*University of Wisconsin-Madison*

**January 1999** **Staff Paper No. 402, Revised**

# Asset Smoothing, Consumption Smoothing and the Reproduction for Inequality under Risk and Subsistence Constraints

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## AGRICULTURAL & APPLIED ECONOMICS

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## STAFF PAPER SERIES

# Asset Smoothing, Consumption Smoothing and the Reproduction of Inequality Under Risk and Subsistence Constraints[*]

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January, 1999

**Keywords**: Subsistence, Poverty Traps, Savings, Dynamic Modeling

**JEL Codes**: D91, I32, O16, O12

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[*] The authors acknowledge the generous support of the IRIS Research Program, University of Maryland under USAID cooperative agreement no. DHR-0015-A-00-0031-00, and the Graduate School of the University of Wisconsin. The authors would like to thank Alain DeJanvry, Jeffrey Williams, and seminar participants who commented on an earlier draft of this paper presented in the development seminars of the Universities of Berkeley and Wisconsin, Université Notre Dame de la Paix at Namur and Stanford University. Any errors remain those of the authors.

# Asset Smoothing, Consumption Smoothing and the Reproduction of Inequality Under Risk and Subsistence Constraints*

***Abstract:***

This paper uses a stochastic dynamic programming model to explore savings and portfolio decisions in an environment in which both yield risk and endogenous asset-price risk exist, and a subsistence constraint creates an inter-temporal link between current consumption and future labor power. Numerical analysis of the model shows that optimal portfolio strategies bifurcate. Initially wealthier agents acquire a higher yielding portfolio and pursue a conventional consumption-smoothing strategy. Initially poorer agents acquire a safer, but less remunerative portfolio and pursue a defensive strategy characterized by asset-smoothing, rather than by consumption-smoothing. The positive correlation between initial wealth and rate of return on wealth that is created by these behaviors implies that inequality reproduces itself over time, and that while asset-based risk-coping is expensive for all agents, it is more so for the poor who forego 18% of their potential income in order manage risk.

# Asset Smoothing, Consumption Smoothing and the Reproduction of Inequality under Risk and Subsistence Constraints

## Section 1 Introduction

This paper models wealth accumulation and portfolio management among rich and poor villagers in the developing world. The model developed here extends the literature on autarchic risk management in the presence of credit market or liquidity constraints, by incorporating four features that are central to this economic environment:

- Households save both in the form of conventional buffer assets (e.g., grain stocks and other safe savings instruments) and in the form of productive assets. Accumulation of the former comes at the opportunity cost of the latter, as well as at the cost of foregone consumption;
- Markets for productive assets (e.g., land) are localized with prices locally determined;
- Household income is subject to covariant as well as to idiosyncratic shocks, creating the possibility that local asset prices may endogenously move with income; and,
- Subsistence risk is non-trivial, especially for poorer villagers.

Starting with an initial distribution of households over a two dimensional asset space, numerical dynamic programming methods are used to solve the model and explore household portfolio management, allowing asset prices to endogenously evolve in response to villager's decisions and in conformity with their rational expectations about the degree of asset price variability and covariation with income shocks that their behavior generates. In this model, portfolio strategies bifurcate. Initially wealthier agents pursue a conventional consumption-smoothing strategy. Initially poorer agents pursue a defensive strategy characterized by asset-smoothing, rather than by consumption-smoothing. Consistent with the suggestion of Sen and Drèze (1989), poorer agents prioritize the preservation of their asset base, smoothing assets at the expense of consumption.

In addition to clarifying the intertemporal rationality of asset-smoothing and consumption-smoothing, the model speaks directly to Atkinson’s (1997) call for more sustained analysis of the economics of accumulation and inequality. In particular, the model identifies what following Lipton (1993) we call the “Micawber threshold,” meaning an initial wealth level below which agents adopt the asset defense strategy and are therefore never able to lift themselves up by their Victorian bootstraps to a higher living standard. The positive correlation between initial wealth and rate of return on wealth that is created by this behavior implies that inequality reproduces itself over time, and is indicative of the costliness of autarchic risk management, especially by poorer agents.

This insight has been captured by Bardhan, Bowles and Gintis (forthcoming), who find in a theoretical model that portfolio returns are positively correlated with initial wealth. As they note, their static framework does not allow for the possibility that individuals may use time and savings to work around missing financial markets. Dercon (1998) develops a model of lumpy asset-investment given subsistence constraints and agent heterogeneity. He shows that the rich accumulate assets more quickly, and that the poor pursue low-risk, low-return activities. This is an important result, revealing as it does the role of risk and subsistence constraints in determining heterogeneous strategies in portfolio management. The work here supports Dercon’s findings and builds on his insights by incorporating the role of endogenous asset-prices—and therefore asset price risk—into the portfolio decision of rich and poor households. As will be shown below, this modification has important implications for the risk-coping behavior of poor households particularly.

The incompleteness of formal financial markets and the ubiquity of credit and liquidity constraints in low-income economies has provoked a number of inquiries into households' capacity to manage risk and smooth consumption in the absence of insurance contracts and in the presence of borrowing constraints. One strand of this literature has examined the variability of consumption *ex post* of informal risk-coping mechanisms (including saving), and has determined that even in the presence of severe borrowing constraints consumption is on average smoother than income, although not by as much as would obtain under complete insurance markets (e.g., Udry, 1994; Townsend, 1994; Paxson, 1992). This literature has not in general, however, attempted to break down these results by household wealth or income levels, nor has it quantified the costs of such informal risk-coping through adverse incentive effects or lost productivity.[1]

The theoretical work of Deaton (1991) suggests that this insurance can be had relatively inexpensively through savings. His intertemporal model of risk-coping under borrowing constraints incorporates stochastic labor income and assumes that households have access to a single buffer asset (such as grain), and that income levels are independent of asset holdings. For "impatient agents" (to use Besley's 1995 term) whose discount rate exceeds the rate of return on the buffer asset, the only costs of such savings as insurance is postponed consumption. However, the costs associated with savings become considerably more complex—and potentially higher—if the savings assets is either a productive asset (as in Rosenzweig and Wolpin, 1993), or if agents have an

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[1] Important exceptions are Morduch (1994) and Kochar (1995; 1994).

opportunity to choose among mulitple assets (as in Dercon, 1998; Rosenzweig and Binswanger, 1993; or Paxson, 1990).

To better understand the costs of autarchic risk management—and how those costs vary for heterogeneously endowed agents—this paper's dynamic portfolio model extends this earlier work in several directions. First, agents in the model allocate their wealth between both a “non-productive,” savings asset (as in Deaton, 1991) and a productive asset (as in Rosenzweig and Wolpin, 1993). Second, the model endows agents with heterogeneous levels of wealth and explores their multiple and interacting strategies. Third, the model incorporates insights from the separate literature on sector-level commodity storage (Brennan, Williams and Wright, 1997; Williams and Wright, 1991; Gardener, 1979) that when agents’ intertemporal asset-management decisions both depend on expectations over future prices, as well as determine those prices in a market, then it becomes critical for models to appropriately endogenize those price movements and market interactions.

Using parameters and shock distributions calibrated on econometric estimates of West African agriculture, this paper relies on iterative numerical dynamic programming methods to fully endogenize relative asset prices and assure that the accumulation behavior of agents is mutually consistent and fully rational (in the sense that agents understand the first and second moments of the joint distribution of the income and asset price shocks they face). In addition, these numerical methods make it possible not only to determine whether or not a particular steady state is locally stable, but also to characterize its quantitative field of attraction over asset space. This approach thus opens the way to a fuller analysis of complex, stochastic programming models in which agent heterogeneity and the possibility of multiple trajectories are central to the analysis.

The remainder of this paper is organized as follows. Section 2 presents the core model. The model is first developed and presented analytically. Because of its complexity, the model is solved numerically. Results of this numerical analysis are presented in Section 3. Two stable attractor points in asset space emerge as optimal portfolio strategies, and their properties and implications are examined in detail. Conclusions, including suggestions for further work in this area and implications for policy, are presented in the paper’s final section.

## Section 2 A Dynamic, Stochastic Model of Asset Accumulation Under Risk and Subsistence Constraints.

This section develops a dynamic programming model of asset accumulation. After laying out the stochastic structure and income generating process, this section specifies the consumption-production linkages that are created by subsistence constraints. The full dynamic programming model is then laid out and an iterative numerical method for implementing the model in a way consistent with rational expectations over endogenous asset prices is detailed.

### 2.1 The Household Utility-maximization Problem.

Household income in this model is generated by a process that combines the savings asset of Deaton's (1991) model, and the productive asset of Rosenzweig and

$$F(T_{it}, M_{it}, q_{it}, q_{vt}) = q_{it} q_{vt} D \cdot (T_{it})^{s} + m M_{it} , \qquad (1)$$

where $T_t$ is the household's land holding in period $t$; $D$ is a land productivity parameter; and $s$ is an output elasticity parameter which represents decreasing returns; and, $m$ is the rate of return on grain.. This diminishing returns specification follows the literature that finds an inverse farm-size/farm-productivity relationship,[2] though more importantly here it represents a force that other things equal will encourage the accumulation of productive assets by low wealth agents.

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[2] There is some conflict in the literature as to whether an inverse farm size-farm productivity relationship exists, and why. (See, for example, Carter, 1984, vs. Benjamin, 1995, for contrasting views). The decreasing returns here are very modest, and are adopted mainly for computational reasons. Relaxing this assumption would strengthen the main results of the paper by raising the returns to the productive asset of wealthier agents.

Production risk is the product of two distinct shocks: an idiosyncratic shock, $q_i$, which captures those risks which affect household *i* only, such as health problems during the work season, or localized flooding or erosion; and a village-level shock, $q_v$, that represents primarily weather-related shocks that affect an entire village. While both types of shocks create household income variability, covariant shocks are unique in that they may create correlated behavior across households and thereby variability in asset prices.

Under this specification, there are thus two assets: a risk-free but low return asset (grain, for example) and a risky, but higher-return asset. The high-return asset is assumed to be land.[3] Land is the asset that is most different from grain in terms of its mean-variance properties, and therefore constitutes a reasonable choice for a productive asset. However, the theoretical insights of the model would hold up equally well if the productive asset chosen were livestock, farm equipment, wells, or even education.

Assets in this setting are distinguished by three properties: their yield-risk; their asset-price risk; and their mean rate of return. Clearly, portfolio management involves a trade-off across all of these properties. In rural areas of the developing world, properties of assets in these three dimensions can be expected to coincide, so that high-return assets also tend to be those assets with high levels of variance in their returns and high levels of negative covariance between their prices and any consumable good price. One implication of such an environment is that a two-asset model adequately captures the full set of portfolio choices available to households. Following other work in the literature, it is assumed that there exist borrowing constraints, so that holdings of both assets must be non-negative.

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[3] Although there is no formal land market in Burkina Faso, land transactions do occur. Long-term land pledging, economically equivalent to a land sale, is a historical feature of land institutions in the Sahel

A number of non-portfolio methods exists to manage risk, from social sharing schemes to formal insurance. (See, for example, Coate and Ravallion, 1993; Coquéry-Vidrovitch, 1985; Platteau, 1991; Reardon, Delgado and Matlon, 1992, Townsend, 1994.) However, these methods in practice fail to eliminate all risk, and the nearly universal existence of portfolio methods (including grain storage) speaks to the need for individual risk management. The risk representations of this model have been carefully parameterized to reflect exposure to risk *ex post* of the effects of institutions such as social reciprocity, labor diversification, and optimal crop management.

The budget constraint that households face is:

$$c_t \pounds \ F(T_{it}, M_{it}, q_{it}, q_{vt}) - P_{Tt}(T_{it+1} - T_{it}) - (M_{it+1} - M_{it}) \ , \qquad (2)$$

where $M_t$ is the household's holding of the non-productive asset (“grain”) in period $t$; and $P_{Tt}$ is the price of land in period $t$. Units are defined so that the prices of grain and the consumption good are equal and numéraires. Agent heterogeneity can be defined both over total wealth level and over the mix of assets. The budget constraint accordingly reflects possibilities of both consumption versus investment choices, as well as portfolio composition choices. As is common in developing countries, and is reflected in the literature, borrowing constraints are assumed to exist (e.g., Dercon, 1998; Rosenzweig and Wolpin, 1993; Deaton,1991).

Subject to (1) and (2) and given initial conditions$T_0$, $M_0$, $q_{i0}$, $q_{v0}$, we assume that agents maximize the present value of their utility over the infinite horizon. The function is additively separable, with a fixed discount parameter:

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(Zimmerman, 1994).

$$\max_{\{\underline{c}_{i0},\underline{T}_{i1},\underline{M}_{t1}\}} E_0\left\{\sum_{t=0}^{\infty} d^{\,t} u(c_{it}) \middle| f\left(P_{Tt}, q_{vt} \middle| \Omega_0\right)\right\} \tag{3}$$

$\underline{c}_s = \{c_s, c_{s+1}, c_{s+2}, \ldots\}$

$\underline{T}_s = \{T_s, T_{s+1}, T_{s+2}, \ldots\}$

$\underline{M}_s = \{M_s, M_{s+1}, M_{s+2}, \ldots\}$

where $d$ is the discount factor; and, $f(P_{Tt},q_{vt}|W_t)$ is the joint distribution of the asset price and the covariant shock. This joint distribution will depend on the land distribution at any given point in time. Agents do not know what this joint distribution will be, but will have rational beliefs about it. Because of the possibility of major structural shifts in the distribution of assets over time, it is important to the integrity of the model that asset prices be fully endogenous.

The magnitude of this covariance depends on several features of the local economy. First, the covariance will be greater the more people are forced—given an adverse shock—to alienate assets to buffer consumption. In general, this will depend on how low average consumable asset stocks are relative to the variation of income. Second, it will depend on the price elasticity of demand for land in the economy. Finally, it will depend on the relative size of the contribution of covariant (as opposed to idiosyncratic) shocks in overall income shocks. In this model, only the third factor is exogenous, and the first two arise out of the endogenous intertemporal choices made by households. Since the covariance itself is one feature these households take into consideration in making such choices, the model will look for a fixed point in the covariation.

## 2.2 Modeling Risk and Subsistence

When consumption drops below a critical subsistence level, households face the possibility of losing labor power through either ill health or death. Consequently, critically low current consumption implies not only low current utility, but also the prospect of utility losses in the future through this consumption-productivity linkage. Speaking to the importance of such dynamic considerations, Drèze and Sen (1989) note that subsistence considerations "are observed even in the behavior of richer people who are not immediately at great risk of starvation."

Put differently, the constraints that subsistence places on asset accumulation may be as important as the more thoroughly researched constraints of nutrition on productivity.[4] While Subramanian and Deaton (1996) question the importance of subsistence considerations based on their findings that the calories necessary for a day's activity cost less than 5 percent of the daily wage, it should be stressed that in a dynamic stochastic context, what is important is not just whether average income suffices for average nutritional requirements, but rather the probability that the income of a particular agent (poor or rich) will fall below subsistence requirements in a particular year (good or bad). Atkeson and Ogaki (1996) exploit this intertemporal significance of subsistence constraints on accumulation to estimate the levels of subsistence requirements in Indian data. Their estimates suggest that people act as if subsistence needs are between one-quarter and one-half of average income, with the more confident estimates being the higher ones. Rosenzweig and Wolpin's (1993) estimates, similarly derived from estimation of an accumulation model, are also at the high end of this range. Available

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[4] The nutrition-productivity literature has had its proponents (e.g., Ray and Streufert, 1993; Dasgupta and Ray, 1990), its sceptics (Bliss and Stern, 1978; Subramanian and Deaton, 1996) and other observers

evidence therefore suggests that subsistence considerations can indeed loom large in low income (and risky) economies.[5]

The consumption-productivity link in this paper is captured by specifying a minimum subsistence level of consumption. Agents who fall below this subsistence level receive zero utility for that period and for every period thereafter. Although this representation is not sophisticated, it does capture the most important intertemporal utility implications of extremely low consumption levels.[6]

Formally, the utility of consumption in period $t$ ($c_t$) is defined as:

$$u(c_{it}) = \begin{cases} (c_{it}/R)^e \text{ if } c_{it} \text{ ³ } R \text{ and } c_{is} \text{ ³ } R \text{ For all } s \text{ Î } \{1,2,...t\text{-}1\} & e < 1 \\ 0, \quad \text{otherwise} & \end{cases} \qquad (4)$$

where $R_0$ is the subsistence minimum, and $e$ is the utility curvature parameter. Note that it is a formal convenience to define the non-separability across periods in terms of utility, instead of in terms of production. One could as easily define $y_t$ equal to zero in periods following (by any distance) zero-consumption periods. Zero-consumption under such a specification would have similar inter-temporal implications to the specification articulated here, and the results of the analysis would be substantially the same.[7]

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(Behrman, 1990).

[5] Subramanian and Deaton's 5% figure is not entirely comparable to Atkeson and Ogaki's subsistence for several reasons. First, of course, one day's wage may have to stretch over several consumer-days worth of consumption, both because there are typically more consumers than wage-earners in a household, and because of unemployment of some of the wage-earners. Second, subsistence needs are broader than the need for calories, or even for nutrition generally. Also important to productive economic life are health care, shelter and household requirements such as soap, energy (batteries, fuelwood, kerosene), and transportation.

[6] This specification is similar to that of Rosenzweig and Wolpin (1993). That paper however, makes the additional assumptions that (a) there is subsistence insurance, so that consumption is always at least an epsilon above subsistence requirements, regardless of income, and (b) that there are no implications in subsequent periods for a subsistence shortfall in a given period. By relaxing both of these assumptions, this paper captures some of the most important dynamic implications of subsistence minima.

[7] As represented below, however, the modeling technique would be greatly complicated by having zero-consumption imply only zero income--rather than zero utility--in future periods.

## 2.3 The Dynamic Programming Model

The intertemporal quality of the portfolio choice arises from agents' need to consider the mean and variability of future asset prices in choosing the levels of those assets in their portfolios. In acknowledging the presence of subsistence risk, an additional intertemporal element is introduced by the fact that future utility may depend on consumption choices in the current period, as discussed above. In this sense, the dynamic choice problem is inseparable in time.

Time-inseparable models must be solved simultaneously for all time periods, and for that reason, the complexity of the problem grows exponentially with the number of time periods involved. Time-inseparable models of many periods are therefore generally not feasible. By contrast, because time-separable models can be solved period-by-period, the complexity of the problem grows only arithmetically with the number of periods involved. The challenge, therefore, is to articulate a model that has the important features of time-inseparability, yet the computational tractability of a time-separable model.

The simplest way of accomplishing this is to add a state variable that captures the important effects of past actions on the structure of current (and future) problems. Define this state variable ($L_s$) as zero if the agent has had zero-consumption in the past (i.e., before period $s$), and one if not. Note that it is in part the irreversibility of $L_{s+1}=0$ that makes the complexity of the problem manageable. This state variable can be thought of as indicating whether the agent is alive in period $s$, which of course depends on the agent's past consumption history.

The key trade-off to be captured in this model is that between present consumption and asset accumulation for future consumption. Capturing this trade-off is now straightforward because the infinite-horizon maximization problem (3) is additively separable in time, breaking into two parts representing present and future consumption. Because the state variable *L* reflects the intertemporal dependence of future utility on past consumption, it emerges as the problem (3) is broken apart:

$$\begin{aligned} &\max_{\{\underline{c}_0,\underline{T}_1,\underline{M}_1\}} E_0\left\{\sum_{t=0}^{\infty} d^{\,t} u(c_t) \middle| f\left(P_{Tt}, q_{vt} \middle| \Omega_t\right)\right\} \quad = \\ &\max_{\{c_0,T_1,M_1,L_1\}}\left\{u(c_0) + d\ \ L_1 \cdot E_0\left\{\max_{\{\underline{c}_1,\underline{T}_2,\underline{M}_2,\underline{L}_2\}}\left\{\sum_{t=1}^{\infty} d^{\,t} u(c_t) \middle| f\left(P_{Tt}, q_{vt} \middle| \Omega_t\right)\right\}\right\}\right\} \end{aligned} \tag{5}$$

$$s.t. \quad (2), (3) \ and \ (4) \qquad (\forall t)$$

$$given \ \ T_0\ , M_0\,, q_{i0}, q_{v0}$$

The value of current consumption is reflected in the first part of equation 5, while the value of foregoing current consumption for the sake of investment is reflected in the second part of equation 5.

Since the second part of equation 5 can be considered the value of the problem in the future, a value function (*J(T,M,L)*) can be defined:

$$J^*(T_0, M_0, L_0) \equiv L_0 \cdot E_0\left\{\max_{\{\underline{c}_{0t},\underline{T}_1,\underline{M}_1,\underline{L}_1\}} \sum_{t=0}^{\infty} d^{\,t} u(c_t)\right\} \tag{6}$$

Here $J^*(T_0,M_0,L_0)$ is the maximum expected discounted present value to be obtained from the asset combination $(T_0,M_0)$ and the state variable $L_0$. Besides being defined over the domain of land and grain stocks, the true value function depends on all the functional parameters of the model, including the distributions of $q_i$, $q_v$. (Because the true value function expresses expected value, it does not depend on particular realizations of $q_{it}$, $q_{vt}$.)

For the sake of simplicity, notation reflecting the dependence of the value function on the joint distribution of asset prices and covariant shocks has been suppressed.

Equation (5) can then be rewritten as:

$$\max_{\{c_0, T_1, M_1, L_1\}} \left\{ u(c_0) + d \; J^*(T_1, M_1, L_1) \right\} \tag{7}$$

*for t = 0 and given* $T_0$ *,* $M_0$ *,* $q_{i0}$ *,* $q_{v0}$ *.*

If the true value function $J^*$ were known, then solving the agents' infinite-horizon problem would be straightforward. Agents allocate available resources in any period to consumption, land investment and grain investment, and thereby also determine *L*. The portfolio choice problem is solved by setting:

$$\frac{1}{P_{Ts}} J_1^*(T_{s+1}, M_{s+1}, L_{s+1}) = J_2^*(T_{s+1}, M_{s+1}, L_{s+1}), \tag{8}$$

and the consumption-investment trade-off would be solved by setting:

$$u'(c_s) = d \, \frac{1}{P_{Ts}} J_1^*(T_{s+1}, M_{s+1}, L_{s+1}) \qquad \left(= d \; J_1^*(T_{s+1}, M_{s+1}, L_{s+1})\right), \tag{9}$$

where $J_i^*$ is the first derivative of $J^*$ with respect to the *i*th argument.

Note that (8) and (9) can be equated in one of two ways. If consumption exceeds the subsistence minimum, so that $L_{s+1}=1$, then the problem is straightforward. If $L_{s+1}=0$, however, then all of the derivatives, *u¢*, *J¢* are zero (except at the discontinuous point where $c_s=R_0$). Equations (8) and (9) still hold, but are zero on both sides. Agents compare these two solutions ($L_{s+1}=1$ and $L_{s+1}=0$) to see which of them yields greater utility.

Bearing in mind that *J* depends on the household's own assets, as well as the asset holding of all other households in the economy through their effects on prices and hence price beliefs, a closed-form solution to the problem is not attainable. Accordingly, the model is simulated, parameterized to data from Burkina Faso. Following Deaton (1991), numerical methods were then employed to calculate the true value function. Details of this process are provided in Zimmerman (1994). Proof that the process converges is provided in Appendix A.

### 2.4 Numerical Implementation of the Model

For the purposes of numerical solution, a stylized village of 100 households was created, within which the productive-asset (land) market clears endogenously, generating a land price. Each household is endowed with land and grain at the beginning of the simulation according to empirically observed asset distributions in Burkina Faso. Each period of the simulation is characterized by a production cycle—in which households realized production according to (3)—and an asset-adjustment and consumption cycle—in which households solved the optimization problem (2) by allocating available resources to consumption and to land and grain accumulation or decumulation. This simulation was repeated for 35 periods, thereby creating a time series of endogenous prices.

Agents in the model are endowed with rational beliefs, meaning that they correctly know the mean and first moments of the distributions of the stochastic variables. Rational beliefs were achieved in the model by an iterative method. Agents were first endowed with a likely trajectory for prices and then the simulation was run for 35 periods to generate new price outcomes. These new price outcomes (or rather the price distributions which they were estimated to represent) then formed the basis for a second run of the entire 35-period simulation. This second run generated in its turn new prices, whose estimated distributions formed the basis of a third run, and so on. This process was pursued until the actual prices realized in the simulation differed by no more than an average for the simulation of 2% from the price beliefs which had generated the result. This process converged after about 10-15 runs of the simulation.

In order to numerically parameterize the model, we turned to data collected in three regions of Burkina Faso from 1981-1985 by the International Crop Research Institute of the Semi-Arid Tropics (ICRISAT). The survey encompassed six sample villages from three agro-climatically distinct regions of Burkina Faso, and is one of the largest and most comprehensive data-sets on West African agricultural production. These data were used to parameterize the initial distributions of the assets, the production function, and the risk structure. This parameterization is discussed in Appendix B.

The parameterization of the risk structure is based on empirical estimates of risk in Burkina Faso by Carter (1997), and is also presented in Appendix B. This risk parameterization takes account of the fact that many forms of risk-mediation exist. Specifically, using the same data-set, Reardon, Delgado and Matlon (1992) find that the coefficients of variation of crop yields are 45%, 52%, and 67% for three regions of Burkina Faso. Households mediate this risk in two principal ways, viz., by pooling income and by diversifying sources of labor income. *Ex post* of these two risk-coping mechanisms, the coefficients of variation of total household income are 12%, 49% and 53% for the three regions.[8] Risk in this model is parameterized to have a coefficient of variation (of crop yields) of 25%. The level of risk in this model thus reflects

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[8] This is a back-of-the-envelope calculation based on data in Reardon, et al.'s paper, and assuming that labor income has no variability. In practice, labor income certainly has at least some variability. Since off-farm labor allocation competes with on-farm labor and is generally decided before harvest shocks are known, off-farm labor income is probably (weakly) positively correlated with crop income.

conservative assumptions about unmediable risk. In the analyses that follow, the parameters specified in Appendix B were varied by 5-10% of their values, with no change to the substance of the results.

## Section 3 Optimal Accumulation and Portfolio Management.

Starting from an initial distribution of agents across the two dimensional asset space, numerical analysis of the model developed in the prior section reveals that each agent gravitates towards one of three types of portfolio strategies. Each strategy is characterized by a pattern of short-term asset management that is followed in the wake of realized shocks, and by a long run equilibrium portfolio position: *i.e.*, a grain-land combination to which the household returns following short term adjustments to shocks.

The first of the portfolio strategies is simply the collapse to a zero wealth position, subsistence crisis and the suffering of the permanent utility loss described above.[9] The second is a defensive portfolio strategy characterized by conservative, relatively low-yielding long-run equilibrium portfolio (i.e., income smoothing) and a pattern of short-term asset management that we will call asset smoothing or asset protection. The third is an entrepreneurial strategy characterized by a relatively high yielding portfolio and a more conventional pattern of liquidating assets to buffer shocks and smooth consumption.

After looking carefully at the characteristics of these strategies, and the economic forces that drive their adoption, we exploit the fact that our numerical analysis permits us to map out those regions of the initial asset space that are attracted to the different portfolio strategies. Identification of these fields of attraction permits us to develop a

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[9] The origin is an absorbing state because at this point agents have no income to allocate to asset investment.

concept of asset poverty and speak to questions about the reproduction of poverty and inequality in the presence of risk and subsistence constraints.

### 3.1 Consumption, Income and Asset Smoothing under Optimal Portfolio Management

Table 1 characterizes the defensive and entrepreneurial strategies using time series generated by a 250-year vector of random shock realizations. The defensive strategy is characterized by both a much lower portfolio value than the entrepreneurial strategy (21,013 grain-equivalent units for the defensive strategy as against 125,427 grain-equivalent units for the entrepreneurial portfolio) and by a less productive portfolio composition (37.1% grain in average value terms versus 0.8%). Although the maximum attainable expected return is higher for lower wealth agents,[10] the expected rate of return for the poorer agents who pursue the defensive strategy (5.3%) is actually lower than that for the wealthier agents who pursue the entrepreneurial strategy (5.9%). Indeed, the lower wealth households who optimally follow the defensive strategy pay an 18% premium (in terms of foregone rate of return on wealth), a figure that dwarfs the 0.4% premium paid by wealthier households. This 18% premium is a stark indicator of poorer households' willingness to pay for insurance, and is in line with empirical studies that have tried to measure the insurance premia implicit in the behavior of low wealth agrarian households.[11]

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[10] This maximum rate of return is what a risk neutral agent would attain by holding a portfolio comprised entirely of land in the case of low wealth agents.

[11] For example, empirical results from von Braun, et al. (1989) suggest that the conservative adoption of risky, non-traditional export crops by small-scale Guatemalan vegetable producers costs these producers as much as 75% of income. Rosenzweig and Binswanger (1993), who find that the poor forego more potential income due to a safer investment strategy than do the rich. Moreover, the order of magnitude of the loss to the poor is the same.

**Table 1**
**Costs and Characteristics of Optimal Stable Self-Insurance Portfolios**

| | **Defensive Portfolio** | **Entrepreneurial Portfolio** |
|---|---|---|
| *Portfolio Value, Composition and Returns* | | |
| Value of Portfolio (using final period price) | 21,013 | 125,427 |
| Portfolio Composition (% grain in value of portfolio) | 37.1% | 0.8% |
| Expected Rate of Return | 5.3% | 5.86% |
| *Cost of Risk Coping* | | |
| Maximum Possible Expected Rate of Return | 6.43% | 5.88% |
| Percent of Maximum Possible Return Foregone | 18% | 0.4% |
| *Consumption Smoothing* | | |
| Standard Deviation of Consumption | 155 | 609 |
| Coefficient of Variation of Consumption | 13.5% | 8.4% |
| *Income Smoothing* | | |
| Standard Deviation of Income | 217 | 1824 |
| Coefficient of Variation of Income | 20% | 25% |
| *Asset Smoothing* | | |
| Coefficient of Variation of Land Stock | 0.7% | 7% |
| Coefficient of Variation of Grain Stock | 1.3% | 66% |

As can be seen in Table 1, both income and consumption are smoother for the poor than for the rich, as measured by the standard deviation of the realized annual levels. However, although income for the poor is very much smoother (the standard deviation of income for the poor is about one-sixth that of the rich), consumption for the poor is only somewhat smoother (the standard deviation of consumption for the poor is about one-half that of the rich). Indeed, the coefficient of variation of consumption reveals that after adjusting for average consumption levels, the poor have *more variable* consumption streams than the rich, even though their income streams are more stable by the coefficient of variation of income measure.

These results reveal that the poor are using *consumption* to buffer *assets* in the wake of shocks—not the other way around. These results go against the common perception that agents with concave utility would want to smooth consumption to the greatest extent possible. In this dynamic model, given the threat to future labor productivity posed by letting consumption fall below subsistence, the poor do not smooth consumption as much as possible, but rather vary consumption to maintain assets. As can be seen in Table 1, agents who pursue this defensive strategy experience coefficients of variation of 0.7% and 1.3% in their stocks of land and grain, respectively, while the coefficient of variation of consumption is 13.5%. By contrast, for the wealthier agents away from the subsistence threat, the coefficients of variation in asset stocks are roughly 10 and 50 times higher than for poorer agents, while the coefficient of variation of consumption is of comparable magnitude.

Graphically, this point is brought out in Figures 1 and 2, where the distinctive buffering strategies that characterize the two strategies come out clearly. The wealthier agents have both greater asset movements as well as more symmetrical ones: poorer agents allow grain stocks to vary slightly, but there is no visible movement in land stocks over the entire 250-period simulation.

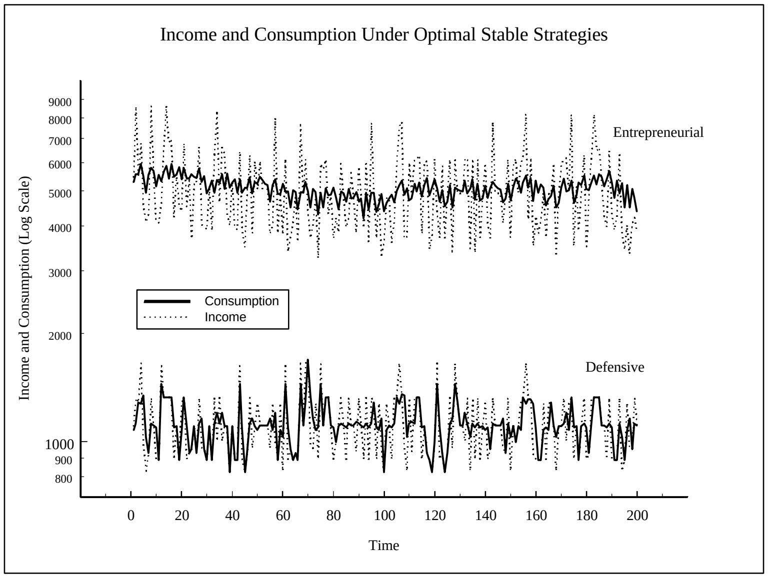

**Figure 1: Income and Consumption under Optimal Portfolios**

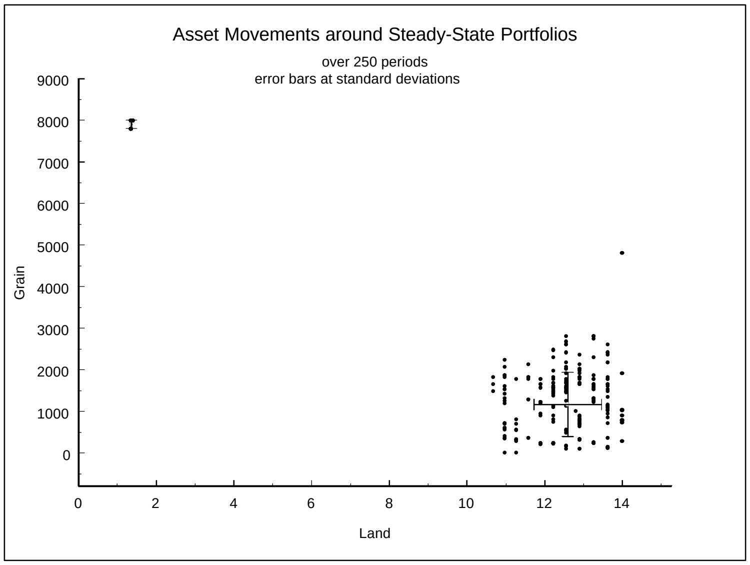

**Figure 2: Asset Movements around Steady-State Portfolios**

The failure of the poor to smooth consumption may seem at first counter-intuitive, especially in the context of a risk-management literature that often describes itself as a consumption-smoothing literature. Consumption smoothing is of course only one possible implication of dynamic utility maximization under risk. This model presents results consistent with empirical findings, such as those summarized by Moser (1998), who notes that in times of stress, "…the preservation of assets often takes priority over meeting immediate food needs." Drèze and Sen (1989) report that "...the reduction of food consumption tends to be an early response to the threat of entitlement failure, apparently motivated, at least partly, by the preservation of productive assets." Such findings suggests that the risk literature's emphasis on consumption-smoothing, as opposed to asset-protection, might not be capturing the full dynamic story of risk management—especially for poor agents.

In the context of the model developed here, it is worth stressing that the low wealth, defensive portfolio emerges as a stable strategy despite the fact that individuals in low wealth positions expect higher returns from the accumulation of additional units of productive land at the margin. Despite their seeming technologically-based competitive advantage in the land market, there are two economic forces that lead these individuals cling to a conservative, grain-intensive portfolio. The first is simply that shifting wealth from land to grain serves to smooth income, since the returns to land are subject to yield risk. The second reasons results from a more subtle interplay between covariant risk, endogenous asset prices and subsistence constraints.

As detailed above, the defensive portfolio strategy is distinguished from the entrepreneurial strategy not only by its *ex ante* portfolio composition, but also by its distinctive pattern of *ex post* behavior in the wake of realized shocks.[12] While income-smoothing motives might explain the first, it cannot account for the second. If there were no asset price risk (i.e., no chance that the price of land relative to consumable foodstuffs would deteriorate), poor and rich agents would be equally inclined to use the productive asset as a buffer, since there would be no reason to buffer with an unproductive asset, and the poor would accumulate assets to the level of the rich. If asset price risk did exist but were not endogenous, then for low levels of exogenous asset price risk, holding the productive asset as a buffer would dominate the unproductive asset for all agents. For high levels of exogenous asset price risk, the unproductive asset would dominate for all agents. In either case agents would migrate from one stable optimal strategy to the other. Only for some range of asset price risk in between would there be multiple equilibria. Endogenizing asset price risk as in this model assures not only that the asset price risk is realistic, but also allows for the asset price risk to be endogenously placed in that intermediate range that drives the multiple stable equilibria.

To see why the endogenous price risk entails the multiple stable equilibria, it is useful to think of the asset-price-risk wedge between the productive and non-productive asset as reflecting the value of asset-based insurance. As with any kind of insurance, this one is in short supply: if everyone tries to buffer consumption against covariant income shocks with transactions in the productive asset market, then covariance between land price and production would climb, and the productive asset loses its value as a buffer. Land, in other words, can only be effectively used by some subset of agents to smooth consumption. This insurance must therefore be rationed, and the rationing mechanism is

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[12] Such results underscore the point made by Eswaran and Kotwal (1990) that credit-constrained households are more likely to pursue safer, less productive portfolios than non-credit-constrained

quantitative—it is the degree of covariant price variation of the productive asset. Since richer people are farther from a subsistence crisis, they are more willing bear some degree of asset price risk, and so they are better able to afford this form of insurance, and are more likely to buffer consumption with productive assets.

This model thus accounts for the emergence and stability of a low wealth defensive portfolio strategy in terms of covariant risk, endogenous asset prices and subsistence constraints. Other students of these phenomena have reached broadly similar conclusions, though often with different conceptual and methodological emphases. For example, Morduch (1994, 1995) has suggested that a defensive portfolio strategy could derive from the poor having worse access to consumption insurance than do the rich. If such stratified access to insurance does exist, then it would tend to reinforce the results presented here. Moreover, differential access to formal or informal insurance is predicated on their ownership of assets, which, as expressed here, arises out of a dynamically endogenous accumulation process.

Second, as Rosenzweig and Binswanger (1993) suggest, the result of wealth-stratified portfolios could arise from decreasing relative risk-aversion. This model provides a clue to why there might be systematically different risk preferences across wealth levels. Here, it is the intertemporal non-separability of the subsistence constraint that drives the poor both to smooth income more than do the rich, and at the same time to take less of this stability in the form of consumption (and more in the form of protecting assets) than do the rich. These results suggest a simple and direct empirical test of the utility-function based versus the subsistence-constraint based conceptions of decreasing relative risk aversion: If the poor simultaneously have smoother income and more

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households.

variable consumption than the rich, then the appeal to utility-based decreasing risk-aversion in explaining wealth-dependent portfolios could be rejected.

### 3.2 The Micawber Threshold and the Reproduction of Inequality

Our reliance upon numerical dynamic programming methods permits us to not only identify asset positions and strategies that are locally stable (as, say, Ray and Streufert, 1992 do), but also to show that they each draw in agents from meaningfully large portions of the asset space.

Figure 3 shows the actual accumulation trajectories of agents over the course of the entire simulation. As can be seen, the trajectories of all of the agents can be grouped into three classes of movements. First, agents in the lower left portion of the asset space stock out over the course of the simulation. Note that even agents who begin the simulation with little land but with grain stocks several times the subsistence minimum eventually draw them down and stock out.

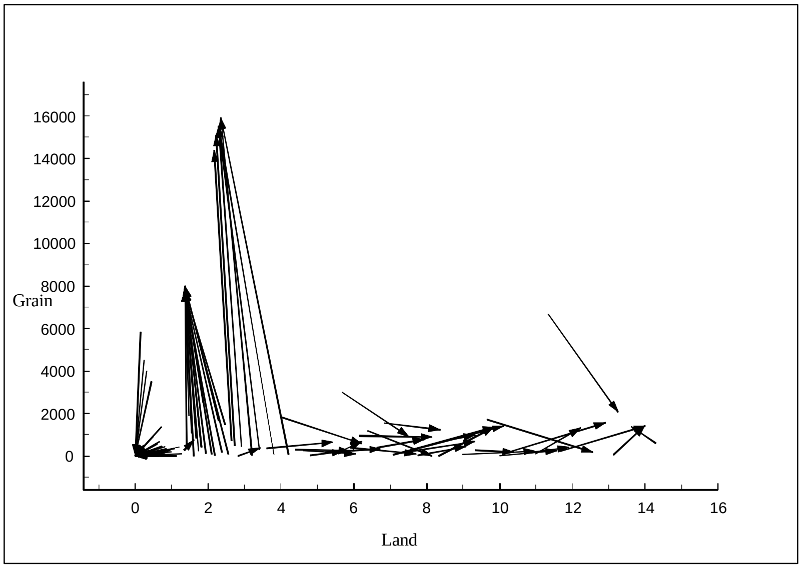

Figure 3: Evolution of Individual Portfolios

Second, agents who begin the simulation with between 1-1/2 and 4 hectares of land readjust their portfolios in the direction of a greater grain-to-land ratio. Here it should be noted that in so doing these agents reduce the average rate of return on their overall asset portfolio, but also reduce its yield riskiness and increase its fungibility. Finally, agents who begin the simulation with more than 4 hectares of land increase their land stocks. Significantly, these agents hold very low levels of grain stocks, as their production levels in even a bad year enable them to avoid a subsistence crisis. These agents experience risk aversion only in the standard, utility-curvature way (*i.e.*, their wealth makes them effectively immune from subsistence crises).[13]

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[13] These zones of attraction are probabilistic, rather than deterministic: near the borders of regions there are exceptionally lucky or unlucky agents who cross into neighboring regions of attraction.

The partitioning of initial asset space in Figure 3 highlights an important implication of the model for the dynamic reproduction of poverty and inequality. In addition to the Rosenzweig and Binswanger (1993) work already discussed, several writers have noted ways in which the dynamic implications of risk differ according to asset levels of households. Michael Lipton (1993) has written of a "Micawber Threshold," below which it is difficult for agents ever to accumulate assets and live up to the Victorian expectations of David Micawber in Charles Dickens' *David Copperfield*. Thomas Reardon and Stephen A. Vosti (1995) write of asset poverty lines, below which inter-temporal behavior and asset use patterns become distorted. Agents who begin with an asset base below the Micawber Threshold shown in Figure 3 are those that retreat northwesterly toward a defensive portfolio strategy. Motivated to avoid the fate of their slightly less well-endowed neighbors, these agents optimally shift toward a portfolio that brings them a rate of return on their wealth that is even less than that experienced by those initially wealthier individuals who pursue an entrepreneurial strategy. With rates of return positively correlated with initial wealth levels, this model implies a pattern in which initially inequality perpetuates and even deepens over time.

In addition to its implications for the reproduction of inequality, Figure 3 also highlights a productivity implication of the model. In general, several sorts of agent heterogeneity could generate differences in the shadow prices of an asset across agents, and hence engender an asset transaction. These differences—related to capital access, human capital, skill and risk—in general underlie a potential pareto improvement from an asset transaction that also implies an increase in aggregate productivity. The interplay between risk and subsistence constraints that drives transactions in this model as agents adjust to their long run portfolio positions is unique as a generator of heterogeneity in that it alone does not necessarily result in a transactions that enhance aggregate output.

This odd feature of risk is due to the fact that households making asset accumulation decisions equate the ratio of present to future marginal utility of consumption to the ratio of the marginal productivity of the asset to its price (to within discounting and expectations operators). Transactions can therefore occur which, though Pareto-improving, decrease aggregate output. While several authors have speculated on the potential importance and productivity implications of risk-induced distress sales (e.g., Braverman and Stiglitz, 1989), the result portrayed in Figure 3 that agents with more than 4 hectares of land accumulate land despite the presence of decreasing returns to land provides evidence of such transactions. The accumulation of land by the wealthy is therefore twinned with the process of stocking out among the poor. Because of the decreasing-returns technology of crop production, this process is deleterious not only to equity, but also to social efficiency. As in the models of Dasgupta and Ray (1987) and Ray and Streufert (1993), a case for redistribution can be made on both income distribution and productivity grounds.[14]

## Section 4 Conclusions

This paper has presented a model that endogenizes asset-based risk-coping in an environment of unmediated risk and subsistence constraints. It has modeled individually rational, intertemporal portfolio decisions in an environment in which both yield risk and

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[14] Bardhan, Bowles and Gintis (forthcoming) observe that asset redistribution may be productivity enhancing without necessarily being Pareto-improving. That observation applies equally to this model in which compensatory payments following an asset redistribution would be infeasible as they would disrupt the stability of the new asset structure.

endogenous asset price risk exist. The results show that poor agents, who are closer to a subsistence minimum—and therefore in danger of losing future labor power if consumption falls too low—pursue a more conservative investment strategy than do wealthier agents. The portfolios of poor agents are heavily invested in consumable assets ("grain"), which offer a low return. The cost of this asset-based risk-coping (i.e., saving) in terms of foregone lucrative investments is high: about 18% of income. These low returns on poor household's portfolios create a poverty trap, since they offer less disposable income, and less opportunity to accumulate assets, than is the case for wealthier agents.

An additional cost of asset-based risk-coping is, paradoxically, increased variability of consumption. Because the poor are concerned about the possibility that their current income plus savings could fall below subsistence requirements in the future, they take steps to stabilize their portfolio and protect assets, even allowing consumption to vary—above the subsistence threshold, to be sure—to maintain asset levels and therefore future income. In this sense, the poor pursue a counterintuitive policy of using consumption to buffer assets, rather than vice-versa, as has commonly been assumed. Rosenzweig and Binswanger (1993) and Dercon (1996) find empirically that agents' willingness to abide profit risk is positively correlated with wealth. This model presents one explanation for such a phenomenon: poor agents are less willing to abide risk because of a non-separability between current consumption and future productive capacity in the form of subsistence requirements.

These results also provide some tentative suggestions for policy. First, the inability of the less well-off to defend a productive asset base—despite their abstract competitiveness in the use of those assets—raises questions about the design if not the feasibility of market-assisted land reform programs (e.g., see Deininger, 1998) that try to enlist land asset markets to shift land from rich to poor. Second, the analysis here provides additional and novel support for *ex post* consumption credit to guarantee a subsistence level of consumption for low-income agents as a way not only of preventing desperation land or livestock sales, but also of attenuating the tendency of the poor to pursue safer but lower-return asset portfolios *ex ante*. Doing so would not only help with poverty alleviation objectives, but also with agricultural output objectives. Third, the role of risk in this analysis suggests that policies to reduce both the overall level of risk to which households are exposed, as well as policies to reduce the component of covariant risk in that overall level would be both equity- and productivity-improving. Such programs would include infrastructural investments in irrigation (to reduce yield risk) and in roads (to reduce asset price covariation), as well as in agricultural research (to reduce yield risk). Finally, the analysis here provides more support for the notion that markets for assets (including land) in general improve the ability of the poor and others to optimally cope with risk.

## Appendix A: Numerical Estimation of the True Value Function

The task of determining the true value function is as conceptually simple as it is computationally intense. First, a lower value function is posited, $J_o(T,M,L)$, which is a known underestimate of $J^*(T,M,L)$ over a grid of points in $(T,M,L)$ space. For any given stochastic outcome $q_v q_i$, this $J_o$ can then be updated for all values of $T$ and $M$ by applying Bellman's operator:

$$J_o'(T_o, M_o | q_{io} \cdot q_{vo}, \Omega_o) = \max_{\{c_o, T_1, M_1\}} \{c_o^{\,e} + d\, J_o(T_1, M_1 | \Omega_o)\}1 \tag{10}$$

Here the notation $x.$ refers to the variable $x$ in any given period, and the notation $x_+$ refers to the variable $x$ in the subsequent period. Note that the form of the value function does not depend on time: the "*o*" subscript refers to the fact that $J_o$ is an underestimate.

The conditional updated value function $\underline{J}$ depends on $q_i$ and$q_{v.}$ because the value of the constraints (2) and (3) depend on the specific realizations of the stochastic shocks. The objective of the iteration process is of course the unconditional value function, which expresses expected present value of utility from a given asset base. The unconditional value function is obtained by summing the conditional value functions over the set of stochastic shocks, and weighting by the appropriate probabilities:[15]

$$J'_o(T_o, M_o | \Omega_o) = \sum_{n=1}^{3} \sum_{m=1}^{3} J'_o\big(T_o, M_o | q_{in} q_{vm}, \Omega_o\big) \cdot Pr(q_i = q_{in} | q_v = q_{vm}) \cdot Pr(q_v = q_{vm}) \tag{11}$$

Since $J_o$ is everywhere a (pointwise) underestimate of the true value function, and since it finds the maximum value---in terms of consumption and accumulation---over its domain, $\underline{J}_o$ will always be at least as large as $J_o$. By repeated applications of Bellman's operator, $\underline{J}_o$ eventually converges to the true value function, $J^*$ (Streufert, 1990). However, since the $J_o(T,M,L)$ are numerical entities, it may not be obvious when they have converged to a limit. For this reason, Bellman's operator is also used to update an upper value function $J^o(T,M,L)$, a known overestimate of $J^*(T,M,L)$. Again, because $J^o$ is

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[15] Mathematically, summing across appropriately weighted stochastic outcomes is no different than

(pointwise) higher than the true value function, successive applications of Bellman's operator will bring $\underline{J}^o$ down toward $J^*(T,M,L)$. When $\underline{J}_o(T,M,L)$ and $\underline{J}^o(T,M,L)$ differ by less than some epsilon for any $(T,M,L)$ combination, then $J^*(T,M,L)$, which is always between the upper and lower value functions, has been identified to within that epsilon.

For this search for a fixed point in value function space to yield the true value function with certainty, the utility function and the production function must together constitute a dynamic problem that is biconvergent. In essence, upper-convergence means that if the largest feasible consumption possibilities are pushed ever further into the future, they matter ever less to present utility. Lower-convergence means that as the lowest feasible consumption possibilities are pushed ever further into the future, they would matter less and less. Upper-convergence and lower-convergence together constitute biconvergence (see Streufert, 1990). Biconvergence guarantees that the transversality condition is met, and that a solution to Koopmans' equation is the same as a solution to the infinite horizon utility maximization problem (i.e., that equation 10 holds). The proof of biconvergence is straightforward, and is presented formally in Zimmerman (1994). Lower-convergence holds automatically because of the form of the utility function. The intuition behind the proof of upper-convergence is that since the profit function under ideal factor allocation is decreasing returns to scale, satiation (in the utility function) and impatience (in the discount rate) imply that agents will not want to infinitely accumulate land or wealth.

## Appendix B Parameterization of the Model based on the ICRISAT Burkina Faso Data

The Data from Burkina Faso were collected from 1981-1985 by the International Crop Research Institute of the Semi-Arid Tropics (ICRISAT). The survey encompassed six sample villages from three different agro-climatic regions of Burkina Faso, and is one of the largest and most comprehensive datasets collected on West African agricultural production. Where possible, parameters were estimated from the ICRISAT data. Utility

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summing across appropriately discounted time periods.

parameters were taken to be reasonable levels. The values of the individual parameters are presented in Table 1.

| **Parameter** | **Notation** | **Value** |
|---|---|---|
| Land productivity | D | 650 kgs/ha |
| Decreasing returns | $\sigma$ | 0.95 |
| Return on non-productive assets | $\mu$ | 0.00 |
| Subsistence requirement | $R_0$ | 800 kgs. |
| Discount factor | $\delta$ | 0.95 |
| Utility curvature | $\varepsilon$ | 0.10 |

**Table 1: Parameter Values for the Numerical Model**

The average land endowment (adjusted for land quality) in the ICRISAT data is 3.5 hectares. Average income is therefore 2137 kgs/year, so that subsistence is 37% of average income. If the poverty line is taken as reasonable proxy for subsistence, then this subsistence line is slightly conservative, but reasonable. Carter (1997) uses the same ICRISAT dataset to estimate production risk in both the Sahel region and the wetter Guinea-Savanna region. The risk parameterization of the model, presented in Table 2, is a stylization of his findings.

|  | Village-Level Shock: $\theta_v$ |  |  |
|---|---|---|---|
|  | **Low** (p=0.3) | **Medium** (p=0.4) | **High (**p=0.3) |
| Idiosyncratic Shock: $\theta_i$ | $\theta_v = 0.75$ | $\theta_v = 1.00$ | $\theta_v = 1.25$ |
| **Low** (p=0.2) | $\theta i = 0.90$<br>$\theta_i\theta_v = 0.675$ | $\theta i = 0.80$<br>$\theta_i\theta_v = 0.800$ | $\theta i = 0.70$<br>$\theta_i\theta_v = 0.875$ |
| **Medium** (p=0.6) | $\theta_i = 1.00$<br>$\theta_i\theta_v = 0.75$ | $\theta_i = 1.00$<br>$\theta_i\theta_v = 1.00$ | $\theta_i = 1.00$<br>$\theta_i\theta_v = 1.25$ |
| **High** (p=0.2) | $\theta_i = 1.10$<br>$\theta_i\theta_v = 0.825$ | $\theta_i = 1.20$<br>$\theta_i\theta_v = 1.20$ | $\theta_i = 1.30$<br>$\theta_i\theta_v = 1.625$ |

**Table 2: Risk Structure**

## *References*

Atkeson, Andrew, and Ogaki, Masao (1996). "Wealth-varying intertemporal elasticities of substitution: Evidence from panel and aggregate data." *Journal of Monetary Economics,* Vol. 38, pp. 507-534.

Atkinson, A.B. (1997). "Bringing Income Distribution in from the Cold," *Economic Journal* 107 (March): 297-321.

Bardhan, P., S. Bowles, and H. Gintis. 1998. "Wealth Inequality, Wealth Constraints and *Handbook of Income Distribution*. A. Atkinson and F. Bourguignon, North Holland.

Behrman, Jere R. (1990). *The action of human resources and poverty on one another: What we have yet to learn.* Living Standards Measurement Study Working Paper, no. 74; Washington, D.C.: World Bank.

Besley, T. (1995). "Savings, Credit and Insurance," in J. Behrman and T.N. Srinivasan (eds.) *Handbook of Development Economics* vol. 3a (North Holland).

Benjamin, Dwayne (1995). "Can Unobserved Land Quality Explain the Inverse *Journal of Development Economics,* Vol. 46, pp. 51-84.

Binswanger, Hans, McIntire, John, and Udry, Chris (1989). "Production Relations in Semi-arid African Agriculture." In "*The economic theory of agrarian institutions* (ed. P. Bardhan). Oxford; New York; Toronto and Melbourne: Oxford University Press.

Bliss, Christopher, and Stern, Nicholas (1978). "Productivity, Wages and Nutrition: Part *Journal of Development Economics*, Vol. 5, pp. 331-62.

Brennan, Donna, Williams, Jeffrey, and Wright, Brian D. (1997). "Convenience Yield without the Convenience: A Spatial-Temporal Interpretation of Storage under Backwardation." *Economic Journal*, Vol. 107, pp. 1009-22.

Carter, Michael R. (1984). "Identification of the Inverse Relationship between Farm Size and Productivity: An Empirical Analysis of Peasant Agricultural Production." *Oxford Economic Papers*, Vol. 36, pp. 131-45.

Carter, Michael R. (1997). "Environment, Technology, and the Social Articulation of Risk in West African Agriculture." *Economic Development and Cultural Change*, Vol. 45, pp. 557-90.

Coate, Stephen, and Ravallion, Martin (1993). “Reciprocity without Commitment: Characterization and Performance of Informal Insurance Arrangements.” *Journal of Development Economics,* Vol. 40, pp. 1-24.

Coquéry-Vidrovitch, Catherine (1985). *Afrique Noire: Permanences et Ruptures.* Paris: Payot.

Cornia, Giovanni Andrea (1989). “Investing in Human Resources: Health, Nutrition and *Journal of Development Planning,* Vol. , pp. 159-187.

Dasgupta, Partha, and Ray, Debraj (1990). “Adapting to Undernourishment: The Biological Evidence and Its Implications.” In “*The political economy of hunger* (ed. J. Dreze and A. Sen), Vol. 1. Oxford; New York; Toronto and Melbourne: Oxford University Press.

Deaton, Angus (1991). “Saving and Liquidity Constraints.” *Econometrica,* Vol. 59, pp. 1221-48.

Dercon, Stefan (1996). “Risk, Crop choice and savings: Evidence from Tanzania.” *Economic Development and Cultural Change*, Vol. 44(3), pp. 485-514.

Dercon, Stefan (1998). “Wealth, risk and activity choice: cattle in Western Tanzania.” *Journal of Development Economics*, Vol. 55, pp. 1-42.

Dixit, Avinash K., and Pindyck, Robert S. (1994). *Investment under uncertainty.* Princeton: Princeton University Press.

Dreze, Jean, and Sen, Amartya (1989). “Famines and Social Response.” In *Hunger and Public Action*. Oxford: Clarendon Press.

Eswaran, Mukesh and Ashok Kotwal (1990). “Implications of credit constraints for risk behaviour in less developed countries.” *Oxford Economic Papers,* Vol. 42, pp. 473-482.

Gardner, Bruce (1979). “Robust Stabilization Policies for International Commodity *American Economic Review,* Vol. 69, pp. 169-72.

Glomm, Gerhard, and Palumbo, Michael G. (1993). “Optimal Intertemporal Consumption Decisions under the Threat of Starvation.” *Journal of Development Economics,* Vol. 42, pp. 271-91.

Kochar, Anjini (1994). “Uncertainty and Poverty: Evaluating the Link in Developing Countries.” Mimeo, Stanford University.

Kochar, Anjini (1995). “Explaining Household Vulnerability to Idiosyncratic Income *American Economic Review* 85(2):159-64.

Lipton, Michael (1993). Growing Points in Poverty Research: Labour Issues. *In* “ILO Symposium: Poverty: New Approaches to Analysis and Policy”.

Morduch, Jonathan (1994). “Poverty and Vulnerability.” *American Economic Review,* Vol. 84, pp. 221-25.

Morduch, Jonathan (1995). “Income Smoothing and Consumption Smoothing.” *Journal of Economic Perspectives,* Vol. 9, pp. 103-14.

Moser, Caroline (1998). “The Asset Vulnerability Framework: Reassessing Urban *World Development* 26(1):1-19.

Ogaki, Masao, and Atkeson, Andrew (1997). “Rate of Time Preference, Intertemporal Elasticity of Substitution, and Level of Wealth.” *Review of Economics and Statistics,* Vol. 79, pp. 564-72.

Paxson, Christina H. (1992). “Using Weather Variability to Estimate the Response of Savings to Transitory Income in Thailand.” *American Economic Review,* Vol. 82, pp. 15-33.

Paxson, Christina H. (1990). “Borrowing Constraints and Portfolio Choice,” *Quarterly Journal of Economics* 105(2):535-43.

Platteau, Jean-Philippe (1991). “Traditional Systems of Social Security and Hunger Insurance: Past Achievements and Modern Challenges.” In *Social security in developing countries* (ed. E. Ahmad and et al.). Oxford; New York; Toronto and Melbourne: Oxford University Press.

Ray, Debraj, and Streufert, Peter A. (1993). “Dynamic Equilibria with Unemployment *Economic Theory,* Vol. 3, pp. 61-85.

Reardon, Thomas, Delgado, Christopher, and Matlon, Peter (1992). “Determinants and Effects of Income Diversification amongst Farm Households in Burkina Faso.” *Journal of Development Studies,* Vol. 28, pp. 264-96.

Reardon, Thomas, Matlon, Peter, and Delgado, Christopher (1988). “Coping with Household-Level Food Insecurity in Drought-Affected Areas of Burkina Faso.” *World Development,* Vol. 16, pp. 1065-74.

Reardon, Thomas, and Vosti, Stephen A. (1995). “Links between Rural Poverty and the Environment in Developing Countries: Asset Categories and Investment Poverty.” *World Development,* Vol. 23, pp. 1495-1506.

*Economic Journal,* Vol. 103, pp. 56-78.

Rosenzweig, Mark R., and Wolpin, Kenneth I. (1993). “Credit Market Constraints, Consumption Smoothing, and the Accumulation of Durable Production Assets in Low-Income Countries: Investment in Bullocks in India.” *Journal of Political Economy,* Vol. 101, pp. 223-44.

Streufert, Peter A. (1990). “Stationary Recursive Utility and Dynamic Programming under the Assumption of Biconvergence.” *Review of Economic Studies,* Vol. 57, pp. 79-97.

Subramanian, Shankar, and Deaton, Angus (1996). “The Demand for Food and Calories.” *Journal of Political Economy,* Vol. 104, pp. 133-62.

Townsend, Robert M. (1994). “Risk and Insurance in Village India.” *Econometrica,* Vol. 62, pp. 539-91.

Udry, Christopher (1994). “Risk and Insurance in a Rural Credit Market: An Empirical Investigation in Northern Nigeria.” *Review of Economic Studies,* Vol. 61, pp. 495-526.

Udry, Christopher (1995). “Risk and Saving in Northern Nigeria.” *American Economic Review,* Vol. 85, pp. 1287-1300.

von Braun, Joachim, Hotchkiss, David, and Immink, Maarten (1989). *Nontraditional export crops in Guatemala: Effects on production, income and nutrition.* Research Report Series, no. 73. Washington, D.C.: International Food Policy Research Institute.

Zimmerman, Frederic J. (1994). *Structural Change and Evolving Institutions in Developing Agrarian Economies: A Dynamic Programming Simulation Analysis,* unpublished Ph.D. Dissertation, Economics Department, University of Wisconsin.