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Risk Rationing and Activity Choice in Moral Hazard Constrained Credit Markets

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Abstract

This paper explores the productivity and income distribution effects of asymmetric information and risk preferences on the credit market. A model of contract design in the presence of moral hazard is developed in which competitive, risk neutral lenders offer contracts to risk averse agents who hold the option to invest capital and labor time in an entrepreneurial activity. The model gives rise to the potential for quantity rationing and an additional form of non-price rationing called *risk rationing*. Both quantity and risk rationed agents would seek credit and carry out the entrepreneurial activity in a first best, or symmetric information world. When information is asymmetric, the menu of available loan contracts shrinks. In equilibrium, neither type of agent ends up with a loan contract, and both undertake a safe, but low return wage labor activity. Quantity rationed agents are involuntarily excluded from the entrepreneurial activity because they are denied any loan contract. Risk rationed agents voluntarily retreat from the credit market and the entrepreneurial activity rather than choose among the limited set of high risk contracts available to them in the presence of asymmetric information. Analysis shows that both quantity and risk rationing are likely to be wealth-biased, inhibiting the activity choice and the income earning potential of low wealth agents, and reproducing initial inequality.

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1 Introduction

In a competitive world of symmetric information and costless enforcement, credit contracts could be written conditional on borrower behavior. Borrowers would then have access to loans under any interest rate-collateral combination that would yield lenders a zero expected profit. However, as a large literature has shown, information asymmetries and enforcement costs that make such conditional contracting infeasible will restrict the set of available contracts, eliminating as incentive incompatible high interest rate, low collateral contracts.[1] As has been emphasized in the literature, this contraction of contract space can result in quantity rationing in which potential borrowers who lack the wealth to fully collateralize loans are involuntarily excluded from the credit market and thus prevented from undertaking high return investments.

The principal contribution of this paper is to show that the contraction of contract space induced by asymmetric information can result in another form of non-price rationing, one that we label "risk rationing." Risk rationing occurs when lenders, constrained by asymmetric information, shift so much contractual risk to the borrower that the borrower voluntarily withdraws from the credit market even when she or he has the necessary collateral wealth to qualify for a loan contract.[2] The private and social costs of risk rationing are similar to those of more conventional quantity rationing. Like quantity-rationed individuals, risk rationed individuals will retreat to lower expected return activities. Moreover, under mild assumptions about the nature of risk aversion, risk rationing will, like quantity rationing, be wealth-biased and predominately affect lower wealth individuals and firms. As in Eswaran and Kotwal's (1990) analysis, initial wealth and activity choice become tightly linked by financial market imperfections.[3]

[1] Summaries of this literature include: (Hillier and Ibrahimo, 1993), (Jaffee and Stiglitz, 1990), (Dowd, 1992) and (Besley, 1995).

[2] Like an interest rate increase, an increase in contractual risk will also help equilibrate the loan market by reducing demand and is thus a form of non-price rationing.

[3] Eswaran and Kotwal assume that quantity rationing exists, whereas the analysis here shows that both quantity and risk rationing are the endogenous result of optimal, competitive loan contracts under asymmetric information and risk aversion. While their work shows that initial wealth differences, not Knightian differences in risk-bearing capacity, explain who becomes the entreprenuers, the analysis here reveals a subtle interplay between wealth, changing risk aversion, optimal contract design and the functioning of the credit market.

The distinction between quantity and risk rationing highlights the fact that a firm's activity choice depends both on the financial feasibility of activities and preference ranking over available activities. The credit rationing literature has focussed primarily on the former. The latter is important, however, because the fact that a positive loan offer makes a high return project financially feasible, does not imply that the project will be chosen over other (safer) activities. When insurance markets are missing, the choice over alternative activities will depend on the nature of risk preferences and on the degree to which credit contracts offer partial insurance through the provision of limited liability. The analysis here will show that the degree to which the credit market fulfills the dual roles of providing liquidity and insurance–and for whom–will depend critically upon the nature of the information environment and producers' risk preferences.

In addition to filling a theoretical lacuna, the distinction between quantity and risk rationing is important from the perspective of empirical work. The econometrics of credit rationing have struggled with the fundamental problem of distinguishing individuals with zero loan demand from quantity rationed individuals. To solve this problem, some studies have resorted to the econometrics of unobserved regime switching (Bell et al., 1997). Others have employed ancillary sample information to distinguish individuals with positive demand from those without. For example, Kochar (1997) uses loan application as a signal of positive loan demand. While the first approach is subject to statistical limitations, use of loan application as a necessary signal of positive demand is highly problematic in the presence of quantity rationing, as Mushinski (1999) argues.[4]

In an effort to obtain more reliable indicators of positive loan demand, several recent enterprise surveys have added questions inquiring why firms do not apply for loans. Not only do such questions reveal significant numbers of discouraged firms that do not apply for loans because they know they will not get them (what Mushinski calls preemptively-rationed), they also reveal

[4] If loan application is costly and individuals know that quantity rationing is a possibility, they will only apply for loans for which they have expect to have a reasonable probabilty of success.

Table 1: Risk and Quantity Rationed Firms

	Peru			*Guatemala*		
	Non-Price Rationed		*Price Rationed*	*Non-Price Rationed*		*Price Rationed*
	Quantity	Risk		Quantity	Risk	
%	36.7	17.2	46.1	31.1	13.7	55.2
Wealth ($)	13,336	9,396	23,771	21,510	6,024	38,972
Input ($/ha)	451	454	868	NA	NA	NA
Income ($/ha)	653	593	919	NA	NA	NA

significant numbers of non-applicant firms that were discouraged from applying for loans by fear of losing required collateral in the event of default. The modeling reported in this paper is an effort to make theoretical sense of this empirical report of fear-driven non-borrowers.

Table 1 reports data on risk-rationing from two recent surveys, one of agricultural enterprises in Peru (Boucher, 2000) and the other of rural farm and non-farm enterprises in Guatemala (Barham et al., 1996). Firms reported as price rationed in the table include both firms that borrowed and those that chose not to because they did not need capital or found the interest rate to be too high. Non-price rationed firms are those that indicated that they would have liked to have borrowed money at the going rate of interest, but that they either could not qualify for a loan (*i.e.*, were quantity rationed), or were afraid to take one because of the risk of collateral loss (risk rationed). As can be seen, risk rationed enterprises constitute 14% to 17% of all surveyed enterprises, and they are 30% of all non-price rationed firms. Failure to account for risk rationed households as non-price rationed would clearly have a major effect on the analysis of the efficiency of credit markets under asymmetric information.

Table 1 also displays some additional information on risk-rationed versus other types of firms. Given the relative homogeneity of agricultural producers in the Peru survey, we can glean a meaningful idea of the activity choice of risk rationed producers by looking at their use of inputs as well as net-income produced per-unit land. As can be seen, the risk rationed firms appear similar to the quantity rationed, with both inputs and income some 30% to 50% below that of price rationed producers. In both the Peru and Guatemala data sets, we see that the mean wealth holdings of both risk-rationed and quantity rationed producers are below the sample means. In the language of this paper, non-price rationing appears wealth-biased in these samples.

We turn now to develop a theory of risk rationing in moral hazard-constrained credit markets. The next section lays out a model of entrepreneurial behavior under uncertainty and describes the structure of credit contracts. Section 3 explores the implications of asymmetric information on the existence and terms of the optimal credit contract, and demonstrates the potential for wealth-biased quantity and risk rationing. Section 4 concludes the paper with a numerical simulation of the model that shows how wealth-biased risk and quantity rationing conspire to create a world in which initial inequality and class structure reproduce themselves over time.

2 Key Assumptions and Model Structure

Every agent has an initial endowment of wealth, $W \in (\underline{W}, \overline{W})$, as well as an endowment of a productive asset which we will call labor. Each agent's labor endowment is identical, and the agent must decide whether to allocate his or her labor to an entrepreneurial activity or to "rent" it out for wages. Wage labor jobs pay a certain income of ω and require a high level of effort. Gross entrepreneurial income, X, is generated according to the following stochastic process:

$$X = x(k, j(e)) = \begin{cases} x_s \; if \; j = s \; and \; k \geq \underline{K} \\ x_f \; if \; j = f \; and \; k \geq \underline{K} \\ 0 \; if \; k < \underline{K} \end{cases} \tag{1}$$

where k is capital input sunk into the entrepreneurial project, e is the agent's level of effort, and j is the state of nature realized after capital and effort are committed. There are two states of nature: success ($j = s$) and failure ($j = f$). Income under success is greater than under failure: $x_s > x_f$. The agent influences the probability of success through choice of effort–which can be either high ($e = H$) or low ($e = L$). Let ϕ^e be the probability of success under effort level e. The probability of success is increasing in effort so that $\phi^H > \phi^L$. The entrepreneurial project has a fixed capital requirement $\underline{K}$. If the capital requirement is not met, output is zero independent of the realized state of nature. Additional capital beyond $\underline{K}$ has zero marginal productivity. Under the fixed capital requirement, agents with insufficient wealth endowments ($W < \underline{K}$) are unable to self-finance production.[5]

2.1 Autarchic Self-Finance

Agents have access to a riskless savings activity, and wealth not invested in the productive activity yields a gross return of $(1 + r^a)$. Net entrepreneurial income under autarchic self-finance would thus be:

$$y_j^a = x(k, j) - (1 + r^a)k \tag{2}$$

Note that liability is unlimited under self-finance: The entrepreneur pays for the full cost of the project irrespective of whether the project fails or succeeds. Letting $\overline{y}_H^a = \phi^H x_s + (1 - \phi^H)x_f - (1 + r^a)k$ denote expected entrepreneurial income under high effort and $\overline{y}_L^a$ denote the same thing under low effort, we make the following assumptions:

$$\overline{y}_H^a > \omega > 0 > \overline{y}_L^a \tag{3}$$

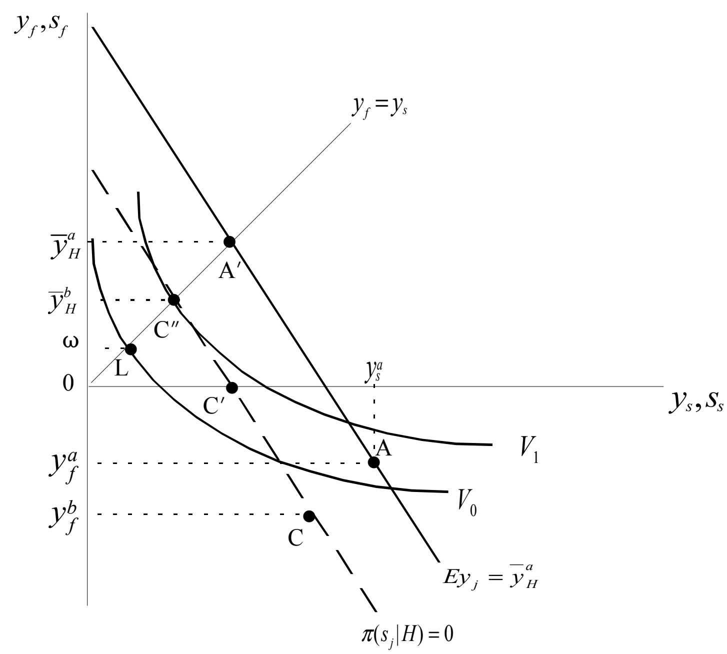
$$y_s^a > 0 > y_f^a \tag{4}$$

Expression (3) indicates that expected net income exceeds the wage rate if high effort is applied but is negative under low effort. By making realized net income negative under failure, expression (4) makes liability a non-trivial issue.

Using these assumptions we can graphically portray these payoffs in state contingent income space as shown in Figure 1. Payoffs under success are measured along the x-axis, while payoffs under failure are shown on the y-axis. Point L along the 45-degree line denotes the certain payoff received under wage labor. More generally, any payoff point along the 45-degree, or full insurance, line yields consumption which is independent of the state of nature. The net income

[5] The assumption of a fixed project size is made for analytical simplicity. The general conclusions are not altered under the less restrictive assumption of a diminishing returns, variable input technology. This continuous input size case is disucssed in chapter four of (Boucher, 2000).

Figure 1: Indifference curves in income/payoff space



pair (y_s^a, y_f^a), shown as point A in the figure, represents the outcome under autarchic self-finance of the entrepreneurial project. Note that we can define a locus of state-contingent payoffs that would yield an expected value identical to the entrepreneurial project under high effort. This locus is given by all state-contingent income pairs, (y_s, y_f), such that $\phi^H y_s + (1 - \phi^H) y_f = \overline{y}_H^a$, or $y_f = \frac{\overline{y}_H^a}{1-\phi^H} - \left(\frac{\phi^H}{1-\phi^H}\right) y_s$. The downward sloping line that passes through point A in Figure 1 illustrates this locus. Note that if $\overline{y}_H^a = \omega$, this locus would pass through point L. However, expression (3) implies that the locus lies to the northeast of point L intersecting the 45 degree line at a point like A'.

2.2 Credit Contracts

The capital costs of the entrepreneurial project can also be potentially funded by loans. We assume that competitive lenders must expect a rate of return of r^b on loans in order to cover the competitive cost of capital. We further assume that $r^b > r^a$ with the difference between the two rates reflecting the cost of intermediation.

One possible loan contract that meets the competitive expected rate of return requirement is the unlimited liability (or fully collateralized) contract that offers the borrower a payoff of the form:

$$y_j^b = x(k, j) - (1 + r^b)k. \tag{5}$$

Irrespective of the realized state of the world, this unlimited liability contract guarantees the lender a gross return of $k(1 + r^b)$ and expected economic profits (π) of zero. With $r^b > r^a$, the payoffs to the borrower under this contract will appear in Figure 1 at a point like C that is strictly to the southwest of the self-finance payoff pair denoted as point A. Following the notational convention established above, let $\overline{y}_H^b = \phi^H x_s + (1 - \phi^H)x_f - (1 + r^b)k$ denote expected net entrepreneurial income conditional on high effort and opportunity cost of capital equal to r^b. To keep the problem meaningful, we will further assume that:

$$\overline{y}_H^b > \omega > 0 > \overline{y}_L^b. \tag{6}$$

Expression (6) implies that, conditional on high effort, the project yields a return greater than the wage rate even at the higher capital cost, r^b. The negative return under failure implied by expression (4) implies that the unlimited liability loan contract requires a transfer of collateral wealth to the lender under failure. A wealth level equal to this liability is obviously necessary for such a contract to be financially feasible for a borrower.

In addition to the unlimited liability loan contract, there is a large menu of loan contracts that (conditional on high effort) will yield expected borrower and lender income identical to that given by the contract represented at point C. While it is conventional to express a loan contract in terms of the nominal interest rate and collateral requirement, a contract can also be expressed in terms of the state-contingent payoffs they offer to the borrower. As with autarchic self-finance, we can define the locus of state contingent payoff pairs, (s_s, s_f), that conditional on high effort yield the borrower an expected entrepreneurial income equal to $\overline{y}_H^b$:

$$s_f = \frac{\overline{y}_H^b}{1-\phi^H} - \left(\frac{\phi^H}{1-\phi^H}\right) s_s \tag{7}$$

Expected gross lender income is $k(1+r^b)$ for every payoff pair along this locus, implying expected economic profits of zero for the lender. This conditional zero lender profit locus (denoted $\pi(s_j|H)$) is illustrated in Figure 1 as the dashed, downward sloping line. Contracts must be on or to the southwest of the zero profit locus for the lender to at least break even. This locus has the same slope and is parallel to the entrepreneur's iso-income line under self-finance.

Note that points along that locus that lie to the northwest of the unlimited liability loan contract C represent higher nominal interest rates (*i.e.,* lower payoff to the borrower under success) and lower liability or collateral (*i.e.,* a higher payoff to the borrower under failure). As such, northwest movements increase the insurance component of the credit transaction as borrower income is smoothed across states of nature while expected income is held constant. The payoff pair denoted C' is a standard debt contract with full default under failure as s_f at that point is zero. Contracts corresponding to points to the northwest of C' are rarely observed as the positive value of s_f indicates that the lender makes a further payment to the borrower in the event of project failure.[6] Finally, note that the payoff pair at C'' is the full-insurance loan contract in which the borrower's income is independent of project success or failure ($s_f = s_s$).

2.3 Agent Preferences

An agent's well being depends on both the level of consumption and the work effort exerted. The utility for an agent with wealth W is $u(c_j) - D(e; W)$, where c_j is consumption in state j and u is a strictly increasing, concave function. We assume that all agents have access to a consumption minimum yielding finite utility which is exogenously guaranteed to the agent by social or other mechanisms.[7] For convenience we set this consumption minimum to zero and normalize utility

[6] Udry (1994), however, gives an empirical example of the existence of such conracts in West Africa.

[7] The consumption minimum prevents the lender from offering contracts which drive the agent's utility under failure towards negative infinity. If the lender could do so, then there would always exist incentive compatible

so that $u(0) = 0$. The disutility of effort, D, depends on both the effort level exerted and the agent's wealth level. Let $d(W) = D(H; W) - D(L; W)$ be the disutility differential of high versus low effort. We make the following assumptions. First, the disutility of low effort is independent of wealth and is normalized to zero so that $d(W) = D(H; W) > 0$. Second, the disutility of high effort is decreasing in wealth so that $d'(W) < 0$. This latter assumption is consistent with the idea that high (physical) work effort is easier to sustain over time for better nourished people as Dasgupta (1997) indicates.[8]

Let $V(c_j, e; W)$ denote an agent's expected utility. Conditional on high effort, the slope of an agent's indifference curve in state-contingent income space is:

$$y_f'(y_s)|_{\overline{V}} = -\left(\frac{\phi^H}{1-\phi^H}\right)\left(\frac{u'(c_s)}{u'(c_f)}\right). \tag{8}$$

Indifference curves are downward sloping and convex to the origin. The marginal rate of substitution (MRS) between income under success and income under failure is decreasing, reflecting the desire of risk averse agents to smooth consumption across states. Note that when consumption is the same in the two states of the world (as it would be under full insurance or wage labor), the MRS is equal to $-\phi^H/(1-\phi^H)$. Finally, because a high initial wealth level would insulate the agent's utility from the success or failure of the entrepreneurial project, we assume that wealth levels are always low enough that the agent prefers high effort to low effort under the unlimited liability conditions of autarchic self-finance.[9]

Figure 1 above displays these indifference curves for two expected utility levels: V_1 that passes through the full insurance credit contract C''; and, V_0 that passes through the certain wage income

contracts and quantity rationing would never occur.

[8] Without this assumption, high wealth people, for whom the marginal utility of consumption is low, will have little incentive to supply high effort levels. As discussed in proposition 2 below, were this the case, collateral requirements for the optimal contract would increase faster than wealth itself, and high wealth agents would be quantity rationed in asymmetric information-constrained loan markets.

[9] Define $\Delta \equiv \phi^H - \phi^L$ and $B(W) \equiv [u(W + y_s^a) - (W + y_f^a)]\Delta$. $B(W)$ is the expected gain in utility that comes from choosing high instead of low effort. The agent will choose high effort under autarky if that gain exceeds the disutility of the higer effort, $B(W) > d(W)$. A sufficient condition which ensures that agents with wealth $W \in (\underline{W}, \overline{W})$ choose high effort is that the following three conditions hold: 1. $B(\underline{W}) > d(\underline{W})$; 2. $B'(W) \leq d'(W)\ \forall W$; and 3. $\overline{W} < \widehat{W}$ where $\widehat{W} : B(\widehat{W}) = d(\widehat{W})$. The first two conditions are essentially a single crossing property for the curves $B(W)$ and $d(W)$, while the third condition restricts the wealth levels considered to those to the left of the crossing point (if it exists).

option, L. To illustrate how risk rationing could occur, the indifference curves in this figure have been drawn with a relatively high degree of curvature. An agent whose preferences were represented by these bowed indifference curves would clearly prefer the entrepreneurial activity under the full insurance credit contract (with payoffs given at C'') to the wage labor contract. However, the agent would prefer the risk free wage labor activity to the entrepreneurial activity financed with the unlimited liability credit contract at point C, despite the higher returns of the latter activity. Such an individual would be risk rationed if the only available credit contract were the unlimited liability contract.

For subsequent analysis, it is useful to define the full insurance risk premium, $p(y_j; W)$, associated with the risky prospect, y_j, for a borrower of collateral wealth, W. Following Pratt (1964), the risk premium is implicitly defined by:

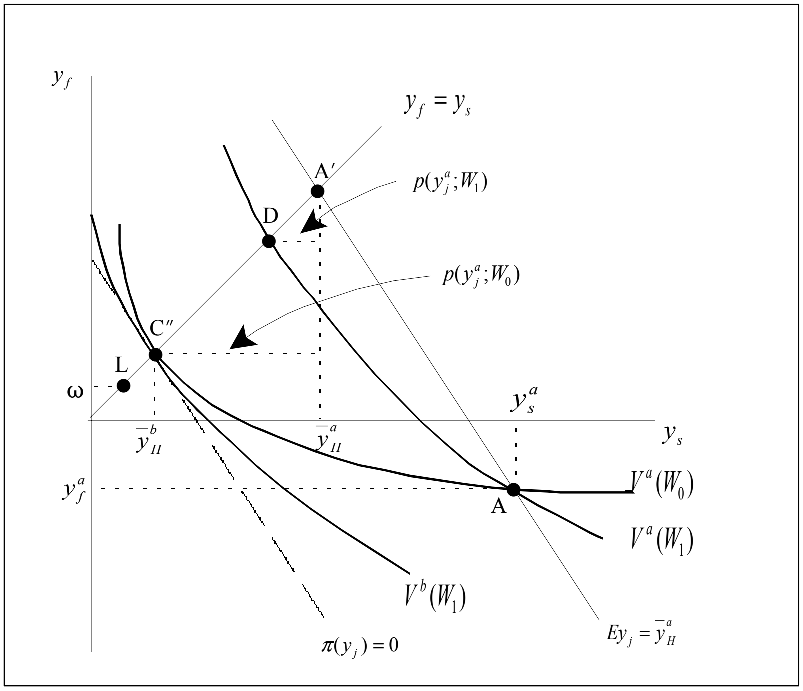
$$EU(W + y_j) = U[W + Ey_j - p(y_j; W)]. \tag{9}$$

The risk premium tells us how much certain consumption the agent is willing to give up to completely eliminate the risk associated with a given income prospect.

Figure 2 provides a graphical representation of the risk premium. $V^a(W_0)$ and $V^a(W_1)$ are the indifference curves through the self-finance option at point A for agents with two different wealth levels. As drawn, the agent with W_0 is indifferent between self-finance and risk free debt-finance under the full insurance credit contract. As such, $p(y_j^a; W_0) = \overline{y}_H^a - \overline{y}_H^b$, and for this agent the risk premium associated with self-finance just equals the finance cost of the full insurance contract $(r^b - r^a)\underline{K}$. In Figure 2, this risk premium is given by the horizontal distance between points C'' and A'.

The agent with wealth W_1, in contrast, strictly prefers self-finance. Equivalently, this agent's risk premium is smaller - represented by the fact that $V^a(W_1)$ crosses the full insurance line to the northeast of C''. If agents' preferences are described by decreasing absolute risk aversion, then it would follow that $W_0 < W_1$.

Figure 2: Symmetric information equilibrium and the self-finance risk premium



3 Price and Non-Price Rationing in Credit Markets

The nature of credit rationing, credit access and activity choice will depend on the set of contracts over state-contingent income space that are offered to agents. This section analyzes the set of contracts that lenders will make available under two informational scenarios. Under the benchmark case of symmetric information, the agent's choice of effort is costlessly observable to lenders and is contractible, meaning that loan contracts that specify (high) effort are enforceable. Under asymmetric information, the agent's choice of effort is a hidden action and contracts cannot meaningfully specify it. Under the assumptions made in Section 2, expected project returns conditional on low effort are negative. The only way a lender could expect to recover costs if low effort were chosen by the borrower would be if the contract offered the borrower negative expected income. Clearly such contracts would be of no interest to borrowers, so under symmetric information, lenders will only offer contracts that specify high effort.

Under asymmetric information, the lender cannot directly specify the agent's effort level and therefore must consider how contractual payoffs indirectly affect the agent's choice of effort. The lender will only offer contracts that are incentive compatible in the sense that their payoff structure makes it optimal for the agent to choose high instead of low effort.

3.1 Credit Markets under Symmetric Information

Under symmetric information a loan contract is a triplet, (s_s, s_f, e), that specifies state contingent borrower payoffs and an effort level. Lenders will only offer contracts specifying the high effort. In a competitive loan market the optimal contract, (s_s^*, s_f^*, H), maximizes the agent's expected utility while guaranteeing the lender non-negative expected profits. The payoffs of the optimal contract solve the following program:

$$\underset{s_s, s_f}{Max} \quad Eu(W + s_j | e = H) \tag{10a}$$

$$subject\ to: \quad \pi(s_j | e = H) \geq 0 \tag{10b}$$

$$-s_j \leq W; \quad j = s, f \tag{10c}$$

Constraints (10b) and (10c) are respectively the lender's zero profit or participation constraint and the agent's wealth or liability constraint. Note that the agent's payoff is not restricted to be non-negative. A negative payoff requires the borrower to hand over some of his assets and thus is equivalent to a collateral requirement.

Combining the first order necessary conditions for the above maximization problem yields:

$$\frac{\frac{\partial V}{\partial c_s}}{\frac{\partial V}{\partial c_f}} = -\frac{\phi^H}{1 - \phi^H} \tag{11}$$

The above expression states that, for a given expected income level, the optimal contract equates the agent's MRS of state contingent consumption with the ratio of the success to the failure probability. Recalling from Equation (8) that the agent's MRS equals $\frac{\phi^H}{1-\phi^H}\frac{u'(c_s)}{u'(c_f)}$, the only solution to Equation (11) for a risk averse borrower is a contract that equalizes consumption across states–*i.e.*, $s_f = s_s$. The optimal contract is thus at the intersection of the forty-five degree full insurance line and the lender's zero profit contour. In Figure 2, the optimal contract is at a point C'' with payoffs of $(\overline{y}_H^b, \overline{y}_H^b)$.

We now turn to the agent's choice among the three activities: debt-finance with the optimal contract, self-finance, and wage labor. Since the optimal contract provides full insurance, the MRS at this contract is independent of agent wealth. As such, the optimal contract itself is independent of agent wealth. Since the optimal contract is identical for all agents and yields a certain income of $\overline{y}_H^b > \omega$, debt-finance will always be strictly preferred to the wage activity.

Next consider whether agents prefer to finance the risky project with their own funds or with a credit contract. By choosing self-finance, producers avoid the finance cost of the credit contract – equal to $(r^b - r^a)\underline{K}$ – and thus earn a higher expected income than under debt-finance. However, self-finance implies greater risk. As depicted in Figure 2, let W_0 denote the wealth level such that an agent is indifferent between self-finance and debt-finance under the optimal, full insurance loan contract, *i.e.*, $p(y_j^a; W_0) = \overline{y}_H^a - \overline{y}_H^b$.[10] With this definition, the following proposition is straightforward to establish.

Proposition 1 *Under symmetric information, all agents undertake the high return, entrepreneurial project. If risk preferences are described by DARA (IARA), agents with wealth $W \leq W_0$ will debt (self) finance the risky project while those with wealth $W > W_0$ will prefer to self (debt) finance.*

Two final points are worth noting under symmetric information. First, if we assume that $\underline{K} < W_0$, then, under DARA, there will exist a class of agents with wealth $W : \underline{K} < W < W_0$ who are able to self-finance the project but instead choose debt-finance. For this group, $p(y_j^a; W) > \overline{y}_H^a - \overline{y}_H^b$, so that their willingness to pay to eliminate the risk of self-finance is greater than the finance cost of the credit contract. For these 'insurance seekers', the credit market provides a substitute (albeit imperfect) for the missing insurance market. Second, even with a perfect credit market, the absence of an insurance market implies a welfare loss. If actuarially fair insurance

[10] Note that risk premium associated with the full insurance loan contract, $p(\overline{y}_H^b; W_0)$, is zero.

were available, then all agents with wealth greater than $\underline{K}$ would purchase insurance, self-finance the project and earn the state independent income, $\overline{y}^b_H$, represented by point A' in Figure 2.

3.2 Credit Markets under Asymmetric Information

The agent's effort levels are non-contractible when information is asymmetric. Because loans can only be profitably made when borrowers choose high effort, lenders will only be willing to offer contracts that are incentive compatible in the sense that they yield the borrower higher expected utility under high than low effort. The difference in expected utility from choosing high versus low effort is:

$$\eta(s_s, s_f, W) = [u(W + s_s) - u(W + s_f)]\Delta - d(W), \tag{12}$$

where $\Delta = \phi^H - \phi^L$. An incentive compatible contract is an (s_s, s_f) pair such that

$$\eta(s_s, s_f, W) \geq 0. \tag{13}$$

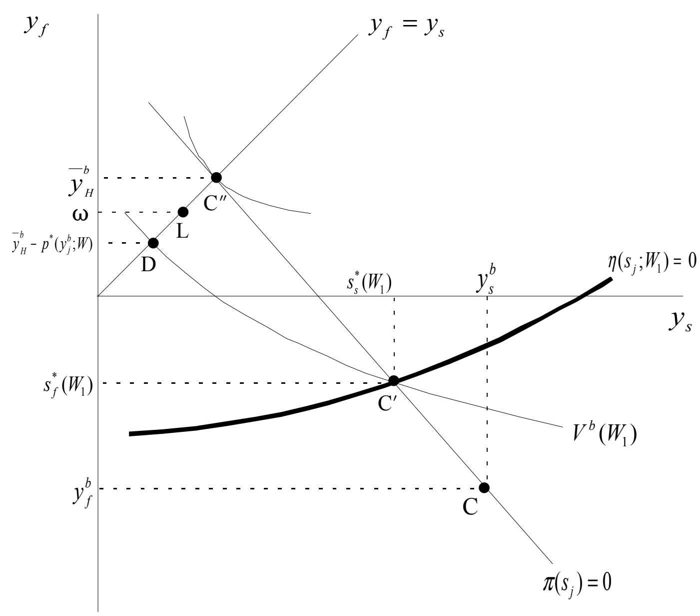
Expression (13) will be called the *incentive compatibility constraint.* Under asymmetric information, the optimal contract will be defined by the optimization program given in Equations 10a - 10c with addition of the incentive compatibility constraint.

Denote the locus defined by $\eta(s_j, W) = 0$ as the *incentive compatibility boundary (ICB).* This locus gives the set of contracts such that the incentive compatibility constraint just binds. Total differentiation of Equation 13 yields:

$$s'_f(s_s)|_{\overline{\eta}} = \frac{u'(c_s)}{u'(c_f)} \tag{14}$$

Equation 14 shows that the ICB is upward sloping with a slope less than unity, as shown in Figure 3. Concavity of the utility function implies that a \$1 increase in the success payoff requires a less than proportionate increase in the failure payoff in order to maintain a constant return to the agent's high effort. Points on or to the southeast of the ICB are incentive compatible; those to the northwest are not.

Figure 3: The Second Best Contract



Assuming for the moment that the wealth constraint does not bind, the positive slope of the ICB, combined with convexity of indifference curves, implies that the optimal contract is unique and occurs at the intersection of the lender's zero-profit contour and the ICB. The positive slope of the ICB also implies that the information-constrained, optimal contract will be the incentive compatible contract that requires the least collateral (*i.e.*, has the highest failure payoff and provides the greatest amount of insurance).

The curve labelled $\eta(s_j, W_1) = 0$ in Figure 3 shows the ICB for an agent with wealth level W_1. The constrained optimal contract for this agent occurs at point C'. While contracts between $C'$and C'' yield higher expected utility, the lender will not make them available because they are not incentive compatible. Asymmetric information has censored the menu of available contracts. For a given wealth level, an increase in the severity of the incentive problem implies that an agent must assume greater liability (risk) in a credit contract.[11] In Figure 3, a worsening incentive problem

[11] The severity of the incentive problem is given by the size of $d(W)/\Delta$. Thus the incentive problem is increasing

would be reflected in a downward shift of the ICB, further reducing the menu of loan contracts and moving the constrained optimal contract farther away from the full insurance contract, C' and towards the full lability contract at C.

3.3 Non-Price Rationing under Asymmetric Information

The censoring of the menu of available loan contracts under asymmetric information creates the potential for both quantity rationing and risk rationing. This section will show that both of these forms of non-price rationing can exist and that they can be wealth-biased in the sense that they will drive lower wealth individuals into the low return, wage labor activity.

A sufficient condition for a positive credit supply is that there is at least one contract that is both incentive compatible and yields non-negative lender profits when the agent pledges their entire wealth as colletaral. If, instead, there are no full collateral contracts that satisfy both of these constraints, then the feasible contract set will be empty and the agent will be quantity rationed. Proposition 2 states the conditions under which quantity rationing will occur and be wealth-biased.

Proposition 2 (Wealth-Biased Quantity Rationing) *Recalling that all agents have a wealth endowment between $\underline{W}$ and $\overline{W}$, then under DARA if:*

$$d(\underline{W}) > \Delta u \left(\frac{\underline{W} + \overline{y}_b^H}{\phi^H} \right) \text{ and} \tag{15}$$

$$d(\overline{W}) < \Delta u \left(\frac{\overline{W} + \overline{y}_b^H}{\phi^H} \right), \tag{16}$$

there there will exist a unique $W^* \in (\underline{W}, \overline{W})$ such that agents with collateral wealth less than W^* will have an empty feasible contract set and will be quantity rationed. Agents with wealth greater than or equal to W^* will have a non-empty feasible contract set.

Appendix A formally proves this proposition. The intuition behind the proposition is straightforward. If the added disutility of providing high effort is small enough for low wealth agents, then the low collateral contracts that are financially feasible for these agents will be incentive compatible and low wealth agents will not be quantity rationed. If, however, the added disutility

in the agent's private benefit of low effort and decreasing in the probability differential of success under high versus low effort.

of high effort exceeds the expression shown on the right hand side of inequality (15), then the minimum collateral requirement for an incentive compatible contracts exceeds the wealth levels of the poorest agents, and these agents will then be quantity rationed.

In addition, if the added disutility of high effort for high wealth agents is too high (greater than the right hand side of inequality (16)), then high wealth agents will be unable to provide the collateral needed for incentive compatible contracts and they too will be quantity rationed. If, however, inequality (16) holds, then there will be at least some higher wealth agents for whom incentive compatible contracts are financially feasible. If both inequalities hold, then there will be wealth-biased quantity rationing in the credit market under asymmetric information.

The asymmetric information induced truncation of the available menu of contracts that results in quantity rationing can also result in another form of non-price rationing that we have labelled as risk rationing, meaning that: (1) The agent would be offered and demand a credit contract in the symmetric information world; (2) The agent is offered a financially feasiable contract in the asymmetric information world; and, (3) The agent chooses not to accept the offered contract in the asymmetric information world, preferring the safe, wage labor activity.[12] Graphically, risk rationing is easy to portray. In Figure 3, the agent with wealth level W_1 would be risk rationed since the indifference curve passing through the optimal contract for wealth level W_1 crosses the full insurance line at point D to the southwest of point L (the certain payoff associated with the wage labor activity). Here we analyze the circumstances under which wealth-biased risk rationing occurs.

As a first step in this analysis, define the critical wealth level W^{**} as follows:

$$W^{**} : p(y_j^b; W^{**}) = \overline{y}_H^b - \omega, \tag{17}$$

where $p(y_j^b; W^{**})$ is the risk premium associated with the constrained optimal contract. An agent

[12] There is another group - that we might label "insurance rationed" - whose credit outcome is altered relative to the benchmark case. These agents are a subset of the "insurance seekers" who have sufficient wealth to self-finance the risky activity but prefer the full insurance contract. The reduction in the implicit insurance of the second best contract leads them to self-finance the risky activity under asymmetric information.

with wealth W^{**} is by definition indifferent between the safe wage activity and bank finance of the risky entrepreneurial activity. We will say that wealth biased risk rationing occurs if agents with wealth W such that $W^* < W < W^{**}$ prefer the low return wage activity over available debt finance (where W^* was defined in proposition 2 as the lowest wealth level where an incentive compatible contract is financially feasible).

More formally, wealth biased risk rationing will occur if (i) $W^{**} \in [W^*, \overline{W}]$ and; (ii) $dp^b(W)/dW < 0$. This first condition simply says that the agent who is indifferent between the optimal contract and wage labor occurs within the relevant range of the wealth continuum. The second condition (that the risk premium associated with the optimal contract be monotonically decreasing in wealth) insures that agents with wealth levels below W^{**} will strictly prefer the safe wage activity, while those with wealth greater than W^{**} will strictly prefer debt finance of the entrepreneurial activity. Proof of the existence of wealth biased risk rationing is made complicated by the fact that changes in wealth affect the optimal activity choice directly via risk preferences, and indirectly via shifts in the terms of the optimal contract (graphically, an increase in wealth will reshape both the ICB and the indifference curves). The following proposition establishes when conditions (i) and (ii) above hold and thus when wealth biased risk rationing will exist.

Proposition 3 (Wealth-Biased Risk Rationing) *Under asymmetric information and agent preferences described by DARA, $W^* < W^{**} < \overline{W}$ if*

$$h\left[\frac{\phi^H d(\underline{W})}{\Delta}\right] < \underline{W} + \omega < \overline{W} + \omega < h\left[\phi^H u\left(\frac{\overline{y}_H^b + \overline{W}}{\phi^H}\right)\right], \tag{18}$$

where $h() = u^{-1}$. The risk premium associated with the optimal contract will be monotonically decreasing in wealth ($dp(y_j^b;W)/dW < 0$) if and only if

$$\frac{\frac{\partial \sigma_s^2}{\partial W}}{\sigma_s^2} < \left|\frac{\frac{\partial \overline{R}}{\partial W}}{\overline{R}}\right| \tag{19}$$

where σ_s^2 is the variance of the optimal contract payoffs and $\overline{R}$ is the coefficient of absolute risk aversion evaluated at the riskless consumption level $W + \overline{y}_H^b$. A sufficient condition for inequality (19) to hold is that:

$$\frac{d'}{\Delta} \leq u_s' - u_f' \tag{20}$$

where u_j' denotes the marginal utility of consumption in state j resulting from the optimal contract. If the inequalities in both (18) and (20) hold, then wealth biased risk rationing will exist.

As shown in Appendix B, Equation 18 is a sufficient condition for the existence of W^{**}. This condition places lower and upper bounds on the certainty equivalents of the optimal contracts for the agents with wealth W^* and $\overline{W}$ respectively. Given the existence of W^{**}, inequalities (19) and (20) are necessary and sufficient conditions, respectively, for the risk premium to be decreasing in wealth (see Appendix B). Agents' risk premia depend upon their degree of risk aversion and the risk implied by their credit contract. Both of these components are functions of agent wealth. Under DARA, wealthier agents are less willing to insure against a *given* contract. If contract terms were independent of wealth, only agents with wealth greater than W^{**} would accept the risk of debt-finance.

The terms of the optimal contract are not, however, independent of agent wealth. As shown in Appendix B, the collateral requirement—and the riskiness of the optimal contract—will increase if $\Delta(u'_s - u'_f) < d'(W)$. On one hand, the utility differential under success versus failure is diminishing in wealth so that wealthier agents have less incentive to choose high effort. To counter this incentive effect and ensure that high effort is chosen, lenders would need to increase the difference between success and failure payoffs and thereby increase the contractual risk facing wealthier agents. The need for lenders to increase contractual risk is, at least partially, mitigated by the declining disutility of high effort for wealthier agents. The net result of the incentive versus the disutility effects of wealth is ambiguous and will depend on the nature of $d(.)$. If the incentive effect dominates, lenders will raise the collateral requirement for wealthier agents. In contrast, if the decreasing disutility of effort effect dominates, then wealthier agents will be offered lower collateral, lower risk contracts. A dominant effort disutility effect is a sufficient condition for wealth biased risk rationing (inequality 20). The necessary condition stated in inequality (19) is that contractual risk must not increase "too much" with agent wealth. More precisely, wealth biased risk rationing occurs if the percentage *change* in the variance in contract payoffs is less than the absolute value of the percentage *decrease* in risk aversion resulting from a unit increase in agent wealth.

4 The Economics of Risk Rationing: Wealth, Optimal Contracts and Activity Choice

The theoretical model developed in this paper has shown that by shrinking the available menu of loan contracts, asymmetric information can result in two sorts of wealth-biased, non-price rationing in credit markets. The first is conventional quantity rationing in which a subset of low-wealth agents find that there is no contract that is made available to them because they lack the minimum collateral necessary to secure a loan. The second is what this paper has labelled risk rationing. Risk rationed agents are able to borrow, but only under relatively high collateral contracts that offer them lower expected well-being than does a safe, wage labor activity. This latter effect is particularly relevant in developing countries where insurance markets are scarce and risk averse agents may seek credit contracts both to overcome liquidity constraints and to obtain insurance against production or price shocks. But when faced with the offer of only a high collateral contract that places their asset base at risk, risk rationed agents choose a safer, lower return activity than they would choose in a symmetric information world. Like quantity rationed agents, the risk rationed are a class for whom decentralized credit markets do not perform well.

From a theoretical perspective, this paper's analysis of optimal credit contracts under risk aversion and asymmetric information suggests several extensions. First, the model could be extended to incorporate the various means by which borrowers and lenders overcome information asymmetries. For example, under monitored lending, the agent's effort level is monitored–either by the lender or by other agents in a group lending scheme–and a penalty is imposed if the agent deviates from the agreed upon effort level.[13] Extending the model in this direction could help explain the frequently observed coexistence of multiple institutional forms of credit delivery. A second and related theoretical extension would be to reconsider the role and logic of informal or

[13] Conning (1996, 1999), for example, has taken initial strides along this line by developing a model which endogenizes the level of monitoring and institutional form under moral hazard for individual credit contracts. Besley and Coate (1995) and Armendariz de Aghion (1999) develop models of endogenous monitoring under group lending.

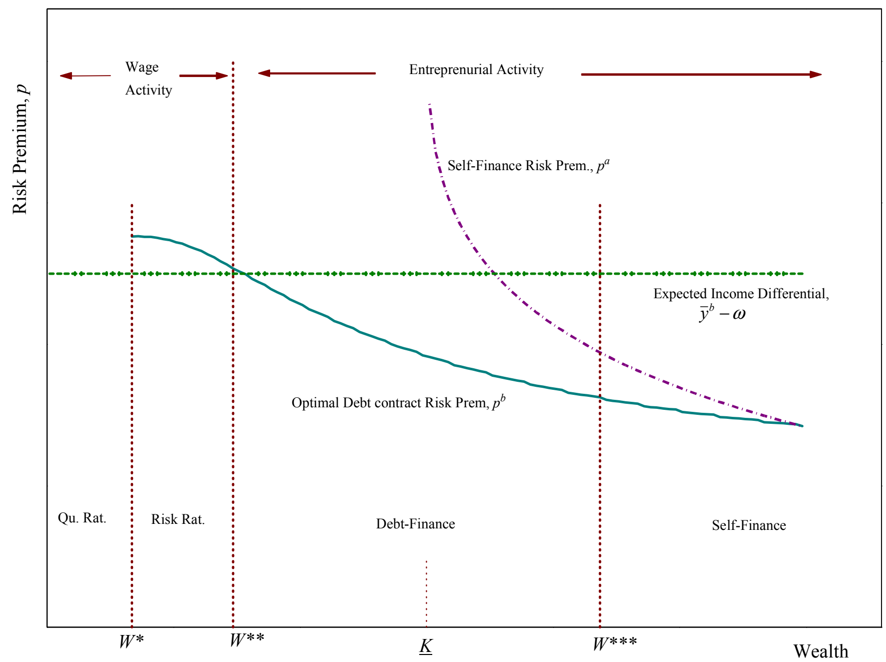
local lenders who are less subject to information asymmetries. These lenders may be able to offer contracts with greater implicit insurance than formal sector contracts. This is consistent with empirical observation that informal lenders rarely require collateral. Even if informal contracts are more expensive in terms of the expected value of loan repayment, agents may prefer them for their implicit insurance.[14]

From an empirical perspective, this paper's analysis suggests that studies that fail to take risk rationed agents into account will overestimate the health of the financial system. While the importance of risk rationing, and the health of the financial system, is ultimately an empirical question, the potential for risk and quantity rationing to create a world in which low wealth agents are systematically excluded form high return entrepreneurial activities can be illustrated by numerically exploring the model developed here. Under the functional forms and parameters given in Appendix C, Figure 4 displays the mapping between wealth, capital access and activity choice under decreasing absolute risk aversion. Agents with wealth less than W^* lack the minimum collateral required to obtain any loan contract. Agents with wealth between W^* and W^{**} have contracts available to them, but the optimal contract exposes them to excessive risk. The risk premium associated with the optimal debt contract, $p(y_j^b; W)$, exceeds the difference between expected income under the entrepreneurial activity and the certain wage activity, $\overline{y}_H^b - \omega$. Like quantity rationed agents, these risk rationed agents undertake the safe, low return wage labor activity.

Beyond wealth level W^{**}, the risk premuim associated with the optimal contract becomes less than the difference between wage and expected entrepreneurial income, and agents accept loan contracts and undertake the entrepreneurial activity. At wealth level $\underline{K}$, agents have enough private wealth to self-finance the entrepreneurial activity, but choose the limited liability of debt

[14] Previous research has tended to view the informal credit market in one of two ways. On one hand, the informal sector is portrayed as the recipient of "spillover" demand from the formal sector ((Bell, 1990), (Bell et al., 1997)). In this view, farmers that are quantity rationed in the formal sector have no alternative except to turn to informal lenders for their credit demand. Alternatively, borrowers may prefer the informal sector to the formal sector because the low transaction costs in the informal sector make it the lowest *effective* cost credit source ((Kochar, 1997), (Chung, 1995)). The consideration of risk presented here raises another interpretation of the informal sector.

Figure 4: Rationing Regime, Risk Premia and Activity Choice



finance. Finally, beyond wealth level, W^{***}, agents choose to self-finance the entrepreneurial activity as the insurance component of the optimal contract is worth less than the cost that must be paid for it in terms of higher interest costs (and lower expected income).[15]

As in Eswaran and Kotwal (1990), initial wealth and activity choice become tightly wedded in the analysis here. Fully endogenous quantity and risk rationing lead the initially poor to choose safe wage labor activity. Those with more favorable initial wealth endowments become entrepreneurs. The expected rate of return on wealth and labor resources will thus be positively related to wealth (as in Bardhan, Bowles and Gintis, 1998), and in the face of moral hazard-constrained credit markets, both class structure and income inequality will tend to reproduce themselves over time.

[15] Formally, W^{***} is such that $p(y_j^a; W^{***}) - p(y_j^b; W^{***}) = \overline{y}_H^a - \overline{y}_H^b$.

Appendix A. Proof of Proposition 2

The proof consists of two steps. First, we describe W^* and show that it is the minimum collateral requirement necessary for a non-empty feasible set. Second, we show that Equations 15 and 16 are necessary and sufficient for the existence of W^*. Define the following success payoffs:

$$s_s^{min}(W): \quad \eta(s_s^{min}, -W; W) = 0 \tag{21a}$$

$$s_s^{max}(W): \quad \pi(s_s^{max}, -W) = 0 \tag{21b}$$

The contract $(s_s^{min}, -W)$ is the full liability contract at which the incentive compatibility constraint (ICC) binds. The contract $(s_s^{max}, -W)$ is the full liability contract which just meets the lender's participation constraint (PC). An agent with wealth W will have a non-empty feasible set if and only if $s_s^{min}(W) \leq s_s^{max}(W)$. Assume not, ie. that $s_s^{min}(W) > s_s^{max}(W)$. Any contract that satisfies the agent's wealth constraint (WC) and the ICC requires $s_s \geq s_s^{min}(W)$ and thus would violate the PC. Similarly, any contract that satisfies WC and PC requires $s_s \leq s_s^{max}(W)$ and thus would violate ICC. Thus, there would be no contracts which simultaneously satisfy all three constraints.

W^* is the wealth level such that all three constraints bind, and is defined by: $s_s^{min}(W^*) = s_s^{max}(W^*)$. From Equation 21a, $\frac{\partial s_s^{min}}{\partial W} = \frac{h'[d(W)/\Delta]d'(W)}{\Delta} - 1 < 0$ where $h(.) = u^{-1}$ and from Equation 21a, $\frac{\partial s_s^{max}}{\partial W} = \frac{1-\phi^H}{\phi^H} > 0$. The monotonicity of these two derivatives implies that, if W^* exists, then any agent with collateral wealth $W < (>) W^*$ will have an empty (non-empty) feasible contract set.

We now take up the issue of the existence of W^*. From the above argument, it is clear that if the poorest agent is *not* quantity rationed, then no agent will be quantity rationed. Similarly, if the wealthiest agent *is* quantity rationed, then all agents will be quantity rationed. Thus we need to see if parameter values exist such that the poorest agent is quantity rationed and the wealthiest is not. Begin by finding the minimum disutilty of high effort such that the lowest wealth agent is quantity rationed. This value, which we denote as $\underline{d}^*$, satisfies: $s_s^{min}(\underline{W}; \underline{d}^*) = s_s^{max}(\underline{W}; \underline{d}^*)$.

Using Equation 21a and recalling that $u(0) = 0$ we have:

$$s_s^{min}(\underline{W}; \underline{d}^*) = h(\underline{d}^*/\Delta) - \underline{W} \tag{22}$$

Similarly, using 21b, it is easy to show that:

$$s_s^{max}(\underline{W}; \underline{d}^*) = \frac{\overline{y}_H^b}{\phi^H} + \frac{(1-\phi^H)\underline{W}}{\phi^H} \tag{23}$$

Combining equations 22 and 23 yields:

$$\underline{d}^* = \Delta u \left(\frac{\overline{y}_H^b + \underline{W}}{\phi^H} \right). \tag{24}$$

Next we find the maximum disutility of high effort such that the wealthiest agent is *not* quantity rationed. This value, denoted by $\overline{d}^*$, is defined by: $s_s^{min}(\overline{W}; \overline{d}^*) = s_s^{max}(\overline{W}; \overline{d}^*)$. Using similar algebra, it is easy to show that:

$$\overline{d}^* = \Delta u \left(\frac{\overline{y}_H^b + \overline{W}}{\phi^H} \right). \tag{25}$$

If inequalities 15 and 16 from Proposition 2 hold, then the poorest agent is quantity rationed and the wealthiest is not and, by the above monotonicity argument, W^* must lie between $\underline{W}$ and $\overline{W}$.

Appendix B. Proof of Proposition 3

The proof consists of two steps. First, assume that W^{**} exists and show that Equations 19 and 20 are necessary and sufficient, and sufficient conditions for risk rationing. Second, show that Equation 18 is a sufficient condition for the existence of W^{**}.

Let p^b be the risk premium associated with the constrained optimal contract. It is straightforward to show that a local approximation to the risk premium associated with this contract is:

$$p^b\left(s_j^*(W); W\right) \approx \frac{\overline{R}\sigma_s^2}{2}. \tag{26}$$

Differentiation of Equation 26 with respect to wealth yields:

$$\frac{dp^b}{dW} \approx \frac{1}{2}\left[\sigma_s^2 \frac{\partial \overline{R}}{\partial W} + \overline{R}\frac{\partial \sigma_s^2}{\partial W}\right]. \tag{27}$$

Wealth biased risk rationing requires $dp^b/dW < 0$ which, by inspection of Equation 27 is equivalent to Equation 19 in Proposition 3.

Under DARA, the first term in square brackets in Equation 27 is negative. The second term may be either positive or negative depending on the relative sizes of the incentive versus the disutility of effort effects of wealth on contract terms. To see this, note that $\sigma_s^2 = \phi^H(1-\phi^H)(s_s^* - s_f^*)^2$. Differentiation with respect to agent wealth yields:

$$\frac{\partial \sigma_s^2}{\partial W} = 2\phi^H(1-\phi^H)\left[\frac{\partial s_s^*}{\partial W} - \frac{\partial s_f^*}{\partial W}\right]. \tag{28}$$

A binding PC implies $\partial\sigma_s^2/\partial W = -2(1-\phi^H)\partial s_f^*/\partial W$, so that an increase in the collateral requirement (decrease in s_f) implies an increase in contract risk. A comparative statics exercise reveals that:

$$\frac{\partial s_f^*}{\partial W} = -\frac{\phi^H\left[\Delta(u_s' - u_f') - d'\right]}{\Delta\left[\phi^H u_f' + (1-\phi^H)u_s'\right]}. \tag{29}$$

Thus, the collateral requirement will decrease if $\frac{d'}{\Delta} \leq u_s' - u_f'$, or if the disutility of high effort decreases more than the utility differential.

We now take up the existence of W^{**}. Assuming that the risk premium is monotonically decreasing in wealth (ie. that Equation 19 holds), then W^{**} will exist if the agent with the minimum collateral requirement, W^*, prefers the wage activity to their contract while the wealthiest agent prefers their credit contract to the wage activity. Consider each of these requirements in turn. The first requirement is that:

$$u(W^* + \omega) > \phi^H u\left(W^* + s_s^*(W^*)\right) + (1-\phi^H)u\left(W^* + s_f^*(W^*)\right). \tag{30}$$

From Proposition 2, W^* is such that: $s_s^*(W^*) = s_s^{min}(W^*)$ and $s_f^*(W^*) = -W^*$. And, by definition, $s_s^{min}(W) = h\left[\frac{d(W)}{\Delta}\right] - W$. Then we can rewrite Equation 30 as:

$$u(W^* + \omega) > \frac{\phi^H d(W^*)}{\Delta}. \tag{31}$$

Note that $W^* > \underline{W}$ implies that $u(W^* + \omega) > u(\underline{W} + \omega)$ and $\frac{\phi^H d(\underline{W})}{\Delta} > \frac{\phi^H d(W^*)}{\Delta}$. Thus a sufficient condition for Equation 30 to hold is that $u(\underline{W} + \omega) > \frac{\phi^H d(\underline{W})}{\Delta}$, which is equivalent to the first inequality of Equation 18 of Proposition 3. This sufficient condition says that the poorest agent prefers wage labor to the full liability contract on their ICB (of course this contract would not be available since it does not satisfy PC).

The second requirement is that:

$$u(\overline{W}+\omega) < \phi^H u(\overline{W}+s_s^*(\overline{W})) + (1-\phi^H)u(\overline{W}+s_s^*(\overline{W})). \tag{32}$$

For agents with $W > W^*$ we know that the contract $(s_s^{max}(W), -W)$ yields lower expected utility than the optimal contract. Thus a sufficient condition for Equation 32 to hold is that the wealthiest agent prefers this full liability contract on the lender's PC to wage labor. Using the definition of s_s^{max}, this sufficient condition reduces to:

$$u(\overline{W}+\omega) < \phi^H u\left[\frac{\overline{y}_H^b + \overline{W}}{\phi^H}\right]; \tag{33}$$

which is equivalent to the final inequality of Equation 18 of Proposition 3.

Appendix C. Functional Form and Parameters

Production and Risk:

Certain Wage income	$w = 25$
Gross entrepreneurial incomes:	$y_s = 100;\ y_f = 0$
Success Probabilities:	$\phi^H = 70\%;\ \phi^L = 25\%$
Capital investment requirement:	$\underline{K} = 15$
Interest rates:	$r_a = 5\%;\ r_b = 30\%$
Expected net entrepreneurial income:	$\overline{y}_H^b = 50.5;\ \overline{y}_H^a = 54.25$

Utility:

$$u(C) - d(W) = c^b - \left(\overline{d} + (\frac{d_1}{1+\frac{W}{d_2}})\right);\ b = 0.5,\ \overline{d} = 0.35;\ d_1 = 1.6;\ d_2 = 5.$$

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