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ESTIMATION OF A U.S. DAIRY SECTOR MODEL BY MAXIMUM SIMULATED LIKELIHOOD

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Abstract:

This paper estimates a multivariate Tobit system of monthly wholesale dairy prices where 4 prices are lower censored by the dairy price support program. Using Maximum Simulated Likelihood (MSL) we test/correct for the effects of simulation noise and discuss the relevance of estimating multivariate versus the single Tobit equations

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Estimation of a US Dairy Sector Model
By Maximum Simulated Likelihood

I. Introduction

There are many economic examples of systems of equations with censored endogenous variables. Among these examples are systems of demand equations in which some consumers report zero consumption of a given commodity (Wales and Woodland, 1983; Lee and Pitt, 1986) and labor supply models (Heckman, 1974). Another example, not often studied, is a system of input demand equations in which some producers choose not to use several inputs at all (Lee and Pitt, 1984).

The econometric treatment of censored endogenous variables in a single equation setting can be traced back to Tobin (1958). In the case of a system of equations with multiple censored variables, it is necessary to take account both of the censoring and the simultaneity. If the endogenous variables in the system are generated simultaneously, it is unlikely that the disturbances of the respective equations are independent. Therefore, there is some efficiency gain from estimating the equations as a system. Of course, the argument in favor of the estimation of the equations as a system is stronger if the theory predicts cross equation restrictions for the parameters.

The estimation of systems of equations with censored endogenous variables has been analytically treated in the literature (Nelson and Olson, 1978; Lee, 1981; Amemiya, 1985). Maddala (1983) and Pudney (1989) present an accessible and thorough discussion of the matter and the problems that remain are basically computational. The likelihood function of the system contains several definite integrals of dimensions that range from one to the number of censored variables in the system. The evaluation of such integrals is not a trivial task for the case of more than three censored variables under the common assumption that the disturbances of the model are distributed multivariate normal. This difficulty remains for any multivariate distribution that, like the multivariate normal, does not have a closed form integral.

The literature on estimation of models with multiple censored variables has searched for alternatives to the evaluation of multiple definite integrals without a closed form solution. Pudney (1989) proposes the use of alternative specifications for the distribution of the disturbances in the system such as the Generalized Extreme Value (GEV) distribution that has integrals with a closed form solution. However, these distributions have rarely been used in empirical work because they impose undue restrictions on the data.

A second solution is to aggregate the censored dependent variables. Aggregation of several censored variables can help to estimate the models in two ways. First, by aggregating, one will be dealing with a smaller number of censored variables. Second, sometimes the censoring disappears altogether with aggregation. The problem is that aggregation can hide some important characteristics of the model and even make the estimation useless for some research objectives.

If these alternative specifications have undesirable properties, then the maximum likelihood estimation of systems of simultaneous equations with multiple censored variables requires a method for evaluating a multiple integral without a closed form solution such as the multivariate normal density function. These two approaches (restrictive specification with a closed form solution versus a more general specification that requires a numerical solution) reflect an ongoing methodological problem in applied economics. Often, we accept quite restrictive assumptions in order to get closed form solutions for the model. In contrast, an alternative, less restrictive specification would lead to a model requiring a numerical solution. Developments in computational hardware and software have fostered the increasing use of numerical methods in many areas of economic research.

Quadrature methods figure prominently among the early numerical solution alternatives to compute the censored, multivariate expectations associated with these problems. Despite many improvements of the initial idea, quadrature methods are still relatively expensive in terms of computer time. Therefore, they are not considered useful for estimating systems of simultaneous equations in which more than three endogenous variables are censored. As an alternative, probability simulation methods evaluate multidimensional integrals keeping a good balance between computation costs and accuracy. The estimation by maximum likelihood where probabilities are simulated rather than calculated is known as Maximum Simulated Likelihood (MSL). One of these probability simulation methods, the Geweke-Hajivassiliou-Keane simulator, is applied in the empirical example estimated in the present paper.

The structure of the paper is as follows. Section II. Section III presents a conceptual model that describes price formation of dairy commodities at the wholesale level in the U.S. In Section IV we present a probability simulation method for the evaluation of high order definite integrals in maximum likelihood estimation of a model with several interrelated censored variables. Section V presents the results of an empirical application of this method to a censored, reduced form system of U.S. dairy prices. Lastly, Section VI provides conclusions and suggestions for further research.

II. The Empirical Context: Multiple Censored US Dairy Prices

The empirical part of this paper estimates a US dairy sector model. There is an extensive literature that analyzes the effects of dairy policies, including Dixon et al. (1991), Liu et al. (1991), Bausell et al. (1992), Cornick and Cox (1994) and Helmberger and Chen (1994). Dixon et al. (1991) evaluated the effects of the Milk Diversion Program and the Dairy Termination Program using a two equation dairy model. Bausell et al. (1992) examined the effects of the Dairy Termination Program and lower support prices using a model with a single aggregate dairy product. This model is estimated using quarterly data. Liu et al. (1991) modeled both the wholesale and retail dairy markets simultaneously. At the same time, they explicitly model government intervention through the price support program. They consider two dairy commodities: fluid milk and aggregated of manufactured products. Helmberger and Chen (1994) analyze the welfare effects dairy programs using an aggregate farm level market model.

These models rely on the aggregation of different dairy products into one or two commodities. Cornick and Cox (1994) analyze the methodological issues raised when several dairy commodities are considered in the model. Of particular interest is the fact that several dairy prices are lower censored by the U.S. dairy support program. In a different study, Cornick (1993) estimates a system of simultaneous equations with three censored endogenous variables using quarterly data. These papers show that modeling the US dairy sector with several interrelated markets at the farm, wholesale and retail levels is a complicated task. The fact that the dairy sector is highly regulated adds even more complexity to the analysis.

The present empirical application expands on the research of Cornick (1993) and Cornick and Cox (1994). However, by using MSL we are able to deal model the US dairy sector model that contains four censored price equations. Another improvement is the use of monthly data of the U.S. dairy sector. Monthly data provides additional degrees of freedom and additional censoring due to the government intervention in the US dairy sector.

Historically, the government, through the price support program and federal milk marketing order system (FMMO), has determined a floor price for farm level manufacturing milk and supported this price through purchasing any quantity of butter, cheese and nonfat dry milk (NFDM) at fixed prices. This governmental intervention implies that the prices of these three dairy products and manufacturing milk are endogenous variables for market prices above the government intervention price and exogenous variables below that price. Therefore, the prices of these three dairy products are lower censored endogenous variables. The econometric censoring problems described earlier appear in this model and therefore provide a good example in which to apply the probability simulation technique mentioned earlier.

Figures 1-4 summarize this price seasonality and censoring in the prices for American cheese, butter, nonfat dry milk, and manufacturing milk, respectively. These figures highlight several important characteristics of US wholesale dairy prices. First, the price supports and import quotas historically have maintained U.S. dairy prices well above world market levels, especially prior to the late 1980's/early 1990's. As well, commodity prices prior to the late 1980's were remarkably stable around the support prices with modest seasonal increases above support for cheese (Figure 1) and butter (Figure 2). Since the late 1980's, butter support has been lowered toward world market prices and considerable seasonal volatility has developed in cheese (Figure 1) and NFDM (Figure 3) prices. The associated impacts on manufacturing milk prices are striking (Figure 4) with considerable seasonal price volatility above the support level since 1988. As well, government price floors have been considerably less binding over this latter period. This partly explains why dairy farmers agreed to the removal of government dairy price supports in the 1996 FAIR Act (Dobson, 1999).

Table 2 summarizes the government intervention induced censoring in key US dairy prices over the January 1985 through December 1994 period. Of the total 117 monthly observations, almost half (57 observations, 49%) display some censoring. Most of this monthly censoring involves only one of the 4 prices (32 observations, 27%), with 15% (17 observations), 3% (3 observations), and 4% (5 observations) involving 2, 3, and 4 of the prices, respectively.

Table 2. Summary of Monthly Censoring by Regime.

Regime definition***						
Butter	**Cheese**	**NFDM**	**Class III**	**Number of Binding Prices**	**Number of Observations**	**Regime % of Total**
nb	nb	nb	nb	0	60	51%
b	nb	nb	nb	1	18	15%
nb	b	nb	nb	1	12	10%
nb	nb	b	nb	1	2	2%
nb	nb	nb	b	1	0	0%
b	b	nb	nb	2	11	9%
b	nb	b	nb	2	0	0%
nb	b	b	nb	2	3	3%
b	nb	nb	b	2	0	0%
nb	b	nb	b	2	2	2%
nb	nb	b	b	2	1	1%
b	b	b	nb	3	3	3%
b	b	nb	b	3	0	0%
b	nb	b	b	3	0	0%
nb	b	b	b	3	0	0%
b	b	b	b	4	5	4%
				Total	117	100%

nb: the support price of this commodity is not binding.
b: the support price of this commodity is binding.

Careful analysis of U.S. dairy prices is increasingly important due to changes in dairy policy towards less market intervention (Cox and Sumner, 1996), increasing interdependence with the global dairy markets, increasing price volatility and the associated emergence of futures and options markets for dairy products (Jesse and Cropp, 1993). Econometric analysis of dairy prices can provide relevant information about the behavior of the conditional mean and conditional variance of prices that are particularly useful in dairy options pricing.

III. A Model of Wholesale Dairy Commodities Prices

The wholesale prices of dairy commodities display the following important features: the prices are functions of a set of exogenous variables; wholesale dairy prices of several commodities are lower censored; and, the dairy prices are simultaneously determined. These suggest that a system of multiple censored dependent variables may be required to properly estimate this system.

The price equations for each commodity can be represented by:

$$p_i^* = x_i \beta_i + u_i \quad i = 1, \ldots, m \tag{1}$$

where p_i^* is a latent variable, x_i is a vector of exogenous variables and u_i is a random disturbance with mean zero and variance σ_i^2. The relationship between the latent variable p_i^* and the price of the commodity can be represented by:

$$\begin{aligned} p_i &= p_i^* \quad \text{if } p_i^* > \bar{p}_i \\ p_i &= \bar{p}_i \quad \text{otherwise} \end{aligned} \tag{2}$$

where, p_i is the price of commodity i and $\bar{p}_i$ represents the price support level for commodity i.

Expression (1) describes the fact that the price of each commodity depends on a set of exogenous variables while expression (2) models the censoring process due to the existence of the U.S. dairy price support program. The model is general enough to apply to the pricing mechanism of commodities that are not subject to the price support program. In that case, the price support is zero and the first equation in expression (2) always holds.

Each price in the model has a "Tobit" structure and the estimation of this model is straightforward. However, since the prices are determined simultaneously, the random shocks of the model are likely to be correlated. If that is the case, there are efficiency gains to be derived from estimating the equations in (2) as a system. There are not many papers in the literature which estimate a system of Tobit equations. A few examples include Hajivassiliou (1993), Cornick et al.

(1994) and Feenberg and Skinner (1994).

The likelihood function of the system of equations in the case in which all prices are above the censoring levels is given by:

$$L_1 = f(u_1,\ldots,u_m), \tag{3}$$

where f is the probability density function of a multivariate normal function with mean zero and variance Ω. The likelihood function for an observation in which the n first prices out of m are censored is:

$$L_2 = \int_{-\infty}^{\bar{p}_1 - x_1 b_1} \cdots \int_{-\infty}^{\bar{p}_n - x_n b_n} f(u_1,\ldots,u_m)\,d\,u_1,\ldots,d\,u_n. \tag{4}$$

Expression (4) represents a portion of the likelihood function with an n-dimensional definite integral. Under the common assumption of multivariate normality of the disturbances of the system this integral does not have a closed form solution. Therefore, estimating the system of prices by maximum likelihood requires an efficient method for evaluating the high dimensional definite integrals. For that reason, in section 3 we analyze available methods for evaluation for such definite integrals.

IV. Probability Simulation Methods versus Quadrature Methods for the Evaluation of High Dimensional Definite Integrals

Among the numerical methods used to approximate the value of a definite integral are the quadrature methods. In these numerical techniques, the integrand is substituted by an approximating polynomial of degree k. For the one-dimensional case, the interpolation of this approximating polynomial needs k+1 evaluations of the integrand. The accuracy of the approximation increases with the order of the polynomial.

However, the number of evaluations increases exponentially with the dimension of the integral. There are some results from numerical analysis that can be used to reduce the number of evaluations needed for a given level of accuracy (Judd, 1996) but, in any case, the computing costs of quadrature methods increases very fast with the dimension of the problem.

For high dimensional problems, the probability simulation methods are an alternative to the more costly quadrature methods. These methods are based on the fact that the integral of interest represents the probability of an event in a population. Lerman and Manski (1981) propose generating a pseudo-random sample of observations from the relevant population and using the relative frequency of the event in the sample to approximate the integral of interest. This simulation method is called a "crude frequency simulator". This method is intuitive and helps to introduce the idea of probability simulation but the simulated probability is not a smooth function of the parameters. As a consequence, it is necessary to use a large number of draws to obtain a reasonable level of accuracy. In addition, this simulation method requires the use of special optimization algorithms in maximum likelihood estimation.

The "crude frequency simulator" was improved in several subsequent papers. Stern (1992) explains the importance of smoothness in a probability simulator and proposes a smooth alternative to the "crude frequency simulator". Geweke (1989) and Borsh-Saupan and Hajivassiliou (1993) proposed the "Smooth Recursive Conditioning" (SRC) simulator. In more recent papers the SRC simulator is called the Geweke-Hajivassiliou-Keane simulator (GHK).

The literature on probability simulation has expanded in the last few years. Hajivassiliou et al. (1996) review a number of available probability simulators. They find that the GHK probability simulator outperforms all other methods by keeping a good balance between accuracy and computational costs. This probability simulator is relatively simple to program for different dimensions of the problem and it can be generalized to any distributional assumption.

The GHK computes the value of the integral:

$$\Pr(\mathbf{a} < \mathbf{u} < \mathbf{b}) = \int_{\mathbf{a}}^{\mathbf{b}} g(\mathbf{u})d\mathbf{u} \tag{5}$$

where, $\mathbf{u}$ is a random vector distributed multivariate normal with mean $\mathbf{0}$ and variance $\mathbf{W}$ and g is the density function of the random vector $\mathbf{u}$. The starting point is that:

$$\Pr(\mathbf{a} < \mathbf{u} < \mathbf{b}) = \Pr(\mathbf{a} < \mathbf{Le} < \mathbf{b}) \tag{6}$$

where, $\mathbf{L}$ is the lower triangular Cholesky factor of $\mathbf{W}$, such that $\mathbf{LL'}=\mathbf{W}$, and $\mathbf{e}$ is a random vector of independent standard normal variables. The right hand side of expression (6) is easier to simulate than the probability in the left hand side due to the triangular structure of the constrains defined by **Le**.

The intervals defining the event in the right hand side of expression (6) can be written as:

$$
\begin{aligned}
& a_1 < l_{11} e_1 < b_1 \\
& a_2 < l_{12} e_1 + l_{22} e_2 < b_2 \\
& \ldots \\
& a_n < l_{1n} e_1 + \ldots + l_{nn} e_n < b_n
\end{aligned}
\qquad (7)
$$

where l_{ij}, a_i and b_i are the corresponding elements of L, **a** and **b**. For notational convenience, arranging terms in (7) the event in the right hand side of equations (6) can be decomposed into the following events:

$$
\begin{aligned}
& A_1 = \left\{ \frac{a_1}{l_{11}} < e_1 < \frac{b_1}{l_{11}} \right\} \\
& A_2 = \left\{ \frac{a_2 - l_{12} e_1}{l_{22}} < e_2 < \frac{b_2 - l_{12} e_1}{l_{22}} \right\} \\
& \ldots \\
& A_n = \left\{ \frac{a_n - l_{1n} e_1 - \ldots - l_{n-1n} e_{n-1}}{l_{nn}} < e_n < \frac{b_n - l_{1n} e_1 - \ldots - l_{n-1n} e_{n-1}}{l_{nn}} \right\}
\end{aligned}
\qquad (8)
$$

Expression (8) shows the recursive nature of the constraints that affect the random vector **e**. As a result, the probability of interest can be written as:

$$\Pr(a < Le < b) = \Pr(A_1)\Pr(A_2 | A_1)\Pr(A_3 | A_1, A_2)\ldots\Pr(A_n | A_1, \ldots, A_{n-1}) \qquad (9)$$

The idea behind the GHK simulator is that while expression (9) can be difficult to calculate directly, it might be relatively easy to simulate instead. To see this, the GHK simulator can be written as:

$$\tilde{\Pr}(a < Le < b) = \frac{1}{R} \sum_{r=1}^{R} \Pr(A_1)\Pr(A_2 | e_{1r})\Pr(A_3 | e_{1r}, e_{2r})\ldots\Pr(A_n | e_{1r}, \ldots, e_{n-1r}) \qquad (10)$$

where the e_{ir}´s are drawn sequentially from independent standard normal distributions truncated by expression (8) and R is the number of simulations. Once the e_{ir}´s are drawn the terms in the product are calculated as:

$$\Pr(A_i \mid e_{1r}, e_{2r}, \ldots, e_{i\text{-}1r}) = \Phi\left(\frac{b_i - l_{1i} e_1 - \ldots - l_{i1i} e_{i1}}{l_{ii}}\right) - \Phi\left(\frac{a_i - l_{1i} e_1 - \ldots - l_{i1i} e_{i1}}{l_{ii}}\right) \qquad (11)$$

where Φ is the cumulative distribution function of a standard normal distribution function.

The truncated random variables e_i can be generated smoothly using the integral transform theorem (Ross, 1988). For example, If y is distributed as a standard normal subject to the constraint $c<y<d$ the cumulative distribution function of y can be written as:

$$G(y) = \frac{\Phi(y) - \Phi(c)}{\Phi(d) - \Phi(c)} = z \qquad (12)$$

where z is distributed uniformly in the interval [0,1]. From (12) we can obtain an expression that relates the truncated random variable y with the uniformly distributed random variable z:

$$y = \Phi^{-1}\left(\left(\Phi(d) - \Phi(c)\right)z + \Phi(c)\right) \qquad (13)$$

Borsch-Saupan and Hajivassiliou (1993) proved that the probability simulator in (10) is an unbiased estimator of the true probability. However, the logarithm of the simulator is not an unbiased estimator of the logarithm of the true probability and therefore the simulated likelihood function is not unbiased. As a consequence, the estimates of the parameters are biased due to simulation noise. The intuition behind this result is that simulation noise cancels out when adding up the likelihood contributions or its derivatives. However, this canceling does not occur when we deal with the logarithm (or any nonlinear function) of the simulated value.

Hajivassiliou (1997) proposes a test for bias generated by simulation noise in MSL estimation. The null hypothesis of the test is:

$$H_0\colon E\left[\frac{\partial \ln L(y, \tilde{\theta})}{\partial \theta}\right] = 0 \qquad (14)$$

against the alternative:

$$H_0:\ E\left[\frac{\partial \ln L(y,\tilde{\theta})}{\partial \theta}\right] \neq 0 \qquad (15)$$

where, L is a simulated likelihood function, y is a vector of observations and $\tilde{\theta}$ is the MSL estimate of the parameter of interest θ.The rejection of the null hypothesis is interpreted as evidence of bias due to simulation noise. The test relies on a simulation of the data generating process $y(\tilde{\theta})$ to calculate the empirical mean (m) and variance (v) of the score variable in (14) and (15). This simulation has to be independent of the simulation used for computation of the GHK simulator. The test can be written as:

$$w = NSm'v^{-1}m \qquad (16)$$

where N is the number of observation in the sample and S the number of simulations per observation. Under the null hypothesis, w is distributed chi-square with degrees of freedom equal to the number of parameters.

The bias created by simulation noise decreases with the number of simulations used for calculation of the GHK simulator. It is very important to distinguish between the number of simulations used to calculate the GHK simulator (denoted by R) and the number of simulations used to compute this test (denoted by S). If the proposed test permits rejection of the hypothesis of negligible bias due to simulation noise, the researcher can increase the number of simulations in the GHK simulator (R) until an acceptable value of w is obtained. The model is then re-estimated with the number of simulations required to generate negligible simulation noise bias.

V. Estimation and Results

An empirical application of the GHK probability simulation estimator to a system of U.S. wholesale dairy prices identified in Table 1 is presented next. This specification follows that of Cornick (1993) and Cornick and Cox (1994) using monthly rather than quarterly data. Monthly data provides additional degrees of freedom and additional censoring due to the government intervention in the US dairy sector. In particular, we analyze data from January 1985 - December

1994.

As discussed above, U.S. dairy price supports provide price floors on three products: American cheese, butter, and nonfat dry milk. As well, classified pricing under the Federal milk marketing orders (MMO) provides a floor price on the manufacturing milk price referred to as the Class III or basic formula price (BFP). These policy interventions provide 4 censored dependent variables in a system of reduced form wholesale price equations that can be used to characterize the US dairy sector (Cornick and Cox). Wholesale demand for ice cream (frozen) and other products completes the wholesale demand for milk. These prices, while not explicitly censored, are determined simultaneously with the other prices in Table 1 and their associated price censoring, policy interventions (Cornick and Cox).

Table 1. Endogenous variables.

VARIABLE	DESCRIPTION
WPB	Wholesale Price of Butter
WPCH	Wholesale Price of Cheese
WPNFDM	Wholesale Price of Nonfat Dry Milk (NFDM)
WPICE	Wholesale Price of Ice Cream
WPOTH	Wholesale Price of Other Products
BFP	Wholesale Price of Manufacturing Milk

The U.S. dairy processing sector is characterized by seasonality in milk production (higher in spring and summer, lower in the fall and winter) and an associated parallel seasonality in production and pricing of products with price supports (butter and nonfat dry milk, in particular). This seasonality is reasonably captured using monthly prices (see Figures 1-4 discussed above).

The Tobit equations for the prices of the censored wholesale dairy products following equations (1) and (2) are estimated by maximum likelihood using the likelihood function specified in equations (3) and (4). Since there are monthly observations with up to four censored endogenous variables, these observations require to simulate rather than calculated the maximum likelihood variable as was discussed in section 3.

The selection of exogenous variables follows Cornick and Cox (1994). The advertising expenditures, personal consumption expenditures, and population variables are demand shifters; the coefficients on these variables are expected to be positive. The wages are a measure of producer costs and are expected to have positive coefficients. The role of the producer price index can be seen as proxy for production costs. As a consequence the coefficient of this variable can be expected to be positive. However, this variable also measures the behavior of the dairy product price relative to other producer prices in the economy. In this case the sign can be undetermined. The indeterminacy of signs in the reduced form of a system of Tobit equations is a well-known result (Hajivassiliou, 1993).

Table 3. Exogenous variables

VARIABLE	DESCRIPTION
ADV	Advertising Expenditures
MP	Milk Production
PCE	Personal Consumption Expenditures
WAGE	Wage Rate in the Food and Kindred Industries
POPU	Population
R	Nominal Interest Rate
PPI	Producer Price Index
D1	First Quarter of the Year Dummy Variable

An increase in the quantity of milk produced can increase the supply of dairy products and hence, decrease price. Therefore the coefficient on this variable is expected to be negative. The interest rate is included in the regression as a measure of the carrying costs of storage. The coefficient on this variable is expected to be negative. A higher interest rate reduces the demand for storage purposes and decreases the price of the dairy product. Each price equation contains several lagged variables that take account of the dynamic structure of dairy product prices.

The estimates of the parameters of the system are presented in Table 4. The standard errors of the parameter estimates are in parenthesis. An "L" before a previously defined variable means that the variable is in logarithms in the model. From the parameters estimates we find that, other things being equal:

1. The population variable, with the exception of the NFDM price equation, captures the long-term trend of decreasing (in real terms) dairy prices.
2. All equations show that prices fall in the first quarter of the year, as expected given the seasonality in U.S. dairy product prices.
3. All dairy prices exhibit a negative relationship with milk produced, as expected.
4. Personal consumption expenditures, a key demand shifter, have a positive effect on all prices.
5. All the producer price coefficients are positive (as expected) and significantly different from zero except for the cheese and manufacturing milk price equations. This suggests that ….
6. The coefficient on interest rate is not significantly different from zero in all equations. This variable proxies storage carrying charges.
7. The dynamic structure of prices is very important in explaining changes in current price.

The results of this model are not easily comparable with previous research in the U.S. dairy sector because this is a reduced form model while other papers rely on a quite simple structural model. The only exception is Cornick (1994). Although he estimates a quarterly reduced form model for a different time period, there are some common findings. For example, the generic advertisement expenditures do not have a significant impact on prices. The effects of

Table 4. Summary of Estimated Parameters Obtained with GHK Probability Simulation Methods.

	LWPB	LWPCH	LWPNFDM	LWPICE	LWPOTH
C	-0.197 (0.011)	-0.071 (0.016)	-0.168 (0.028)	0.000 (0.002)	-0.003 (0.002)
LADV	-0.004 (0.015)	0.014 (0.019)	-0.039 (0.025)	-0.005 (0.003)	-0.001 (0.004)
LWAGE	-0.786 (0.036)	-0.016 (0.060)	-0.495 (0.079)	-0.013 (0.023)	-0.010 (0.024)
LMP	-0.117 (0.025)	-0.125 (0.046)	-0.071 (0.044)	0.001 (0.013)	-0.009 (0.015)
LPCE	0.639 (0.028)	0.5601 (0.029)	0.571 (0.041)	0.167 (0.020)	0.072 (0.024)
LPOPU	-1.677 (0.044)	-1.391 (0.062)	0.502 (0.053)	-0.420 (0.023)	-0.243 (0.082)
LR	-0.013 (0.014)	-0.005 (0.014)	0.002 (0.021)	-0.000 (0.002)	0.000 (0.002)
LPPI	-0.010 (0.031)	0.248 (0.032)	0.675 (0.113)	0.114 (0.020)	0.067 (0.019)
D1	-0.052 (0.010)	-0.024 (0.009)	-0.022 (0.014)	0.000 (0.001)	-0.001 (0.001)
L1	1.148 (0.027)	0.967 (0.068)	0.975 (0.084)	0.884 (0.041)	1.392 (0.025)
L2	-0.655 (0.034)	-0.353 (0.061)	-0.463 (0.064)	-0.012 (0.041)	-0.450 (0.025)
L3	0.288 (0.026)				

milk produced and population are similar too.

However, these findings contrast with the results in Liu et al. (1991) where the advertisement expenditures are found to be very important in explaining demand of both fluid and manufacturing milk and therefore are likely to have an important impact on the price of milk and other dairy products.

As it was mentioned before, it is difficult to interpret the coefficients of a reduced form model. However, if the research objective is the impact on endogenous variables of a change in exogenous variables a reduced form model is all that is needed. If the sector is difficult to model, then reduced form models might have an additional advantage over structural models. There are numerous structural models that share the same reduced form model. In this sense, if what we need is provided by the reduced form model, we reduce the risk of estimating the "wrong" structural model by estimating a reduced form.

The dairy model is important in itself and can be useful in analyzing dairy policies that involve changes in support prices or other exogenous variables. The model can be relevant as well in analyzing issues related to the emergence of dairy futures that price some of the commodities analyzed here. In this context, we could pursue a thorough discussion of the current characteristics of the model and a set of potential improvements. However, at this point we choose to analyze the methodological issues concerning the estimation of a system of equation with multiple censored variables.

In the introduction we claim that if the endogenous variables in the system are generated simultaneously, it is unlikely that the disturbances of the respective equations are independent and therefore, there is some efficiency gain from estimating the equations as a system. However, the use of probability simulation methods can produce bias in parameter estimates due to simulation noise. Therefore, it is very important to test for the existence of simulation noise bias in the estimation of the parameters. We run several tests similar to the one in expression (16). Results from these tests, summarized in Table 5, suggest that for a modest number of simulations in the GHK simulator (R=100) the simulation noise is negligible. This finding is consistent with results presented in Hajivassiliou et al. (1996) that suggest the use of a number of simulations (R) equal to the number of observations in the sample.

Table 5. Results of the simulation noise test.

NS	R	W	Critical value c^2_{89} a=0.95
200	20	90.28	113
200	100	88.6	

The estimation of a system of equations containing severalTobit equations is far from trivial and to our best knowledge the computer code is to some extent case specific. In the other hand, the estimation of single Tobit equations is an almost trivial task. The natural question to ask is what we are really accomplishing estimating the system of equations containing Tobit versus estimating single equations separately.

The question is not well treated in the literature but it is not difficult to come with two tentative answers:

1) From an statistical point of view, single equation estimation can be seen as the result of imposing some parametric restrictions in the system. More precisely, the single equation estimation is equivalent to the estimation of a system of equations with a variance-covariance matrix where all the off-diagonal elements are equal to zero. This parametric restriction can be tested following Hajivassiliou (1997). In the present case, we can not reject the restricted model (i.e., single equation estimation) -- see Table 6. However, this test may suffer from the problems that arise in the estimation of the variance-covariance elements (Greene, 1997).

Table 6.Likelihood-ratio test of the null hypothesis that off diagonal terms of the variance covariance matrix are zero (i.e., no need for systems estimator).

	LnL	Critical value c^2_{6} a=0.95
Restricted model	17.05	12.59
Unrestricted model	18.10	
Likelihood ratio test	2.11	

2) From an economic point of view, we can check if the unrestricted model provides results that differ from the restricted one. Under inspection of the result, we find as a regularity that the parameters that are significantly different from zero tend to have similar values on the single

equation and in the system. For example, this is the case with the parameters on the Population and Personal Consumption Expenditures variables. These parameters are significantly different from zero in all equations and the estimates are very similar under both approaches. The parameters on the Wage variable are similar in both approaches in equations where the parameters are significantly different from zero.

The importance of the methodology used in this paper should not be obscured by the statistical result in Table 5 that suggest that single equation Tobit estimation cannot be rejected in favor of MSL estimation of a system of Tobit equations in the present empirical example. The importance of the correlation between the disturbances of the system is an empirical issue and we are able to statistically test this empirical correlation using MSL. Without the MSL methodology, the potential importance the estimating a censored system of equations versus single equation Tobits would be an open question. As well, the current empirical example did not use cross-equation restrictions witch are likely to increase the importance of systems based estimation.

VI. Conclusions

This paper explores the use of probability simulation methods in maximum likelihood estimation of a system of equations with multiple censored endogenous variables. The empirical application consists of a system of six dairy price equations where up to four prices are lower censored by the U.S. dairy price support program and federal milk marketing orders.

The GHK probability simulator proves to be computationally tractable in the maximum likelihood estimation of models with more than three limited dependent variables. This probability simulator is a smooth continuous function on the parameter space, it is easy to program and computational costs increase only linearly with the number of censored variables. It is important to stress that the estimation of models with more than three censored endogenous variables was considered unfeasible until recently.

It is important to analyze the bias generated by simulation noise when estimating a model by MSL. In the present case, the bias was negligible for a modest number of simulation in the computation of the GHK simulator.

The estimation of a model with several related Tobit equations is far from a trivial task. Therefore, it is natural to evaluate the relevance of estimating a system of Tobit equations versus the estimation of single Tobit equations. For the empirical application in this paper, we can not reject statistically the restricted model made of single Tobit equations.

In summary, the estimation of systems of equations with multiple censored endogenous variables by maximum simulated Maximum Simulated Likelihood using the GHK probability simulator seems to be a feasible task. This result suggests that demand systems where some consumers choose no to consume several of the goods can be estimated using this procedure. The same can be said of multiple choice models prevalent in the economic literature and a number of models with likelihood functions that contain high dimension definite integrals.

VII. References

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Figure 1: U.S. Monthly Cheese Prices, 1985-94.

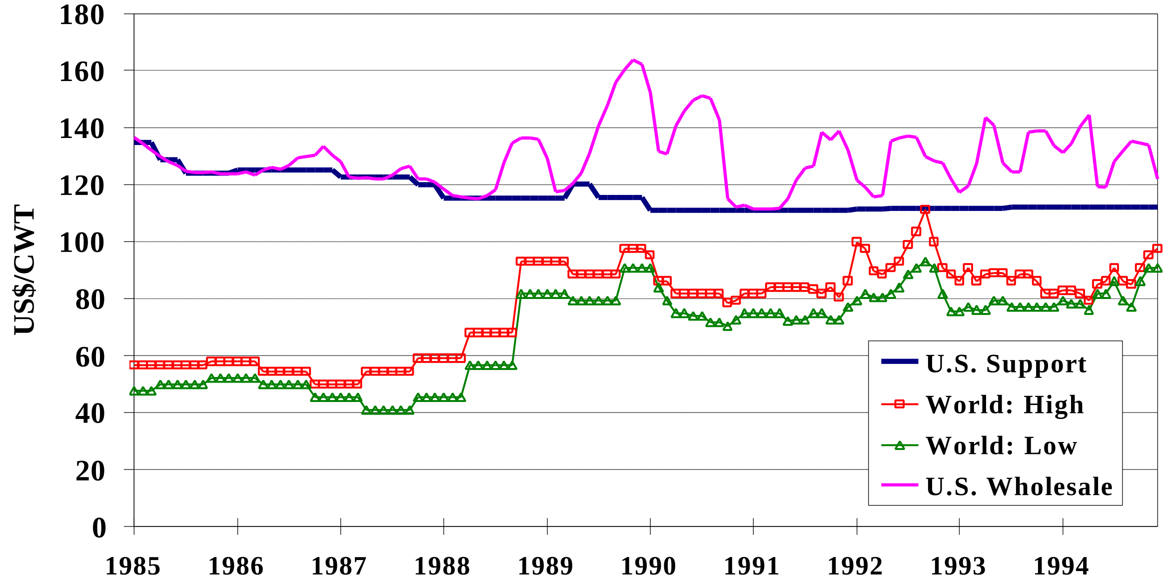
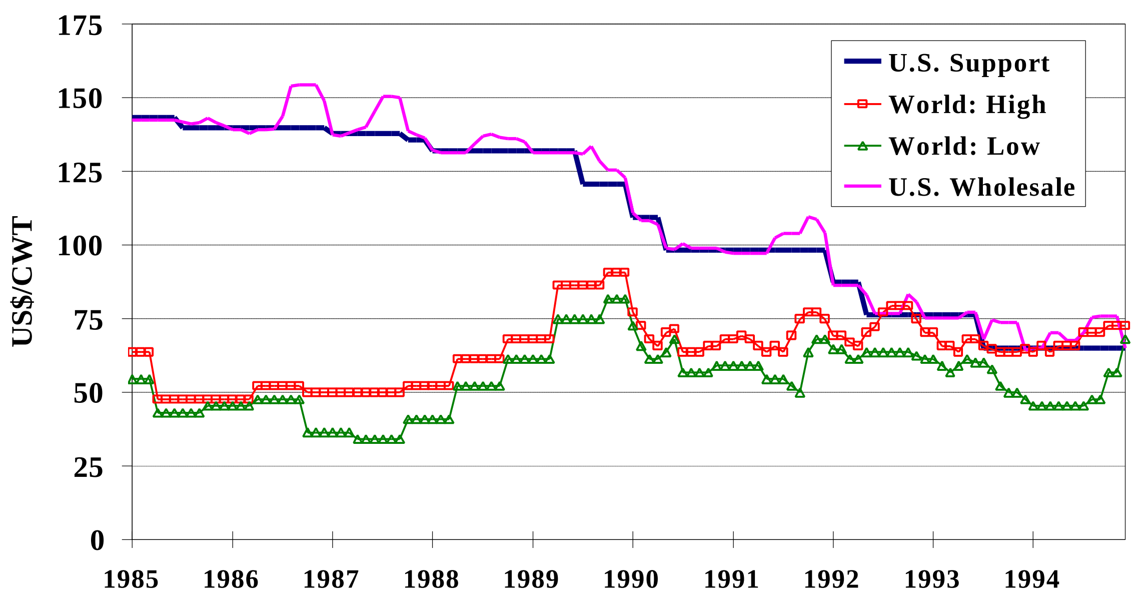


Figure 2: U.S. Monthly Butter Prices, 1985-94.



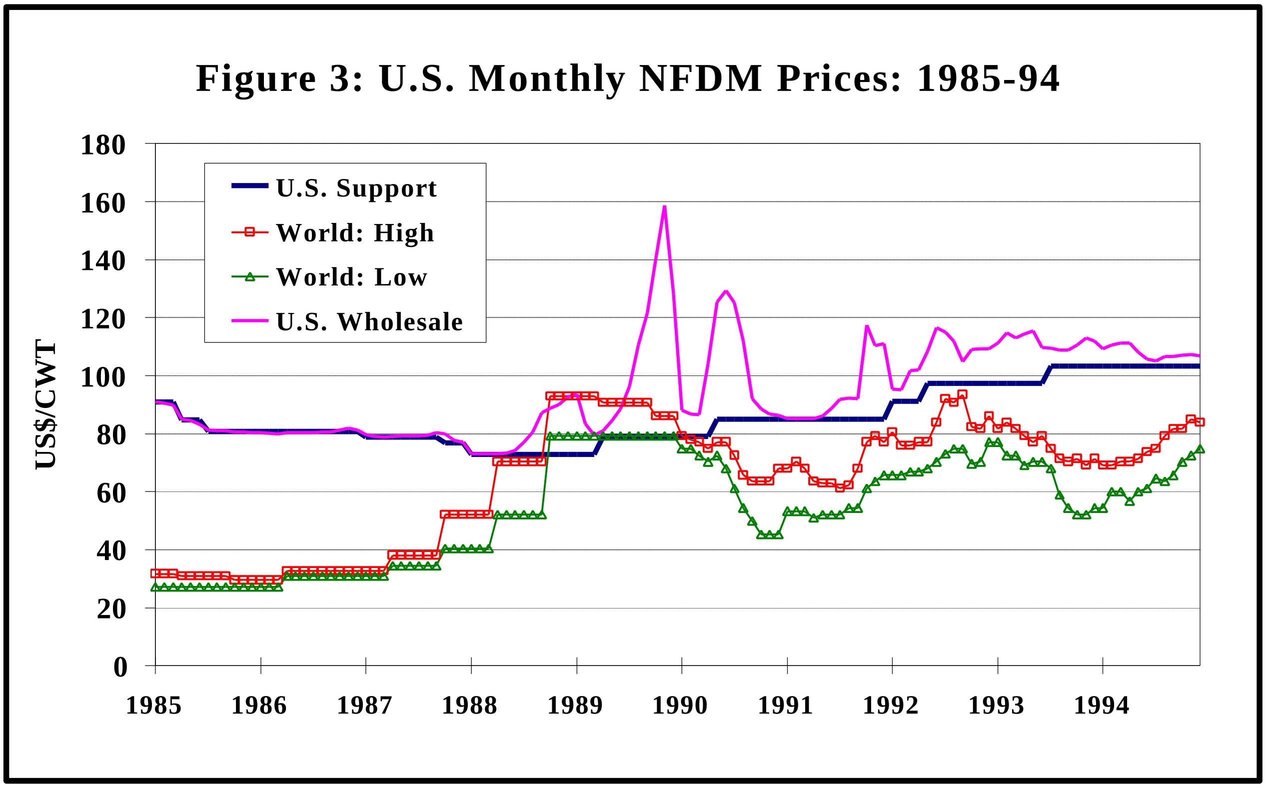


Figure 3: U.S. Monthly NFDM Prices: 1985-94
180
160
140
120
100
80
60
40
20
0
US$/CWT
U.S. Support
World: High
World: Low
U.S. Wholesale
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994

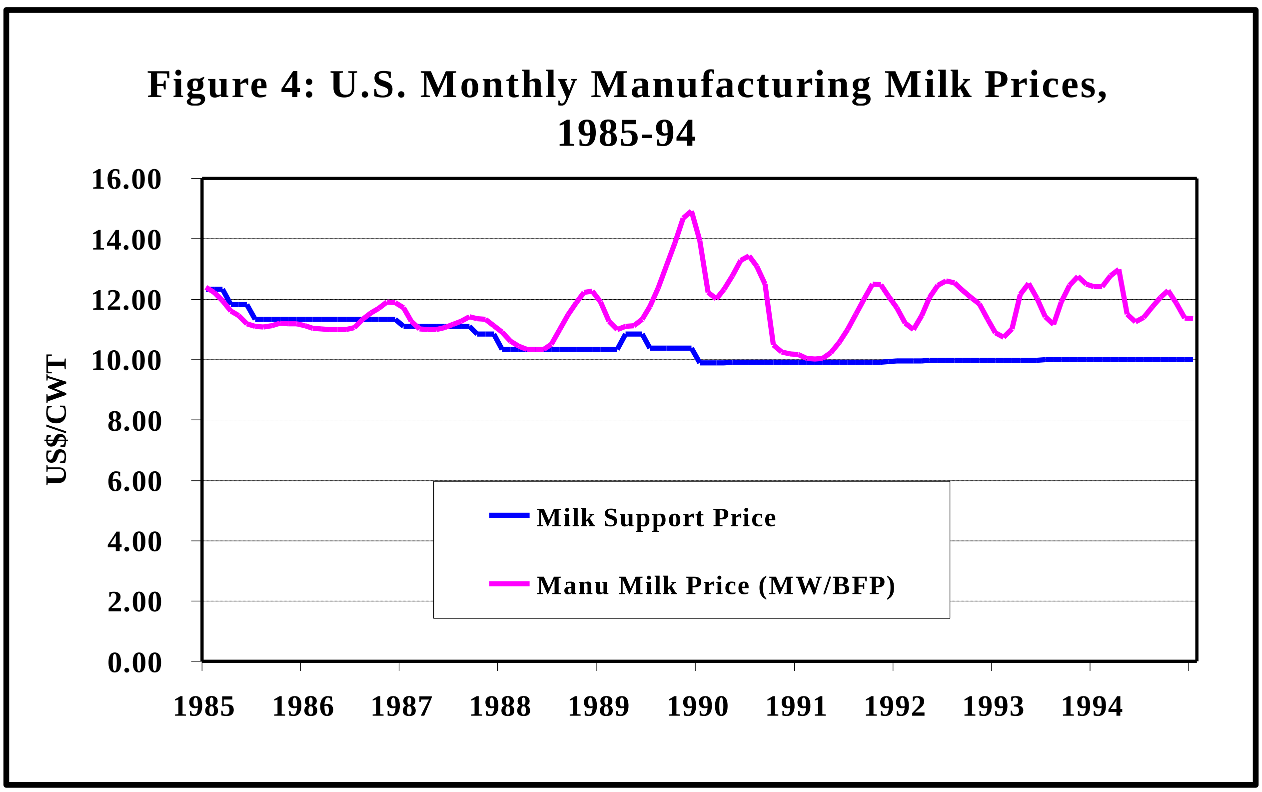


Figure 4: U.S. Monthly Manufacturing Milk Prices, 1985-94
16.00
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
US$/CWT
Milk Support Price
Manu Milk Price (MW/BFP)
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994