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Consumer Promotion and Purchase Timing: The Case of Cheese

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Abstract

The dynamics of cheese purchases is analyzed by estimating a series of econometric models of duration based on a 170 week household panel. Besides purchase quantity and price data, information with respect to coupon use and household demographic characteristics are used in a variety of models which build upon each other in terms of assumed distribution of interpurchase time, effect of previous purchases, role of demographic characteristics and effect of unobserved interpurchase time heterogeneity. Likelihood ratio tests clearly reject the null hypothesis that coupon use has no impact on cheese purchase timing.

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Consumer Promotion and Purchase Timing: The Case of Cheese

The U.S. dairy industry has responded to decreasing per capita dairy product consumption by adopting industry-wide advertising and promotion campaigns. Examples of these programs include the 1995 $52 million Fluid Milk Processor Education Program and the cheese, butter and other dairy product promotion efforts of the National Dairy Promotion and Research Board. Recent evaluations of several dairy advertising campaigns have indicated a positive net effect of such programs (Forker and Kinnucan, 1991; Cheese Reporter, 1995; Sun, Blisard and Blaylock, 1995). For example, Kaiser and Roberte(1995) evaluate generic fluid milk advertising in New York City over the January 1986 - December 1992 period and found a significant impact on whole and low fat milk demand. Larson(1992) argues for continued expansion of such generic agricultural commodity promotion programs given the positive secondary benefits of increased effectiveness of associated branded promotion efforts and protection of market share from processed substitute products.

Previous evaluations of the effect of promotion efforts on U.S. dairy product demand have tended to focus on generic advertising programs at the aggregate commodity or market level (Blaylock and Blisard, 1988, 1990; Kinnucan and Forker, 1991; Sun, Blisard and Blaylock, 1995; Ward and Dixon, 1989; Ward and McDonald, 1986).[1] Little work has been done at the household level to analyze the impact of dairy product promotion or to analyze the impacts of non-advertising promotion efforts. With proposed changes in U.S. dairy policy to a more market oriented system, the use of both advertising and non-advertising promotion efforts to maintain industry revenues can be expected to increase. Understanding household impacts is important to understanding the potential benefits of such efforts.

Household level analyses of the impacts of commodity promotion programs typically hypothesize impacts on both quantity and timing of product purchases (Neslin, Hendersen, and Quelch, 1985; Gupta, 1988,1991). Figure 1 provides one representation of the relationship between product promotion, quantity purchased and the dynamics of these purchases. With commodity promotion effort, there may be a direct positive impact on quantity purchased and a negative impact on the length of time between purchases. With less time between purchases, this implies that household stocks may be larger. With larger household stocks, the lower the amount purchased. The net effect of promotion, therefore depends on the direct purchase impacts and indirect stock effects. The possible conflict of the direct and indirect quantity impacts implies that unless shorter interpurchase time is recognized, comparing quantity consumed with and without product promotion may overestimate the true amount by which a particular promotion campaign increases demand (Neslin, Hendersen, and Quelch, 1985, p.150). In terms of dairy food manufacturers, dairy product promotion efforts may result in increase sales during a promotion period but the net impact will depend on the extent to which this increase is due to consumers switching from other products versus an acceleration of purchases by current consumers but with no increase in total purchases (Blattberg, Eppen, and Lieberman, 1981).

The phenomenon of purchase acceleration may be viewed positively or negatively from the retailers view depending on marketing environment. For example, in a competitive marketing environment, purchase acceleration may be an explicit objective undertaken by retailers so as to counter-act anticipated marketing efforts of competitors. Alternatively, without such a predatory situation, purchase acceleration may result in consumers stockpiling the commodity that they would have purchased regardless of the promotion (Neslin, Hendersen and Quelch, 1985).

In the present analysis and as a first step in investigating the relationships shown in Figure 1, we focus on the dynamics of the consumer purchase process. In particular, we examine the effect of coupon-based price deals on interpurchase times. We apply econometric models of duration to a frequently purchased food commodity, cheese. We conduct this analysis by using a data set which consists of a household panel observed over 170 consecutive weeks from March, 1991 to June 1994. Besides purchase quantity and price, information with respect to coupon use and household demographic characteristics are available. The models presented below build upon each other in terms of assumed distribution of interpurchase time, effect of previous purchases, role of demographic characteristics and effect of unobserved interpurchase time heterogeneity.

Description of the Model of Interpurchase Time

Early approaches used to model non-durable product interpurchase time ignored the effect of time varying marketing and demographic variables and assumed the number of product purchased over successive time periods were independent and followed a Poisson distribution (Chatfield and Goodhardt, 1973, p.828; Dunn, Reader, and Wrigley, 1983, p. 250). Under the Poisson assumption, the consumer's interpurchase time follows an exponential distribution. Given this exponential distribution, interpurchase time has the following density, **f(t)**, survivor function, **S(t)**, and hazard rate **H(t)**:

$$(1)\quad \begin{aligned} f(t) &= \lambda\, e^{(-\lambda t)} \\ S(t) &= P(T > t) = e^{(-\lambda t)} \\ H(t) &= \lim_{\Delta t \to 0} \frac{P(t + \Delta t > T \geq t \mid T \geq t)}{\Delta t} = \frac{f(t)}{S(t)} = \lambda \end{aligned}$$

where λ is the distribution's shape paramater, T a random variable of duration of interpurchase time, t interpurchase time, **S(t)** the probability of not purchasing before interpurchase time t, and **H(t)** the purchase probability between the time t and t+Δt given that there has not been a previous purchase (Yamaguchi, 1991).

The distribution paramater, λ, can be estimated using the log-likelihood function (LL_i):

$$(2)\quad \begin{aligned} LL_i(I|\lambda) &= \ln\left(\prod_{j=1}^{n_i+1} H(t_{ij})^{\rho_{ij}}\, S(t_{ij}) \right) \\ &= n_i \ln(\lambda) - \lambda(t_{is} + t_{ic}) \quad (i = 1, \ldots, N) \end{aligned}$$

where I is the vector of interpurchase time, t_{ij} the interpurchase time for the i^{th} individual on the j^{th} event, n_i the number of events (purchase occasions), t_{is} the sum of n_i interpurchase times and N households. To account for the possibility that the last observation may be right censored (i.e., no purchase has occurred by the last time period of observation for the i^{th} individual), ρ_{ij} is the Kronecker delta equal to 1 if there is a purchase on the j^{th} event, 0 if there is right censoring, and t_{ic} censored time which is zero if the i^{th} observation is not censored (Gupta, 1991, p.3; Yamaguchi, 1991).

From (2), if at time t_{ij} a purchase occurs, the contribution to the likelihood function of each purchase occasion is the probability density function of the occurrence of the event at time t_{ij}, $\mathbf{f(t_{ij})}$, the product of $\mathbf{H(t_{ij})}$ and $\mathbf{S(t_{ij})}$ from (1). If an observation is right censored, the contribution to the overall likelihood function is the probability of not having the event occur between 0 and t_i, $\mathbf{S(t_{ij})}$ (Yamaguchi, 1991, p.11).[2]

As shown by the hazard rate in (2), one shortcoming of assuming an exponentially distributed interpurchase time is that a consumer's purchase probability for the next period is unaffected by the length of time since last purchase (Gupta, 1991; Chatfield and Goodhardt, 1973; Jeuland, Bass and Wright, 1980). To overcome this shortcoming many researchers have adopted the gamma distribution and in particular the Erlang-2 form in analyses of non-durable purchase duration times (Herniter, 1971; Chatfield and Goodhardt, 1973; Zufryden, 1978; Jeuland, Bass and Wright, 1980; Gupta, 1988).[3]

When interpurchase times are distributed according to Erlang-2, the density function, survivor function, and hazard rates are:

$$(3) \quad \begin{aligned} f(t) &= \lambda^2 t\, e^{(-\lambda t)} \\ S(t) &= (1 + \lambda t) e^{(-\lambda t)} \\ H(t) &= \frac{f(t)}{S(t)} = \frac{\lambda^2 t}{(1 + \lambda t)} \end{aligned}$$

where λ is the distribution location paramater (Gupta, 1991). Thus from (3), the hazard rate is now dependent on interpurchase time. The associated likelihood function used to estimate λ is:

$$(4) \quad \begin{aligned} L_i(I|\lambda) &= \prod_{j=1}^{n_i} f(t_{ij}) S(t_{ic}) = \left[\prod_{j=1}^{n_i} \lambda^2 t_{ij}\, e^{(-\lambda t_{ij})}\right] (1 + \lambda t_{ic}) e^{(-\lambda t_{ic})} \\ &= \lambda^{2n_i} t_{ip}\, e^{(-\lambda(t_{is} + t_{ic}))} (1 + \lambda t_{ic}) \quad (i = 1,\ldots,N) \end{aligned}$$

where t_{ip} the product of interpurchase times. The associated log-likelihood function is:

$$(5) \quad LL_i(I|\lambda) = 2\, n_i \ln(\lambda) + \ln(t_{ip}) - \lambda(t_{is} + t_{ic}) + \ln(1 + \lambda t_{ic}) \quad (i = 1,\ldots,N)$$

The above model assumes that all consumers have the same hazard rate function given the constant λ distribution paramater. In reality differential purchase rates exist across consumers due to household and market characteristics (Gupta, 1988; Helsen and Schmittlein, 1992, 1993; Ward and Davis, 1978a; Neslin, Hendersen and Quelch, 1985). A method that has been used to allow for heterogeneity in hazard rates is to assume that λ is randomly distributed across interpurchase time. If we assume λ to have a gamma distribution with paramaters, α and ω, the unconditional likelihood function in (4) for the i^{th} consumer becomes:

$$(6) \quad L_i(I) = \int_0^{\infty} L_i(I|\lambda)\,\Psi(\lambda)\,d\lambda$$

where $L(I|\lambda)$ is the conditional likelihood shown in (3), $\Psi(\lambda)$ is the gamma distribution:

$$(7) \quad \Psi(\lambda) = \frac{\alpha}{T(\omega)}(\alpha\lambda)^{(\omega-1)}e^{(-\alpha\lambda)}$$

and $T(\omega)$ is the gamma function (McDonald and Butler, 1987, p.232; Wagner and Taudes, 1986).[4] The resulting log-likelihood function with the assumption of interpurchase time being distributed Erlang-2 with gamma heterogeneity is:

$$(8) \quad LL_i(t|\lambda) = \omega\ln(\alpha) + \ln t_{ip} + \sum_{j=0}^{2n_i-1}\ln(\omega + j) + \ln\left(1 + \frac{(2n_i + \omega)t_{ic}}{t_{is} + t_{ic}}\right) - (2n_i + \omega)\ln(t_{is} + t_{ic} + \alpha) \quad (i = 1,\ldots,N)$$

Although (6) allows λ to have a distribution, the effect of exogenous factors on the distribution of interpurchase time has still not been incorporated. As an alternative to (6) and (7), market and household characteristics can impact interpurchase time via the following:

$$(9) \quad \lambda_i(t) = \lambda_0 e^{(\beta X_{it})}$$

where X_{it} are time dependent explanatory variables and β and λ_0 are paramaters to be estimated. Using (9) along with (5) results in interpurchase time being the result of a nonhomogeneous Poisson process (Gupta, 1991). Assuming λ_0 time invarient, the survivor and density functions are:

(10)
$$S(t) = \left(1 + \int_0^t \lambda(\tau)d\tau\right) \exp\left(-\int_0^t \lambda(\tau)d\tau\right) = (1 + \Delta(t))\exp(-\Delta(t))$$
$$f(t) = \lambda(\tau)\left(\int_0^t \lambda(\tau)d\tau\right) \exp\left(-\int_0^t \lambda(\tau)d\tau\right) = \lambda(t)\Delta(t)\exp(-\Delta(t))$$

where: $\Delta(t) = \lambda(1)d_1 + \sum_{w=2}^{k-1} \lambda(w) + \lambda(k)\big(t - d_1 - \gamma(k - 2)\big)$

and d_1 is the proportion of the first week included in the first purchase, k is the curret week of purchase (with the week of last purchase set to 1), γ equal to 1 if k equals 1 and $\lambda(\cdot)$ defined by (9) (Gupta, 1991, p.6). The resulting log-liklihood function can be shown to be:

(11) $$LL_i(I|\lambda_0) = 2n_i\ln\lambda_0 + \ln\Gamma_i^p + \ln\Omega_i^p - \lambda_0\left(\Omega_i^s + \Omega_i^c\right) + \ln\left(1 + \lambda_0\Omega_i^c\right)$$

where: $\Gamma_i^p = \prod_{j=1}^{n_i} \Gamma_{ij} = \prod_{j=1}^{n_i} e^{(\beta X_{ij})}$; $\Omega_i^p = \prod_{j=1}^{n_i} \Omega_{ij}$; $\Omega_i^s = \sum_{j=1}^{n_i} \Omega_{ij}$

$$\Omega_{ij} = \left(\Gamma_{i1}d_{i1} + \sum_{w=2}^{j-1} \Gamma_{iw} + \Gamma_{ij}\left[t_{ij} - d_{i1} - \gamma(j - 2)\right]\right)$$

$$\Omega_i^c = \left(\Gamma_{i1}d_{i1} + \sum_{w=2}^{j_c-1} \Gamma_{iw} + \Gamma_{ij_c}\left[t_{ic} - d_{i1} - \gamma(j_c - 2)\right]\right)$$

The above model assumes no unobserved heterogeneity in the hazard rate function given that λ_0 is time and household invarient. Unobserved heterogeneity in consumers' hazard rates can be accounted for by allowing λ_0 to be distributed across the population. If we assume λ_0 to be distributed according to the gamma distribution shown in (7), the likelihood function which allows for both unexplained heterogeneity and time-varying covariates can be represented as:

(12) $$L_i(I) = \int_0^\infty L_i(t|\lambda_0)g(\lambda_0)d\lambda_0 = \frac{\alpha^\omega \Gamma_i^p \Omega_i^p}{\Omega_i^s + \Omega_i^c + \alpha^{(2n_i + \omega)}}\left[1 + \frac{(2n_i + \omega)\Omega_i^c}{\Omega_i^s + \Omega_i^c + \alpha}\right]\left[\prod_{j=0}^{(2n_i - 1)} (\omega + j)\right]$$

with log-likelihood:

$$(13) \quad LL_i\left(I|\lambda_0\right) = \omega \ln(\alpha) + \ln\Gamma_i^p + \ln\Omega_i^p + \sum_{j=0}^{2n_i-1} \ln(\omega + j)$$

$$\ln\left(1 + \frac{(2n_i + \omega)\Omega_i^c}{\Omega_i^s + \Omega_i^c + \alpha}\right) - (2n_i + \omega)\ln\left(\Omega_i^s + \Omega_i^c + \alpha\right) \quad (i = 1,\ldots,N)$$

With this formulation, differences in interpurchase times are accounted for by two components: one which allows for heterogeneity in base hazard rate which is time invariant and a second component which assumes homogeneity of the effect of changes in time-dependent covariates on interpurchase time but allows for differences in these time-dependent covariates (X_{it}) across households.

Description of the Consumer Panel and Explanatory Variables

We apply the theoretical models represented by (5), (8), (11), and (13) to an analysis of U.S. cheese purchases. The purchase data used are obtained from a March, 1991-June, 1994 U.S. consumer panel maintained by Nielsen Marketing Research (NMR). Only cheese purchased for at-home consumption is included in this data.[5] On each purchase occasion a panel member records: date, UPC code, expenditures and quantity purchased. This recording process is conducted at home via the use of hand held UPC scanners. Households notify NMR if no purchases had occurred during the previous week because of not purchasing during a given week or the result of being away from home due to vacation, business trip, etc. For this analysis we include households that reported continuously over 170 weeks. This does not imply that households in the panel purchased each week but during weeks where cheese was not purchased for at-home consumption, NMR was given this information. Given the size of the household panel we randomly selected households from the continuous panel. In order to avoid extremely long interpurchase times, we include households that have more than 2 purchase occasions each year over the 170 week period. Purchase opportunities and occasions are defined on a weekly basis.

We apply our econometric model to four different cheeses: *all, processed, natural cheddar,* and *cottage cheese*.[6] Table 1 provides an overview of the samples' cheese purchase characteristics. For all cheese, over 18 percent of the purchase weeks occurred with some type of cents-off coupon. This is similar to that observed for processed cheese. Only 3 percent of cottage cheese purchase occasions involve the use of some type of coupon. In contrast to our initial hypotheses, we see reduced "coupon week" interpurchase times only for all cheese. This result may be deceiving given that there are other market and demographic characteristics impacting interpurchase time as well as coupon use. The duration model estimated here isolates the impact of each of these variables on purchase timing.

Purchase Characteristics Affecting Timing

For interpurchase time models represented by (11) and (13) we incorporate household and market variables as distribution shifters. Table 2 provides the definition of these variables along with sample means and expected direction of the impact of changes in these variables on purchase hazard rate. From Figure 1, beginning period inventory is hypothesized to have a direct impact on interpurchase time (Neslin, Henderson and Quelch, 1985; Gupta 1988, 1991). Beginning household inventory equals last periods inventory plus purchases made last period minus consumption. Jain and Vilcassim(1991) outline several empirical problems with estimating household inventory from a data set that does not explicitly collect such information. In order to avoid potential biases in making assumption concerning initial inventories and estimating household consumption rates, we follow Jain and Vilcassim(1991) by including lagged volume purchases, PURCH_LAG, as an explanatory variable (Helsen and Schmittlein 1992, 1993). We hypothesize that interpurchase time would increase (and hazard rate would decrease), the greater the amount purchased on the last occasion. Thus we expect a negative coefficient for this variable in the empirical applications of (11) and (13) (Jain and Vilcassim, 1991).

As suggested by Gupta(1988, 1991) we would expect that a price drop may cause the consumer to purchase earlier than usual (increase the rate of purchase). To examine this effect we include the variables POS_CHANGE and NEG_CHANGE. The first variable represents the relative change in product price since the last purchase when there is a shelf price increase. NEG_CHANGE is similarly defined except for a price decrease. Following, Rajendran and Tellis, we could have used previous purchase price as a point of comparison. Instead, we use a measure of the consumers "reference price". For the present analysis we define reference price as:

$$\text{(14)} \quad \text{Ref\_Price}_c \equiv 0.571 * \text{Price}_{c-1} + .286 * \text{Price}_{c-2} + .143 * \text{Price}_{c-3}$$

where c refers to purchase occasion, and Price is per pound shelf price. The use of the above declining weights approximates a geometric function with a common ratio of .5 (Rajendran and Tellis, p.27).

From (14), POS_CHANGE and NEG_CHANGE are defined as:

$$\text{(15)} \quad \text{POS\_CHANGE} \equiv \begin{cases} \dfrac{\text{Ref\_Price}_c - \text{Price}_c}{\text{Ref\_Price}_c} & \text{if } \left(\text{Ref\_Price}_c - \text{Price}_c\right) < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$\text{NEG\_CHANGE} \equiv \begin{cases} \dfrac{\text{Ref\_Price}_c - \text{Price}_c}{\text{Ref\_Price}_c} & \text{if } \left(\text{Ref\_Price}_c - \text{Price}_c\right) > 0 \\ 0 \text{ otherwise} \end{cases}$$

With a price increase relative to the reference price, we would expect an increase in interpurchase time, decreased purchase probability and therefore a negative POS_CHANGE coefficient . Conversely, a positive coefficient is hypothesized for NEG_CHANGE. Using these two variables we can test the hypothesis that the impacts of a price increase or decrease are symmetric but of the opposite sign.

The variable used to reflect the effect of coupon redemption on interpurchase time is COUP_VALUE which represents the value of these coupons. Ward and Davis(1978b), using a similar variable, found that coupon redemption rates are determined, in part, by coupon face value. If coupon redemption induces a stockpiling type of behavior in which consumers purchase sooner than usual than we would expect a positive coefficient for this variable in (11) and (13).

Previous analyses of cheese demand have shown that during the holiday period, cheese demand increases (Sun, Blisard and Blaylock; 1995, Blaylock and Smallwood; 1983, 1986). The variables SUMMER and HOLIDAY are used to account for hypothesized differential hazard rates during the summer months, June, July and August, and over the Thanksgiving and Christmas holiday period, respectively. We hypothesize increased probabilities of purchase during these times, shorter interpurchase times and therefore positive coefficients for these variables.

Household Characteristics Affecting Purchase Timing

Demographic characteristics such as household size, composition, ethnicity and income have been shown to be important determinants of cheese demand (Blaylock and Blisard, 1983, 1986; Heien and Wessells, 1990). In the present analysis the variable INV_HHSIZE represents the inverse of the number of resident household members.[7] One would expect that the greater the household size, the more quickly household inventories will be depleted, ceteris paribus, implying a shorter interpurchase time and an increased hazard rate. Thus we would expect a negative coefficient for this variable (Jain and Vilcassim, 1991). We control for household composition by including the variables PER≤5, PER6_13, and PER14_18. Since our data includes birth month and year for each household member, we update these variables on a monthly basis. It is unclear as to the impact of having young children in the household on overall cheese purchases. Young people eat less compared to adults but many adults face dietary restrictions on the amount of fat and cholesterol that can be consumed. Three separate composition variables are included so as to allow for differential impacts of children on household interpurchase time as children age.

The characterization of the ethnicity of each household is based on characteristics of the main meal planner which was assumed to be the female head, if present. Previous Tobit-based analysis of U.S. household cheese consumption have found that non-white households had a lower probability and consumed less than white households (Blaylock and Smallwood, 1983, 1986) The variables BLACK and HISPANIC are used to capture differences across ethnic groups. If this decreased in demand is reflected in both the number of purchases and purchases per occasion, we would expect negative coefficients for these variables in (11) and (13).

Household pre-tax income in the data set is reported in 16 categories ranging from less than $5,000 to more than $100,000. To convert these categorical data to a continuous form, we assumed the midpoint of each category to be household income. For households with income above $100,000 an income of $150,000 was assumed. To control for household size, composition *and* income, the variable POV_RATIO is the ratio of household pre-tax income to poverty threshold income as defined by the Bureau of Census (Department of Commerce, 1995). Poverty income is used by the Bureau of Census to estimate the number of individuals and families in poverty. Poverty threshold income levels are dependent on number and age distribution of household members. We are unsure of the effect of this variable on interpurchase time. For low income households, the ability to purchase large amounts per purchase occasion may be limited, thus implying relatively short interpurchase times, ceteris paribus. Alternatively, given limited income, they may purchase smaller total amounts, thus implying longer interpurchase time. The variables SUBURB and RURAL are included to account for differences in purchase patterns for households that live in suburban and rural areas when compared to central cities. These unique purchase patterns may be the result of differences in opportunity costs of shopping trips, real incomes, etc.

Characterization of the Distribution of Interpurchase Times

Sixteen models were run: four cheeses under four distribution assumptions of interpurchase time and gamma heterogeneity. Estimation was undertaken using the Maximum Likelihood module within the GAUSS software system. From the above discussion, the Erlang-2 model (5) is not nested within the Erlang-2 with heterogeneity. As such the use of a likelihood ratio test is not appropriate for evaluating the performance of one model over the other. In order to evaluate heterogeneity versus no-heterogeneity assumption we adopt the method suggested by Schwarz and reviewed by Rust and Schmittlein. The test statistic developed by Schwarz is:

$$(15) \quad \zeta \equiv \ln(LL) - \tfrac{1}{2}(\ln N)\,P^{*}$$

where P^* is the number of estimated parameters. In comparing non-nested models, larger values of ζ indicate the preferred model.

Table 3 presents the likelihood function values for the 16 estimated models. The combination of gamma heterogeneity and time-varying covariates are presented for each cheese. Given the nested nature of the models with and without covariates, likelihood ratio tests of the null hypothesis of no impact of covariates is presented in the column (4). For example, the χ^2

statistic of 23764.2 is the result of testing the null hypothesis of the Erlang-2 model for All Cheese without gamma heterogeneity or time varying covariates (5) versus a similar model with time-varying covariates (11). In order to compare non-nested models the ζ statistic is presented in the last column of Table 3. The results of both the likelihood ratio tests and relative values of the ζ statistics indicate, that similar to Gupta(1991), the preferred model for each cheese is the version where gamma heterogeneity and time-varying covariates are included. Table 4 presents the paramater estimates for this model for each cheese.

Impact of Purchase Characteristics

Purchase characteristics coupon use, price change, lagged inventories and seasonality were found to impact interpurchase time although their importance varied by characteristic and cheese. Of particular importance for this analysis is the hypothesis test concerning coupon redemption and interpurchase time. From the parameter estimates presented in Table 4, the COUP_VALUE coefficients are of the expected sign for all cheese types. Likelihood ratios were used to test the null hypothesis that coupon redemption does not accelerate cheese purchases. This hypothesis is clearly rejected for all cheese types given the large χ^2 statistics (Table 5). The impact of changes in shelf-price is as hypothesized given negative POS_CHANGE and positive NEG_CHANGE coefficients for all cheeses. Interpurchase time was found to increase with shelf price increases while a price decrease increases the hazard rate and therefore decreased interpurchase time. We tested the null hypothesis that the price response was symetric. The resulting χ^2 statistic in Table 5, indicates a rejection of this hypothesis providing justification for the differentiation of price changes.

Using the POS_CHANGE, NEG_CHANGE, and COUP_VALUE coefficients in Table 4, we estimate the effect of changes in these variables on cheese specific hazard rates. Putting these in elasticity form, the impacts are presented in Table 6.[8] All of the price change elasticities are inelastic. Cottage cheese appears to be the lease sensitive to price changes. The coupon-value elasticities are inelastic except for cheddar cheese.

For processed and cheddar cheese, larger lagged purchases increases interpurchase time. Surprisingly, we find a positive impact of lagged purchases on cottage cheese hazard rates and negative impact on interpurchase time. Seasonality was found to impact purchase timing with all and cheddar cheese having longer interpurchase times during summer months and cottage cheese having shorter interpurchase time. The only cheese where interpurchase time varied during the holiday season was cottage cheese with increased interpurchase times. The above results may be viewed in conflict with previous analyses of the cheese market. Differences in how the dynamics of the purchase process are incorporated, the focus on quantity demanded, the use of more aggregated cheese definitions and length of time encompassed by the data set used in these analyses may explain the results obtained here versus these previous analyses. For example, Sun, Blisard, and Blaylock use time series (monthly) data to estimate conditional per household cheese demand. They found that household demand for natural (cheddar) cheese increased significantly during November and December. The insignificant HOLIDAY impacts are not in conflict with Sun, Blisard and Blaylock's results given our focus on interpurchase time and the above study's concern with quantity demanded and no consideration of purchase dynamics. Our results do not rule out that there may be purchased more per visit during the holiday periods, just that the timing of visits is not impacted.

Impact of Household Characteristics

In Table 2, we hypothesized the sign of the impact of demographic characteristics on the distribution of interpurchase time. As expected, household size has a negative impact on interpurchase time. For each cheese type, the greater the proportion that are young children, the shorter the interpurchase time. For all and cottage cheese, the presence of older children has a positive impact on the hazard rate and decreased interpurchase time while the opposite impact was obtained for cheddar cheese. The effect of ethnicity varied across cheese type. Relative to non-minority households, black households tend to exhibit shorter interpurchase times for cheddar cheese. Hispanic households show shorter interpurchase times for cheddar and cottage cheese and longer interpurchase time for processed cheese. In terms of the impact of household income the greater household income relative to poverty threshold income, the larger the interpurchase time for processed and cheddar cheese.

Blaylock and Smallwood (1983) use a single period cross-sectional data set to analyze cheese demand. Again, this study was not able to investigate the dynamics of purchases. They found that non-white households were less likely to purchase all types of cheese investigated.

They also found that households with children differed in their cheese purchase patterns. The effect of children were found to differ across variety. Similar to the comparison to the results of Sun, Blisard and Blaylock, a comparison with our results is difficult given the difference in endogenous variables and time period encompassed by the data set used in the analysis.

Simulated Hazard Rate Profiles and Survival Functions

The results of the likelihood ratio tests and the ζ statistics indicate that the duration model incorporating household characteristics and gamma heterogeneity represented by (13) is the preferred model. Using the results from these models we simulate hazard rates profiles for the above four cheeses. These hazard rate profiles show the instantaneous probability of a household purchasing given the number of weeks since last purchase. Figure 2 shows hazard rates for non-minority, Black, Hispanic, and single person households.[9] From this figure we see that as interpurchase time increases, Hispanic households exhibit the largest instantaneous probability (hazard rate) of purchase for all cheese, while single person households show the lowest. For all cheese, after 4 weeks, the hazard rate for the simulated single-person household is slightly more than .48 compared to more than .67 for Hispanic households. Single-person households have the lowest hazard rates except for cottage cheese where Black households exhibit the lowest profile.

In contrast to the above hazard rate profiles, Figure 3 shows "survival" probability profiles based on (3). These profiles show the probability of a household not purchasing a particular cheese as interpurchase time increases. Given (3) the relative position of each household type will be the opposite of that observed in Figure 2. For all cheese, after approximately 6 weeks, all households have less that a 10% probability of not purchasing by this time. At 3 weeks, there is a 41% probability that single-person households will not have purchased any cheese. This compares with 36% for Black, 28% for non-minority, and 28% for Hispanic households. A similar pattern was observed for processed cheese with 3-week "survival" probabilities of 58%, 54%, 47% and 46% for Single, Black, non-minority, and Hispanic households respectively.

Summary and Areas of Future Research

The use of coupon-based incentive programs continue to be an important marketing tool. The present analysis investigates one facet of coupon usage, namely its impact on the timing of purchases for a frequently purchased non-durable commodity, cheese. We estimate a series of duration models for four cheese classifications: *all, processed, natural cheddar,* and *cottage* cheese. We incorporate within these duration models household demographic and purchase characteristics that allow for the distribution of interpurchase times to vary across households. The results of likelihood ratio tests indicate that these characteristics are statistically significant factors impacting the distribution of interpurchase time.

A likelihood ratio test of the null hypothesis that coupon use has no impact on the timing of cheese purchases is clearly rejected. As hypothesized, the use of coupons result in reduced interpurchase times for all cheeses. This impact, however varies across cheese type, especially when considering the type of household doing the purchasing. Elasticity estimates indicate inelastic hazard rate responses to price changes and to the use of coupons. Only in the use of coupons in the purchase of cheddar cheese, is the hazard rate elasticity greater than 1.

The present analysis has been concerned with one facet of determining the effect of coupon use on overall commodity demand. Previous analyses have shown, coupon use has direct impacts on purchase time and quantity purchased and indirect impacts on quantity purchased that may counteract the direct impacts (Gupta, 1988; Neslin, Henderson and Quelch, 1995). An area of future research is one of developing a model which takes into account the simultaneous decisions of coupon use, interpurchase time **and** quantity purchased. Previous analysis have not recognized the simultaneous nature of these consumer decisions. The present analysis provides the foundation for this future research activity necessary to determine the net effect of coupon promotion on commodity demand and whether the costs of such promotion are justified.

Table 1. Cheese Purchase Characteristics

Cheese Type	Number of Households	Mean Purchase Weeks	Percent of Purchase Weeks With Coupon Use	Interpurchase Time[a]		
				All Weeks	Without Coupons	With Coupons
All Cheese	658	52.5	18.8	3.2	3.3	2.9
Processed Cheese	526	35.0	18.7	4.8	4.7	5.0
Natural Cheddar	324	28.5	13.3	5.8	5.7	6.4
Cottage Cheese	529	34.9	3.4	4.8	4.7	6.1

[a]These means are calculated over all purchase occasions and households.

Table 2. Definition and Mean Values of Demographic and Cheese Purchase Characteristics

Variable Name	Definition	Expected Sign	Type of C		
			All Cheese	Processed	C
Purchase Characteristics					
PURCH_LAG[a]	Amount purchased on last purchase occasion (lb.)	–	1.2	1.2	
COUP_VALUE[b]	Value of coupons used per occasion ($/lb.)	+	0.80	0.74	
POS_CHANGE[a]	Ratio of change in price to reference price when there is a price increase (#)	-	-0.156	-0.112	
NEG_CHANGE[a]	Ratio of change in price to reference price when there is a price decrease (#)	+	0.116	0.088	
SUMMER[a]	Dummy variable for June, July and August (0/1)	+	0.279	0.263	
HOLIDAY[a]	Dummy variable for November and December (0/1)	+	0.127	0.149	
Household Characteristics					
INV_HHSIZE[c]	Inverse of household size (1/# of Members)	–	0.477	0.466	
BLACK[c]	Dummy variable =1 if meal planner is Black (0/1)	?	0.055	0.049	
HISPANIC[c]	Dummy variable =1 if meal planner is Hispanic (0/1)	?	0.033	0.061	
PER$\leq$5[c]	Percent of household members less than 6 years (%)	–	0.034	0.055	
PER6_13[c]	Percent of household members between 6 and 13 years (%)	–	0.065	0.075	
PER14_18[c]	Percent of household members between 14 and 18 years (%)	–	0.046	0.047	
POV_RATIO[c]	Ratio of household income to poverty threshold income (#)	?	3.43	3.35	
SUBURB[c]	Dummy variable = 1 if household resides in suburb (0/1)	?	0.120	0.131	
RURAL[c]	Dummy variable = 1 if household resides in rural area (0/1)	?	0.064	0.086	

Note: The "Expected Sign" column refers to hypothesized coefficient signs for variables in equations (11) and (13).

[a] indicates mean taken over all purchase occassions

[b] indicates mean taken over all purchase occassions where there is a coupon used

[c] indicates mean calculated over the number of households consuming each cheese .

Table 3. Likelihood Function Values for Erlang-2 Interpurchase Time Models With and Without Covariates and Gamma Heterogeneity

Cheese Type	Model Description		Likelihood Funciton Value	χ^2-Statistic (15 d.f.)	ζ
	Gamma Heterogeneity	Purchase and Household Covariates			
All Cheese	No	No	-73789.4	23764.2*	-73792.6
	No	Yes	-61907.3		-61959.2
	Yes	No	-70737.0	25475.4*	-70743.5
	Yes	Yes	-57999.3		-58054.5
Processed	No	No	-47968.3	13953.6*	-47971.4
	No	Yes	-40991.5		-41041.6
	Yes	No	-45787.5	17091.2*	-45793.8
	Yes	Yes	-37241.9		-37295.2
Cheddar	No	No	-26213.6	7287.6*	-26216.5
	No	Yes	-22569.8		-22616.0
	Yes	No	-25874.3	10326.0*	-25880.1
	Yes	Yes	-20711.3		-20760.4
Cottage	No	No	-48880.4	8734.2*	-48883.5
	No	Yes	-44513.3		-44563.5
	Yes	No	-46612.9	12634.6*	-46619.2
	Yes	Yes	-40295.6		-40348.9

Note: *indicates significance at the .001 level.

Table 4. Parameter Estimates of Erlang-2 Interpurchase Time Models With Time Varying Covariates and Gamma Heterogeneity

Coefficient	All Cheese	Processed	Cheddar	Cottage
α	6.075* (0.627)	8.908* (0.795)	9.346* (1.012)	6.292* (0.690)
ω	3.420* (0.310)	3.723* (0.297)	3.505* (0.376)	2.361* (0.257)
Purchase Characteristics				
PURCH_LAG	-0.004 (0.003)	-0.031* (0.005)	-0.106* (0.009)	0.025* (0.005)
COUP_VALUE	0.354* (0.002)	0.538* (0.003)	0.682* (0.009)	0.648* (0.019)
POS_CHANGE	-0.092* (0.003)	-0.140* (0.006)	-0.352* (0.018)	-0.076* (0.007)
NEG_CHANGE	0.315* (0.017)	0.438* (0.002)	0.595* (0.037)	0.452* (0.028)
SUMMER	-0.029* (0.009)	0.004 (0.011)	-0.074* (0.018)	0.051* (0.010)
HOLIDAY	-0.002 (0.012)	-0.008 (0.015)	0.009 (0.022)	-0.120* (0.016)
Household Characteristics				
INV_HHSIZE	-0.450* (0.035)	-0.438* (0.056)	-0.402* (0.067)	-0.191* (0.042)
PER≤5	0.340* (0.072)	0.303* (0.075)	0.688* (0.075)	0.119 (0.089)
PER6_13	-0.047 (0.059)	0.066 (0.075)	-0.217* (0.079)	-0.005 (0.078)
PER14_18	0.172* (0.061)	-0.168* (0.058)	-0.066 (0.073)	0.159* (0.067)
BLACK	-0.069 (0.096)	-0.126 (0.065)	0.302* (0.084)	-0.210 (0.225)
HISPANIC	-0.075 (0.043)	-0.055 (0.027)	0.170* (0.059)	0.310* (0.070)
POV_RATIO	0.105 (0.052)	-0.159* (0.066)	-0.208* (0.072)	0.138* (0.038)
SUBURB	-0.025 (0.039)	-0.000 (0.063)	0.061 (0.075)	-0.022 (0.049)
RURAL	-0.049 (0.074)	0.135 (0.076)	-0.008 (0.123)	0.020 (0.061)

Note: Numbers in parentheses are standard errors.

*indicates significance at the .001 level.

Table 5. Chi-Square Statistics Obtained From Likelihood Ratio Tests

Null Hypothesis	All Cheese	Processed Cheese	Cheddar Cheese	Cottage Cheese
$\beta_1 = \beta_2 = . . . = \beta_k = 0$	25475.4(15)*	17091.2(15)*	10326.0(15)*	12634.6(15)*
$\beta_{COUP\_VALUE} = 0$	1283.8(1)*	1229.3(1)*	807.0(1)*	89.8(1)*
$\beta_{POS\_CHANGE} = -\beta_{NEG\_CHANGE}$	7138.0(1)*	4344.7(1)*	768.9(1)*	4566.1(1)*

*Significant at the .001 level

Table 6. Hazard Rate Price and Coupon-Value Elasticities.

Variable	All Cheese	Processed Cheese	Cheddar Cheese	Cottage Cheese
POS_CHANGE	0.183	0.197	0.281	0.075
NEG_CHANGE	0.465	0.485	0.416	0.359
COUP_VALUE	0.445	0.654	1.138	0.464

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Footnotes

. Two studies that have examined the effect of coupon use on a specific food commodity are those of Ward and Davis(1978a) and of Lee and Brown(1985). They examine the effect of coupon use on frozen concentrated orange juice demand. Both studies using monthly household level purchase data found significant effects of coupon redemption on quantity of juice purchased. Neither study investigate the effect of such promotion on purchase timing.

. It is important to include observations that are right censored. If we omit observations that are right-censored or assume an arbitrarily large duration value for these censored observations biased distribution parameter estimates may be obtained. For these right censored observations, information concerning survival up to t_i are accounted for without making assumptions about the timing of the event's future occurrence (Yamaguchi, 1991; Tuma and Hannan, 1979).

. We can represent the gamma distribution (f_G) as:

$$f_G(t) = \frac{\lambda^r t^{r-1} e^{-\lambda t}}{r-1}$$

When the distribution parameter **r** takes only integer values this distribution is referred to as an Erlangian distribution. When **r** is set equal to 2, this is referred to as an Erlang-2 distribution (Cox and Lewis, 1966).

. Other methods to include heterogeneity can be found in McDonald and Butler(1987) and Butler and Worrall(1991).

5. Given that our analysis covers 170 weeks of potential purchases, an examination of aggregate U.S. cheese supply and demand characteristics revealed modest annual growth rates in supply and per capita consumption. For example, over the study period both the U.S. supply and per capita consumption of cheese increased between 1-3% annually over the four year period 1991-1994 (Putnam and Allshouse).

. Following industry standards, cottage cheese is not included in the all cheese category. A detailed listing of cheeses in each category can be obtained from the author upon request.

7. The inverse is used so as to avoid potential scaling problems when maximizing the various likelihood functions.

8. For each elasticity, they are calculated at mean values of the exogenous variables except for the exogenous variable that is changing. For this exogenous variable conditional means are used.

9. The profiles in Figures 2-3 are generated by using mean values of the demographic variables for the various subgroups. For example, Hispanic hazard rate profiles for all cheese, are calculated based on mean values of the demographic variables for Hispanic households only. Non-minority households for these figures are assumed to be households that are classified as being neither black nor hispanic.