of our approach by applying it to an analysis of mergers in the gasoline market. We note that our methodology is particularly applicable to analyses of those merger policies that involve upstream or downstream divestiture.

Key Words: Vertical strategy; imperfect competition; concentration indices

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Market Concentration and the Analysis of Vertical Market Structures

1. Introduction

Multi-product companies are common in today's economy. Such companies may operate in both horizontal and vertical markets. Firms get involved in vertical structures to capitalize on efficiency gains or to take advantage of opportunities to act strategically under imperfect competition. Vertical organization can involve arms-length transactions, contracting, coordination and/or vertical ownership (i.e., forward and backward vertical integration).

A great deal of literature focuses on evaluating the effects of market power. Economists commonly use the Herfindahl-Hirschman Index (HHI) of market concentration to analyze markets and assess market power (e.g., Whinston, 2006). More recent work has attempted to extend this approach to vertical structures. Gans (2007), for example, has explored the use of HHI to study standard products under vertical organization.[2] However, given that vertical structures are often associated with product differentiation, there is a need to examine how to use HHI-type measures of market concentration to analyze vertical markets.

Our paper investigates how vertical market structure affects the pricing of differentiated products under imperfect competition. Using a Cournot model, we propose concentration indices that extend the classical HHI to consider vertical structures and differentiated products. We term these concentration indices VHHI (for Vertical HHI), and explore how they relate to pricing.[3]

We also compare our VHHI with Gans' (2007) proposed vertical concentration measures. Unlike Gans (2007), our analysis includes differentiated products. It illustrates how

[2] See also Hendricks and McAfee (2009) for a discussion about measurements of concentration in a vertical sector.
[3] See Shi and Chavas (2010) for a further illustration of the utility of the VHHI in an econometric application to the pricing of genetically modified soybean seeds.

substitution/complementarity relationships across vertical channels affect pricing. Our comparison also points to an error in Gans' analysis.

Finally, our proposed VHHI approach provides a means to assess how vertical structure mergers affect pricing under alternative divestiture scenarios. The usefulness of the approach is illustrated by an application to mergers in the gasoline market.

Our paper is organized in the following manner: In Section 2 we present a conceptual model that examines multiproduct pricing under imperfect competition within a vertical sector. In Section 3 we compare our proposed indices with those of Gans. In Section 4 we discuss how our approach may be used to evaluate the connections between market structure and pricing. In Section 5 we apply our approach to an analysis of mergers in the gasoline market. In Section 6 we discuss our conclusions and explore avenues for further study.

2. Conceptual Model

Consider a vertical sector comprised of n firms that produce multiple upstream (intermediate) outputs that are used to construct multiple downstream (final) outputs. (Our notation explicitly distinguishes between intermediate and final products.[4]) The j-th firm produces r intermediate outputs $\mathbf{y}_j^{\mathrm{O}} = \sum_i \mathbf{y}_{ji} \in \mathbf{R}_+^{\mathrm{r}}$, and $\mathbf{y}_{ji} \in \mathbf{R}_+^{\mathrm{r}}$ denotes the vector of intermediate outputs that the j-th upstream firm produces and sells to the i-th downstream firm, $r \geq 1$. The vector of r intermediate outputs $\mathbf{y}_i^{\mathrm{I}} = \sum_j \mathbf{y}_{ji} \in \mathrm{R}_+^{\mathrm{r}}$ is purchased by the i-th firm, which produces m final outputs $\mathbf{z}_i = (z_{i1}, \ldots, z_{im}) \in \mathbf{R}_+^{\mathrm{m}}$, $m \geq 1$. The i-th firm purchases other inputs $\mathbf{x}_{\mathrm{i}}$ at prices $\mathbf{w}$ under a technology that is represented by the feasible set $\mathbf{F}_{\mathrm{i}}$, where $(\mathbf{x}_i, \mathbf{y}_i^{\mathrm{I}}, \mathbf{y}_i^{\mathrm{O}}, \mathbf{z}_i) \in \mathbf{F}_i$

[4] This distinction is particularly relevant to our discussion of Gans' (2007) work. We cover this in greater detail in Section 3.

means that inputs $\mathbf{x}_i$ and purchased intermediate outputs $\mathbf{y}_i^{\mathrm{I}}$ are feasible to produce intermediate outputs $\mathbf{y}_i^{\mathrm{O}}$ and final outputs $\mathbf{z}_i$.

Our analysis assumes efficient contracting among firms within a vertical sector. Letting $\mathbf{z} = (\mathbf{z}_1, \ldots, \mathbf{z}_n)$, this supposes that firms chose inputs $\mathbf{x}$ and intermediate products $\mathbf{y}$ in order to minimize aggregate cost:

$$C(\mathbf{z}) \;=\; \mathrm{Min}_{\mathbf{x},\mathbf{y}}\ \{\mathbf{w}\cdot\left(\sum_i \mathbf{x}_i\right) : \sum_i \mathbf{y}_i^{\mathrm{O}} = \sum_i \mathbf{y}_i^{\mathrm{I}};\ (\mathbf{x}_i, \mathbf{y}_i^{\mathrm{I}}, \mathbf{y}_i^{\mathrm{O}}, \mathbf{z}_i) \in \mathbf{F}_i\},$$

which yields $\mathbf{x}_i^*(\mathbf{z})$, $\mathbf{y}_i^{\mathrm{O}*}(\mathbf{z})$ and $\mathbf{y}_i^{\mathrm{I}*}(\mathbf{z})$, where $\mathbf{q}^*(\mathbf{z})$ denotes the shadow prices of the constraints $\sum_i \mathbf{y}_i^{\mathrm{O}} = \sum_i \mathbf{y}_i^{\mathrm{I}}$. The i-th firm's associated (net) cost function is $c_i(\mathbf{z}) = \mathbf{w}\cdot\mathbf{x}_i^*(\mathbf{z}) + \mathbf{q}^*(\mathbf{z})\cdot[\mathbf{y}_i^{\mathrm{I}*}(\mathbf{z}) - \mathbf{y}_i^{\mathrm{O}*}(\mathbf{z})]$. Under efficient contracting among all firms and given $\sum_i \mathbf{y}_i^{\mathrm{O}*} = \sum_i \mathbf{y}_i^{\mathrm{I}*}$, the efficient choice of the $\mathbf{y}_{ij}$'s implies that the firm cost functions $c_i(\mathbf{z})$ satisfy $\sum_i c_i(\mathbf{z}) = C(\mathbf{z})$.[5] We denote final demand by the price dependent demands $\mathbf{p}(\mathbf{Z}) = (p_1(\mathbf{Z}), \ldots, p_m(\mathbf{Z}))$, where $\mathbf{Z} = (Z_1, \ldots, Z_m) = \sum_i \mathbf{z}_i$ is the vector of m aggregate final outputs. Furthermore, we let $\mathbf{M} = \{1, \ldots, m\}$. Thus, the i-th firm's profit is:

$$\pi_i = \sum_{k\in\mathbf{M}} p_k(\mathbf{Z})\cdot z_{ik} - c_i(\mathbf{z}). \tag{1}$$

Under a Cournot game, the optimal choice of z_{ik} satisfies both $\pi_i \geq 0$ and the Kuhn-Tucker conditions:[6]

[5] Although our characterization of firm cost relies on uniform prices $\mathbf{q}^*(\mathbf{z})$ for intermediate goods, the analysis could easily be extended to include non-linear pricing. For example, under two-part tariffs, the payment made (received, if negative) by firm i to firm j would be: $t_{ij} + \mathbf{q}^*(\mathbf{z})\,[y_{ji} - y_{ij}]$, where t_{ij} is the lump-sum payment (receipt, if negative) made by firm i to firm j, subject to the restrictions $t_{ii} = 0$ and $t_{ji} = -t_{ij}$ for $i \neq j$. The (net) cost function of firm i then becomes $c_i(\mathbf{z}) = \sum_j t_{ij} + \mathbf{w}\cdot\mathbf{x}_i^*(\mathbf{z}) + \mathbf{q}^*(\mathbf{z})\cdot[\mathbf{y}_i^{\mathrm{I}*}(\mathbf{z}) - \mathbf{y}_i^{\mathrm{O}*}(\mathbf{z})]$, which again satisfies $\sum_i c_i(\mathbf{z}) = C(\mathbf{z})$.

[6] Our focus on efficient contracting among firms in the vertical sector means that we neglect issues related to the exercise of market power in markets for intermediate products. This exercise of market power can lead to additional inefficiencies under successive oligopoly (as in the case of double marginalization, as analyzed by Salinger (1988) and discussed in Gans (2007, p. 671-673)). The exercise of market power in markets for intermediate products can create inefficiencies by increasing aggregate costs C, which affect firm costs and intermediate goods pricing. In order to evaluate these issues, one would utilize Equations (1) and (2a) – (2c) in order to assess the effects on firm costs c_i. Unfortunately, it is not possible to obtain simple measures of these effects without imposing additional

$$p_k(\mathbf{Z}) - \tfrac{\partial c_i}{\partial z_{ik}} + \sum_{k' \in \mathbf{M}} \tfrac{\partial p_{k'}}{\partial Z_k} \cdot z_{ik'} \leq 0 , \tag{2a}$$

$$z_{ik} \geq 0 \tag{2b}$$

$$z_{ik} \cdot [\, p_k(\mathbf{Z}) - \tfrac{\partial c_i}{\partial z_{ik}} + \sum_{k' \in \mathbf{M}} \tfrac{\partial p_{k'}}{\partial Z_k} \cdot z_{ik'}] = 0 . \tag{2c}$$

Equation (2a) illustrates the impacts of cost (as captured by the term $\tfrac{\partial c_i}{\partial z_{ik}}$) and the exercise of market power (as described by the term $\sum_{k' \in \mathbf{M}} \tfrac{\partial p_{k'}}{\partial Z_k} \cdot z_{ik'}$). Equation (2c) is the complementary slackness condition for profit maximization. Equation (2c) holds in cases when firm i is involved only in the upstream market for the k-th product (with $z_{ik} = 0$) as well as when it also produces in the downstream market (with $z_{ik} > 0$).

Under a Cournot game, Equations (2a) – (2c) support an arbitrary configuration of firms in the vertical sector. These include those firms involved only in upstream markets (when $z_{ik} = 0$ for all k), firms involved only in downstream markets (when $\mathbf{y}_i^{O} = 0$) and vertically integrated firms that are involved in both the upstream and downstream markets (with $\mathbf{y}_i^{O} \neq 0$ and $z_{ik} \neq 0$ for some k). We note that such configurations may be motivated by efficiency gains (such as when vertical economies of scope mean that vertical integration contributes to cost reduction) and/or by strategic behavior under imperfect competition (as in situation where dominant players implement strategies that lead to foreclosure in some markets (see Rey and Tirole (2008)).

Equation (2c) provides a basis to evaluate the role of market power. Given $Z_i > 0$, let $s_{ik} \equiv \tfrac{z_{ik}}{Z_k} \in [0, 1]$ denote the i-th firm's share in the market for the k-th final good. Dividing (2c) by Z_k and summing across all firms yield the following pricing equation:

$$p_k(\mathbf{Z}) - \sum_i s_{ik} \cdot \tfrac{\partial c_i}{\partial z_{ik}} = -\sum_{k' \in \mathbf{M}} \tfrac{\partial p_{k'}}{\partial Z_k} \cdot H_{kk'} \cdot Z_{k'} , \tag{3}$$

restrictions on the model (such as linear demand and cost functions (see Salinger (1988)). It is important to note that the analysis presented below proceeds without imposing such restrictions.

where $H_{kk'} \equiv \sum_i s_{ik} \cdot s_{ik'} \in [0, 1]$. We use the following term to express the right-hand side of Equation (3):

$$M_k = -\sum_{k' \in \mathbf{M}} \tfrac{\partial p_{k'}}{\partial Z_k} \cdot H_{kk'} \cdot Z_{k'} \; .$$

In Equation (3), M_k measures the *k*-th final good's departure from marginal cost pricing. Indeed, under constant marginal cost (where $\frac{\partial c_i}{\partial z_{ik}}$ is a constant), Equation (3) reduces to $p_k(\mathbf{Z}) - \frac{\partial c_i}{\partial z_{ik}} = M_k$, with $M_k = 0$ if and only if the price of the *k*-th good equals its marginal cost. In this context, $\frac{M_k}{p_k}$ is the Lerner Index that reflects the strength of departure from marginal cost pricing for the *k*-th final good.

M_k can be interpreted as the "market power component" of price for the *k*-th final good. Thus, M_k can be used to characterize the exercise of market power in a vertical sector under product differentiation and Cournot behavior. And it establishes linkages between pricing and market concentrations (as measured by $H_{kk'}$).[7]

When $k = k'$, $H_{kk} \equiv \sum_i s_{ik}^2$ reduces to the classical Hirschman-Herfindahl Index (HHI) of concentration in the *k*-th market (e.g., Whinston, 2006). When demands are downward sloping (with $\frac{\partial p_k}{\partial Z_k} < 0$), Equation (3) yields the standard result that an increase in the HHI in the *k*-th market, which reflects increased concentration in that market, contributes to higher market power component M_k and to a higher Lerner Index $\frac{M_k}{p_k}$. This intuitive result links increased market concentration with higher consumer costs.

[7] Given $H_{kk'} \equiv \sum_i s_{ik'} \cdot s_{ik} \in [0, 1]$, any increase in $H_{kk'}$ is associated with an increase in market concentration. , At one extreme, $H_{kk'} \to 0$ under perfectly competitive markets (where the number of firms is large). At the other extreme, $H_{kk'} = 1$ when monopolies are present in the *k*-th and *k*'-th final good markets.

When $k \neq k'$, $H_{kk'} \equiv \sum_i s_{ik} \cdot s_{ik'}$ is a cross-market index of concentration. Any increase in $H_{kk'}$ contributes to a higher market power component M_k when $\frac{\partial p_{k'}}{\partial Z_k} < 0$, and to a lower M_k when $\frac{\partial p_{k'}}{\partial Z_k} > 0$. We define goods k and k' to be substitutes (complements) when increases in Z_k reduce (stimulate) consumers' willingness-to-pay for good k' (i.e. when $\frac{\partial p_{k'}}{\partial Z_k} < 0$ (> 0) (see Hicks, 1939)). Hence, we obtain an important result: An increase in cross-market concentration $H_{kk'}$ contributes to higher market power component M_k when final goods k and k' are substitutes, and to a lower M_k when they are complements. Our approach therefore illustrates how complementarity/ substitution impacts market power effects in a multi-market context. When vertical integration contributes to differentiation among final products, the $H_{kk'}$s in Equation (3) measure vertical market concentration. On that basis, we term the concentration indices $H_{kk'}$ "vertical HHI" or VHHI.

We are particularly interested in exploring how Equation (3) performs in situations where final products are perfect substitutes. Indeed, as we discuss in detail in the next section, Gans (2007) implicitly makes this assumption. If there is perfect substitution among final products, the law of one price applies (with $p_k(\mathbf{Z}) = p(\mathbf{Z})$ for all $k \in \mathbf{M}$), and the price-dependent demand takes the form $p(\mathbf{Z}) = p(Z)$, where $Z = \sum_k Z_k = \sum_k \sum_i z_{ik}$. We use $S_i = \frac{\sum_k z_{ik}}{Z}$ to denote firm i's aggregate market share. Then multiplying M_k by $\frac{Z_k}{Z}$ and summing over all k yield:

$$
\begin{aligned}
M \equiv \sum_k \tfrac{Z_k}{Z} \cdot M_k &= -\sum_k \tfrac{Z_k}{Z} \cdot \sum_{k'} \tfrac{\partial p}{\partial Z} \sum_i \left(\tfrac{z_{ik}}{Z_k} \tfrac{z_{ik'}}{Z_{k'}} \right) \cdot Z_{k'} \\
&= -\tfrac{\partial p}{\partial Z} \cdot \sum_i \sum_k \sum_{k'} \left(\tfrac{z_{ik}}{Z} \tfrac{z_{ik'}}{Z} \right) \cdot Z \\
&= -\tfrac{\partial p}{\partial Z} \cdot HHI \cdot Z,
\end{aligned}
\qquad (4)
$$

where HHI is the classical Hirschman-Herfindahl Index: $HHI = \sum_{i \in \mathbf{N}} S_i^2$.

In (4), M captures the effects of market power under perfect substitution. Equation (4) defines the market power component M as a weighted average of the market power components M_k's across all final goods. Then, (4) shows that, under perfect substitution, market power may be analyzed *as if* there is a single market. In this case, the market power component M is proportional to the classical HHI. Additionally, we note the following relationship between our $H_{kk'}$s and the classical HHI: $HHI = \sum_k \sum_{k'} \frac{Z_k}{Z} \cdot \frac{Z_{k'}}{Z} \cdot H_{kk'}$. This indicates that the classical HHI is a weighted average of the $H_{kk'}$s, with market shares serving as weights. Finally, from (3) and comparing M and M_k, our analysis clearly shows how the presence of differentiated products can affect the evaluation of market power effects.

3. Relationship to Gans' Approach

Gans (2007) analyzed market power effects in vertical structures. His work focused on two concentration measures: one under vertical contracting and one in successive Cournot oligopoly. Our approach focuses on the case of efficient contracting (rather than on successive oligopoly, as we discuss in footnote 6). We therefore compare our methodology with that of Gans' approach under vertical contracting.

The results from Equations (2) – (3) correspond to Gans' analysis, in that Gans suggests that the exercise of market power given by M_k depends only on downstream market shares (2007, p. 667). However, other results differ. In order to examine these differences, we observe that Gans (2007) makes the following assumptions:

A1: All final goods are "perfect substitutes" (Gans, 2007, p. 665);

A2: Firms i and j are in collusion, and make decisions that maximize joint profit in the upstream inputs market (Gans, 2007, p. 666);

A3: The quantity of final goods is equal to “the sum of all upstream inputs” (Gans, 2007, p. 665), which implies that $\sum_{k\in\mathbf{M}} z_{ik} = \sum_j y_{ij}$ and $\sum_{k\in\mathbf{M}} Z_k = \sum_i \sum_j y_{ji}$;

A4: $\mathbf{y}_{ii} = \min\{s_i, \sigma_i\} \cdot Z$ (as A1 already reduces Z to single output), where $s_i \equiv \frac{\sum_j y_{ij}}{Z}$ is the i-th firm’s market share in the downstream market and $\sigma_i \equiv \frac{\sum_j y_{ji}}{Z}$ is the i-th firm’s market share in the upstream market (Gans, 2007, p. 667). Gans (2007) justifies this assumption by arguing that a firm that maintains both upstream and downstream production capacities will always supply inputs to itself first.

Assumption A1 implies that our model reduces to a single final good, with $m = 1$ and $Z = \sum_j z_j \in R_+$, a situation that does not allow for final goods to exhibit horizontal or vertical product differentiation. Assumptions A2 and A3 strengthen the postulation that firms i and j are in collusion in the final goods market too. Once the numbers of upstream inputs are chosen, A3 then determines the final output. We also observe that although Assumption A3 applies under fixed-proportion production technologies, it is restrictive in that it does not apply under non-Leontief technologies in general. Finally, Assumption A4 imposes restrictions on upstream and downstream firms’ trading patterns.

Under Assumption A1, Equations (1) and (3) become:

$$\pi_i = p(\textstyle\sum_j z_j) \cdot z_i - c_i(\mathbf{z}), \tag{1'}$$

$$p(Z) - \textstyle\sum_i s_i \cdot \frac{\partial c_i}{\partial z_i} = -\frac{\partial p}{\partial z_i} \cdot H \cdot Z, \tag{3'}$$

where $H \equiv \sum_i (\frac{z_i}{Z})^2$ is the classical HHI, and the market power component is $M = -\frac{\partial p}{\partial z_i} \cdot H \cdot Z$.

Thus, under Assumption A1, the classical HHI accurately represents market concentration in a vertical sector.

However, if this is the case, why do our results so markedly differ from those of Gans (2007)? Assumptions A2, A3 and A4 can help answer this question. Assumptions A2 and A3 imply that firms *i* and *j* collude. Equation (1') expresses firms *i* and *j*s' joint profit as:

$$\pi_i + \pi_j = p(Z) \cdot (z_i + z_j) - c_i(\mathbf{z}) - c_j(\mathbf{z}) .$$

Additionally, given Cournot behavior, the first-order condition for $z_i > 0$ is:

$$p(Z) - \tfrac{\partial c_i}{\partial z_i} - \tfrac{\partial c_j}{\partial z_i} = -\tfrac{\partial p}{\partial Z} \cdot (z_i + z_j) . \tag{5}$$

We then consider Assumption A3, which states that $z_i = \sum_j y_{ij}$. If we presume that assumptions A1, A2 and A3 hold, Equation (5) then becomes:

$$p(Z) - \tfrac{\partial c_i}{\partial z_i} - \tfrac{\partial c_j}{\partial z_i} = -\tfrac{\partial p}{\partial Z} \cdot \sum_k (y_{ik} + y_{jk}) = -\tfrac{\partial p}{\partial Z} \cdot (s_i + s_j) \cdot Z . \tag{6}$$

Our Equations (5) – (6) correspond with Gans' Equation (3) (Gans, 2007, p. 667). Equation (5) differs from our Equation (3') above in that (5) involves [$z_i + z_j$], and (3') involves only z_i. This is intuitive: Given that A2 and A3 imply that firms *i* and *j* are in collusion, in (5) and (6) we add their shares together because the two firms behave as if they were a single firm. This analysis proves that Gans' Equation (3) applies only under assumptions A2 and A3. However, because collusion is not common in vertical sectors, Gans' approach (2007, p. 666 – 667) seems to have limited usefulness.

Using $s_i \equiv \sum_j \tfrac{y_{ij}}{Z}$ and $\sigma_j \equiv \sum_i \tfrac{y_{ij}}{Z}$, multiplying the right-hand side of Equation (6) by s_i and summing across all ***i*,** Equation (4') becomes:

$$\begin{aligned}
M &= -\tfrac{\partial p}{\partial Z} \cdot [\sum_i s_i \cdot (s_i + s_j)] \cdot Z \\
&= -\tfrac{\partial p}{\partial Z} \cdot [\sum_{i \in \mathbf{N}} (s_i \cdot s_i) + \sum_{i \in \mathbf{N}} (\sum_{j \in \mathbf{N}} \tfrac{y_{ij}}{Z}) \cdot s_j] \cdot Z \\
&= -\tfrac{\partial p}{\partial Z} \cdot [\sum_i (s_i \cdot s_i) + \sum_j (s_j \cdot (\sum_i \tfrac{y_{ij}}{Z}))] \cdot Z \\
&= -\tfrac{\partial p}{\partial Z} \cdot [\sum_i (s_i \cdot s_i) + \sum_j (s_j \cdot \sigma_j)] \cdot Z \\
&= -\tfrac{\partial p}{\partial Z} \cdot [\sum_i (s_i \cdot s_i) + \sum_i (s_i \cdot \sigma_i)] \cdot Z
\end{aligned}$$

$$= -\tfrac{\partial p}{\partial Z} \cdot [\textstyle\sum_i s_i \cdot (s_i + \sigma_i)] \cdot Z \,. \tag{7}$$

Given $\frac{M}{p}$ as the Lerner Index, it follows that the Lerner Index associated with (5) – (6) is:

$$L = -\tfrac{\partial \ln p}{\partial \ln Z} \cdot [\textstyle\sum_i s_i \cdot (s_i + \sigma_i)] \,. \tag{8}$$

Equation (8) differs from the Lerner Index given in Gans' proposition 1 (2007, p. 667), which reports that $L = -\frac{\partial \ln p}{\partial \ln Z} \cdot [\sum_i s_i \cdot (s_i + \sigma_i - \frac{y_{ii}}{Z})]$. Our result shows that, when compared with (8), Gans' Lerner Index includes one additional term that involves $\frac{y_{ii}}{Z}$. Since the derivation of Equation (8) from Equation (7) is straightforward, there identifies an error in Gans' analysis and in the way he arrived at his Proposition 1.

Additionally, when we employ Assumptions A1, A2 and A3, our Equation (8) suggests the following measure of market concentration: $H = \sum_i \{s_i \cdot (s_i + \sigma_i)\}$. Yet, under Corollary 1, Gans proposes a different concentration index: $\sum_i (s_i \cdot \max\{s_i, \sigma_i\})$ (Gans, 2007, p. 667). These two indices clearly differ, and this is due, in part, to the error we have identified in Gans' Proposition 1.[8] Moreover, Gans' Corollary 1 imposes Assumption A4, which states that $y_{ii} = \min\{s_i, \sigma_i\} \cdot Z$ (Gans, 2007, p. 667). Assumption A4 appears strong and is unlikely to apply when upstream and downstream firms are engaged in extensive trade.

Conversely, from our Equation (3), VHHI concentration indices, $H_{kk'} \equiv \sum_i s_{ik} \cdot s_{ik'}$, hold without requiring us to impose any of the restrictions given in A1 – A4. Our indices allow for the horizontal and/or vertical product differentiation in final goods that exists in many markets. Additionally, our indices do not assume that upstream and downstream firms collude; nor do we restrict trading patterns. Furthermore, under Cournot behavior, such concentration indices apply

[8] Gans' Corollary 1 relies on his Proposition 1. Thus, the error we have identified in his Proposition 1 carries over to, and invalidates, his Corollary 1.

to a variety of technologies in any type of vertical organization. Thus, the concentration indices in Equation (3) have broad application and would appear attractive in the analysis of market power in vertical sectors. In Section 5, we further illustrate the usefulness of our approach.

4. Implications for Market Structure Analysis

In Equation (3), $p_k(\mathbf{Z}) - \frac{\partial c_i}{\partial z_{ik}} = M_k = -\sum_{k' \in \mathbf{M}} \frac{\partial p_{k'}}{\partial z_{ik}} \cdot H_{kk'} \cdot Z_{k'}$ expresses how market concentration impacts price p_k. This effect involves the interaction between three terms: 1) The magnitude of the demand response (as represented by $\frac{\partial p_{k'}}{\partial z_{ik}}$); 2) degree of market concentrations (which are measured by the $H_{kk'}$ s); and 3) the market size for each final product (which is measured by $Z_{k'}$). Each term describes a different piece of the market power/pricing puzzle. Equation (3) also indicates that this impact involves both own-market effects (when $k = k$') and cross-markets effects (when $k \neq k$'). As noted above, in the presence of differentiated products, these cross-market effects are particularly relevant.

We begin by considering the own-market effects. When $k = k$', and given $\frac{\partial p_k}{\partial z_{ik}} < 0$, we obtain the standard result: The degree of price enhancement increases with a more inelastic demand (i.e., with an increase in $|\frac{\partial p_{k'}}{\partial z_{ik}}|$), a more concentrated own-market (as measured by a rise in H_{kk}) and an expanding market (as measured by a greater market size Z_k).

Equation (3) also captures cross-market effects $-\frac{\partial p_{k'}}{\partial z_{ik}} \cdot H_{kk'} \cdot Z_{k'}$ when $k \neq k$'. The sign of $\frac{\partial p_{k'}}{\partial z_{ik}}$ determines the sign of these effects. As discussed in Section 2, $\frac{\partial p_{k'}}{\partial z_{ik}} < 0$ (> 0) when products k and k' are substitutes (complements). Given $H_{kk'} \cdot Z_{k'} > 0$, it follows that cross-market effects contribute to price enhancements when products are substitutes; but these spur price reductions when products are complements. This analysis illustrates how substitution/complementarity impacts market power's effects on pricing.

The magnitude of cross-market effects also shifts in proportion to cross-market concentration $H_{kk'}$ and to market size $Z_{k'}$. Observe that cross-market concentration and market size have multiplicative effects on pricing. Thus, cross-market concentration $H_{kk'}$ is relevant only when $Z_{k'} > 0$; and is likely to be important only when the market size, $Z_{k'}$, is sufficiently large. Therefore, an increase in cross-market concentration $H_{kk'}$ contributes to a lower price p_k when products *k* and *k*' are complements and the market size for product *k*' is sufficiently large. Alternatively, a rise in cross-market concentration $H_{kk'}$ can enhance price p_k when products *k* and *k*' are substitutes and the market size for product *k*' is sufficiently large.

Finally, the multiplicative market size effects suggest that a vertically integrated firm may have an incentive to adopt a market foreclosure strategy in an upstream or downstream market if consumers perceive the products to be complementary. In this case, foreclosure in *k*' market could mean that the relevant market size $Z_{k'}$ can be reduced to zero (or to a negligibly small amount). Under this scenario, the firm would no longer experience the downward pressure on pricing product *k* generated by cross-market effects under complementarity.

In summary, our approach can be used to analyze changing market structures and market sizes in vertical sectors, and is particularly relevant to those firms that exhibit horizontal or vertical product differentiation. As such, our methodology can be employed to investigate the impacts of alternative mergers along with possible divestiture requirements in upstream or downstream markets. It can also be utilized to explore how entries/exits and foreclosures alter market sizes and the exercise of market power. In the following section, we illustrate the usefulness of our approach with a specific application.

5. Analyzing the Effects of Mergers: An Example

Like Gans (2007) and Hendricks and McAfee (2009), we use data about the Exxon-Mobil merger and the California gasoline market to illustrate the utility of our approach. We also evaluate how alternative mergers impact retail gasoline prices. Moreover, we use the same data to compare our approach with the classical HHI and with Gans' VHHI (2007). Table 1 lists market share information for petroleum refining and gasoline retailing in California.[9]

Both Gans (2007) and the classical HHI assume that all final goods are perfect substitutes, (i.e., that consumers consider gasoline to be a homogenous good). We, however, treat gasoline marketed through vertically integrated channels and through non-integrated channels as differentiated products. This is supported by Hastings (2004), who examined the gasoline market in southern California in 1997 and found evidence that price competition in gasoline markets follows a model of differentiated products: Consumers view branded high-share gasoline as different from independently-sold gasoline. Vertically integrated producers typically supply branded high-share gasoline, and independent stations sell gasoline via non-integrated channels. We follow Hastings' finding and allow for product differentiation at the retail level.

Our model does not make any assumptions in regard to upstream and downstream firms' specific trade patterns (in contrast with Gans (2007)). However, the data in Table 1 do not provide detailed information about these patterns. For the purposes of comparison, we follow Gans (2007) and impose the following assumptions: vertically integrated firms always use their own inputs first, sell to other downstream firms only if they have excessive input supplies, and buy from other upstream firms only if they have shortages in input supplies.

[9] The data are taken from Gans (2007), which were obtained from an earlier version of Hendricks and McAfee (2009).

Table 1. Market Shares (%), West Coast CARB Gasoline.[a]

Company	Upstream (Refining)	Downstream (Retailing)
Chevron	26.4	19.2
Tosco	21.5	17.8
Equilon	16.6	16.0
Arco	13.8	20.4
Mobil	7.0	9.7
Exxon	7.0	8.9
Ultramar	5.4	6.8
Paramount	2.3	0
Kern	0	0.3
Koch	0	0.2
Vitol	0	0.2
Tesoro	0	0.2
Petro-Diamond	0	0.1
Time	0	0.1
Glencoe	0	0.1

For example, Chevron's share in the downstream market (19.2%) is smaller than its segment of the upstream market (26.4%); thus, we categorize all of its retail market operations as vertically integrated. Alternatively, Arco's piece of the downstream market (20.4%) is larger than its portion of the upstream market (13.8%); we therefore divide its downstream market share accordingly: We list 13.8% in the vertically integrated market and 6.6% in the non-integrated market. Note that by doing so, we also adopt Gans (2007) assumption that downstream firms utilize Leontief (fixed-proportion) production technologies. We have made these assumptions merely to illustrate our points.[10] Below, we use *v* to denote the vertically integrated market and *u* to refer to the non-integrated market.

In order to compare our approach with that of Gans (2007), we construct our concentration measures by simply summing Exxon and Mobil's relevant shares under different merger scenarios. We follow Gans (2007) and label the concentration measures as follows:

[10] As such, these assumptions should not be used to validate (or invalidate) our conceptual approach.

Classical HHI, Gans' proposed vertical indices under contracting (hereafter termed Gans-VHHI) and our VHHI.

In Table 2 we present the constructed measures. As opposed to the single measures that the classical HHI and Gans-VHHI capture, our VHHI contains three measures: One encapsulates the non-integrated gasoline market's within-market concentration ($VHHI_{uu}$), another describes the vertically integrated gasoline market's within-market concentration ($VHHI_{vv}$) and a third denotes vertically integrated and non-integrated markets' cross-market concentrations ($VHHI_{uv}$).

The Exxon-Mobil merger unambiguously increases concentration measures under all three approaches. The increases are as follows: 11% for the classical HHI, 9.7% for the Gans-VHHI, and 18.1% for $VHHI_{uu}$, 7.8% for $VHHI_{vv}$, and 24.6% for $VHHI_{uv}$. Our VHHI details how the merger affects the within- and cross-market concentrations, and therefore provides important information in the evaluation of alternative merger scenarios.

Our method allows us to distinguish between mergers that require no divestiture and those that entail upstream or downstream divestiture. This approach contrasts with that of the classical HHI. Indeed, the classical HHI does not account for upstream divestiture when evaluating a post-merger downstream situation. Furthermore, with the classical HHI, a downstream divestiture eliminates a merger's impact on the downstream market; thus, the HHI is restored to the pre-merger level. The Gans-VHHI is similar to the classical HHI, in that it suggests that the upstream divestiture does not change the non-divestiture post-merger downstream VHHI. Additionally, the downstream divestiture requirement mitigates, but does not fully offset, the increase in the post-merger Gans-VHHI measurement.

Our VHHI approach, shown in Table 2, illustrates how alternative merger/divestiture scenarios impact vertically integrated and non-integrated markets and cross-market interactions.

In the case of upstream divestiture, vertically integrated and non-integrated within-market concentration measures ($VHHI_{vv}$ and $VHHI_{uu}$) increase in comparison to those for non-divestiture mergers, and the cross-market concentration measure ($VHHI_{uv}$) decreases. When downstream divestiture occurs, within-market concentration measures also increase in relation to those for non-divestiture mergers, but these increases are smaller than they are for the upstream divestiture scenarios. Cross-market concentration measures are also lower than they are for upstream divestiture mergers.

We also computed the simulated markups M and Lerner Indices $\frac{M}{P}$ under these alternative merger/divestiture scenarios. As showed in Equation (3), the simulation requires parameter values for the demand coefficient $\frac{\partial p_{k'}}{\partial Z_k}$ and information about pricing and market size.

In order to illustrate our point, we set total market size at $Z = 10$. This figure includes both the non-integrated and vertically integrated markets (Z_u and Z_v). Hendricks and McAfee (2009) assume that the price demand elasticity of gasoline is 1/3, and in 2000 the inferred pump price of gasoline was \$1.45 per gallon. We use this information and the assumed market size of 10, and obtain $\frac{\partial p}{\partial Z} = -0.44$. Thus, we presume $\frac{\partial p_v}{\partial Z_v} = \frac{\partial p_\ell}{\partial Z_\ell} = -0.44$. With regard to differentiated products, we assume that retail products are "mild-substitutes", with $\frac{\partial p_v}{\partial Z_\ell} = \frac{\partial p_\ell}{\partial Z_v} = -0.22$ (a departure from the assumption of perfect substitution, given that 0.22 < 0.44).[11]

In order to construct the Lerner Indices for differentiated gasoline markets, we need to assume vertically integrated and non-integrated gasoline pump prices. Hastings (2004) estimated that independent brand gasoline retailed at approximately 7 cents per gallon lower than did the high-share brand. In our simulations we therefore set $p_v = \$1.485$ and $p_u = \$1.415$.

[11] We have found no solid empirical evidence that evaluates the strength of substitution relationships in gasoline retail markets. We have chosen $\frac{\partial p_v}{\partial Z_\ell} = \frac{\partial p_\ell}{\partial Z_v} = -0.22$ for illustration purposes only.

Table 2. Concentration Measures.

Concentration Measure		**Pre-merger**	**Post-merger**	**Post-merger with Exxon Refinery Divestiture**	**Post-merger with Exxon Retail Divestiture**
Upstream HHI		0.1758	0.1856	0.1758	0.1856
Downstream HHI		0.1577	0.1750	0.1750	0.1577
Gans-VHHI Under Contracting		0.1791	0.1964	0.1964	0.1833
Our VHHIs	$VHHI_{uu}$	0.2975	0.3514	0.4168	0.3815
	$VHHI_{vv}$	0.1694	0.1826	0.1929	0.1871
	$VHHI_{uv}$	0.1100	0.1371	0.1092	0.0665

We report our simulation results in Table 3. Table 3 compares the markups and Lerner Indices (expressed in percentage term as $\frac{M}{p}\cdot 100$) generated by the classical HHI, the Gans-VHHI under contracting and by our VHHI. The pattern of changes in markups and Lerner Indices for the classical HHI and Gans-VHHI is similar to that of Table 2. This reflects the fact that both approaches assume that the demand coefficient and market size variables remain constant before and after a merger, regardless of its divestiture requirements. Thus, only the concentration measures change. We note that for all the scenarios listed in Table 3 the Gans-VHHI approach generates higher markups and Lerner Indices than does the classical HHI.

Conversely, with our VHHI approach, divestiture requirements impact vertically integrated and non-integrated gasoline market sizes, which implies that the final effects are not necessarily aligned with the change in VHHIs. Our simulated markups and Lerner Indices are significantly lower for the non-integrated market than for the vertically integrated market. Moreover, all of our figures are lower than those generated by the classical HHI and Gans-VHHI.

Table 3. Simulated Impact of Merger.

	Pre-merger	Post-merger	Post-merger with Exxon Refinery Divestiture	Post-merger with Exxon Retail Divestiture
Perfect Substitutes Across Vertical Structures: Classical HHI				
M, Departure from Marginal Cost Pricing ($)	0.69	0.77	0.77	0.69
$\frac{M}{P}\times 100$, Lerner Index	47.6	53.1	53.1	47.6
Formula: $M=-\frac{\partial P}{\partial Z}HZ$, where $H=\sum_i s_i^2=VHHI_{uv}\cdot\frac{Z_u}{Z}\cdot\frac{Z_v}{Z}+VHHI_{uu}\cdot\frac{Z_u}{Z}\cdot\frac{Z_u}{Z}+VHHI_{vu}\cdot\frac{Z_v}{Z}\cdot\frac{Z_u}{Z}+VHHI_{vv}\cdot\frac{Z_v}{Z}\cdot\frac{Z_v}{Z}$ Parameter Values: $\frac{\partial P}{\partial Z}=-0.44$, $Z=Z_u+Z_v=10$, $p=\$1.45$				
Perfect Substitutes Across Vertical Structures: Gans-VHHI Under Contracting				
M, Departure from Marginal Cost Pricing ($)	0.79	0.86	0.86	0.81
$\frac{M}{P}\times 100$, Lerner Index	54.5	59.3	59.3	55.9
Formula: $M=-\frac{\partial P}{\partial Z}HZ$ Parameter Values: $\frac{\partial P}{\partial Z}=-0.44$, $Z=Z_u+Z_v=10$, $p=\$1.45$				
Differentiated Products Across Vertical Structures				
M_u, Departure from Marginal Cost Pricing ($)	0.39	0.47	0.57	0.42
$\frac{M_u}{P_\ell}\times 100$, Lerner Index	27.6	33.2	40.3	29.7
M_v, Departure from Marginal Cost Pricing ($)	0.68	0.73	0.72	0.70
$\frac{M_v}{P_v}\times 100$, Lerner Index	45.8	49.2	48.5	47.1
Formula: $M_u=-\frac{\partial P_u}{\partial Z_u}\alpha_{uu}VHHI_{uu}Z_u-\frac{\partial P_v}{\partial Z_u}VHHI_{uv}Z_v$, $M_v=-\frac{\partial P_v}{\partial Z_v}VHHI_{vv}Z_v-\frac{\partial P_u}{\partial Z_v}VHHI_{vu}Z_u$ Parameter Values: $\frac{\partial p_v}{\partial Z_v}=\frac{\partial p_u}{\partial Z_u}=-0.44$, $\frac{\partial p_v}{\partial Z_u}=\frac{\partial p_u}{\partial Z_v}=-0.22$, $Z_u+Z_v=10$, $p_v=\$1.485$ and $p_u=\$1.415$				

All three approaches suggest that, from the perspective of anti-competitive concerns, merger policies that employ downstream divestiture requirements result in the most preferred outcomes. As compared to the pre-merger Lerner Index, the classical HHI approach estimates these impacts as a full offset, and the Gans-VHHI reports a partial offset. With our VHHI approach, the non-integrated market's Lerner Index increases by 2.1 percentage points. (In comparison, for a non-divestiture merger, the increase is 5.6 percentage points and is 12.7

percentage points for an upstream divestiture merger). Additionally, we note a 1.3 percentage point increase in the Lerner Index for the vertically integrated market. (This compares to an increase of 3.4 percentage points for a non-divestiture merger and 2.7 points for an upstream divestiture merger). These results illustrate the level of detail that our VHHI approach provides.

Most importantly, we note that neither the classical HHI nor the Gans-VHHI can capture any of the impacts of an upstream divestiture merger policy. Our VHHI approach, however, suggests that under merger policies that require upstream divestiture, vertically integrated gasoline markups and Lerner Indices decrease slightly (from \$0.73 to \$0.72, and from 49.2 to 48.5), and non-integrated gasoline markups rise from \$0.47 to \$0.57 (and the corresponding Lerner Indices increase from 33.2 to 40.3).

This simple example demonstrates the usefulness of our VHHI concentration indices in analyzing the impacts of different merger/divestiture policies. Our approach captures both within- and cross-market concentration effects, and accounts for the changes in market size that are associated with different merger/divestiture scenarios.[12] Finally, our approach is empirically tractable, and offers scholars an opportunity to more deeply explore the variety of factors that shape how market power affects vertical sector market structures.

6. Concluding Remarks

Our paper has investigated how vertical firms price differentiated products. We have developed a Cournot model that captures how vertical organization impacts differentiated product pricing under imperfect competition. Our analysis proposes concentration indices that

[12] And as discussed above, our approach also captures complementarity/substitution effects through cross-market effects.

extend the classical HHI to include vertical structures under differentiated products. We also identify limitations in Gans' (2007) indices.

Our work illustrates how cross-product substitution/complementarity relationships across vertical channels affect pricing. We identify the ways in which market size interacts with market concentration and cross-product relationships. Furthermore, our approach offers a broadly applicable tool that can help analyze the market power effects in vertical sectors under differentiated products.

We have established the viability of our approach by applying it to an investigation of mergers in the gasoline market. This application has also allowed us to detail how our VHHIs capture the impacts of different merger/divestiture policies. Neither the classical HHI nor Gans-VHHI has this property.

Thus, our approach helps better understand how changing market structures affect pricing in vertical sectors. As such, it can inform the design and evaluation of antitrust policy. Our approach is, however, limited by its static nature. It may be further refined in order to more deeply explore the market dynamics that impact the exercise of market power in vertical sectors.

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