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Abstract: This paper develops the formulation of a spatial equilibrium model with intermediate products. The presence of intermediate products brings a better representation of the real world, accounting for the technological relationships among commodities in the several stages of production. Instead of the usual assumption of constant costs of transformation, positively sloped cost of transformation functions are assumed, reflecting increasing marginal costs of transformation. The model is implemented and validated in the analysis of the optimal allocation and pricing of animal products, grains and oilseeds in the MERCOSUR.

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SPATIAL EQUILIBRIUM WITH INTERMEDIATE PRODUCTS:
IMPLEMENTATION AND VALIDATION IN THE MERCOSUR

Introduction

During the 1980’s and 1990’s, several Latin American countries increasingly adopted market-oriented policies to foster the development of open and competitive economies, as the region’s policy-makers reevaluated the import substitution strategies. Nations moved towards economic integration and greater participation in world trade, following a global tendency for trade liberalization. Various regional blocs involving Latin American countries were formed, with the goal of economic integration. Among these regional groupings is the *Mercado Común del Sur* (MERCOSUR). Originally, it was formed by Argentina and Brazil in the mid 1980’s. On March 26, 1991, the agreement was formalized by the *Tratado de Asunción*, including two other nations: Paraguay and Uruguay. The agreement aims at establishing a single market among the four nations, based on the free movement of goods and services; the establishment of common external tariffs and trade policies; the coordination of macroeconomic and sectoral policies; and the harmonization of legislations to strengthen the process of integration. It is effective since the beginning of 1995, although with a list of exceptions, and shall be completely operative by the end of the year 2000.

As Perez del Castillo (1993) suggests, the MERCOSUR represents an historic opportunity for these nations to pool and harness their vast potential for the greater prosperity of their inhabitants. He points out that although there have been some difficulties, most observers consider that the negotiations are progressing reasonably well. Ala Rue and Lavergne (1992) indicate that historically trade within the region has been small and limited to certain commodities. They consider that the increase in trade flows is a result of regional agreements, and one of the characteristics of the MERCOSUR is the possibility to sustain this increase over time.

Table 1 presents some of the basic statistics for the MERCOSUR countries. In 1990, the four countries had a total population of over 190 million and a total area of nearly 12 million square kilometers. Except for Paraguay and Uruguay, each pair of countries shares common borders. The sum of Gross Domestic Product (GDP) of Argentina, Brazil, Paraguay and Uruguay was almost US$ 600 billion in 1990. It can be noted in Table 1 that Argentina and Brazil constitute the largest part of the market.

Agriculture is a crucial and dynamic sector in the region, and demonstrates considerable specialization. The MERCOSUR is a major producer and net exporter of grains and oilseeds, fruits, coffee, beef, poultry and other agroindustrial products. Table 2 presents the agriculture’s share of total exports, illustrating the importance of the agricultural sector to these economies. In 1992, the exports of agricultural products accounted for 25% of total exports in Brazil, and 38% in Uruguay. Agricultural exports accounted for larger shares of total exports in Argentina and Paraguay, respectively 58% and 75%. In this table, it can also be observed that all the MERCOSUR countries are net exporters of agricultural products. Thus, not only the intraregional trade linkages are important, but also the linkages between the MERCOSUR and the rest of the world. The formation of a regional bloc brings economies of scale created by the enlargement of the markets, appropriate specialization and foreign investment, resulting also in a improved international bargaining position.

Several approaches have been taken to study the implications of trade liberalization and the formation of regional blocs on the agricultural sector. The approach taken in this paper is based on the mathematical programming models developed by Samuelson (1952) and Takayama and Judge (1964a, b, 1971), to analyze the allocation of resources among spatially separated markets. Several extensions and empirical applications have been implemented to different sets of commodities and sets of regions, following those studies. Thompson (1981) and Hertel (1990) present surveys of the developments in international agricultural trade models.

In multi-commodity models, the relationships between commodities may be very important, and are usually considered through cross-price effects. Nonetheless, the incorporation of stages of production with the presence of intermediate products brings a closer representation of reality. In the real world, final commodities are produced by using not only primary factors of production, but also intermediate products. Vanek (1963) introduces the issue of intermediate products by supposing that each product in the economy can be used both as an intermediate and as a final commodity. Samuelson (1966) suggests that the productive system can be considered as a “black box”, with an input of primary factors of production and an output of the net quantity of final commodities. Using this approach, it has been demonstrated that the traditional theorems of the theory of international trade (Heckscher-Ohlin, Stolper-Samuelson, Rybczynski) are still valid even in the presence of intermediate products.

In this paper, a spatial equilibrium model is developed allowing for the occurrence of stages of production with intermediate products. It is considered that there are several stages in the process of production. In each stage, the commodities produced are destined as intermediate products for the production of new commodities in the next stage, being possible that one commodity keeps its form from one stage to another. In the last stage, all commodities are destined to final consumption. Also in each stage, commodities are assumed to be transportable between regions.

Previous studies dealing with intermediate products (Takayama and Judge, 1964b; Thore, 1992; Bishop, Pratt and Novakovik, 1993) assume constant costs of processing, which may differ among regions. In this case, it is necessary to consider the regional capacities in the set of constraints. The present study innovates in the sense that it assumes positively sloped cost of transformation functions, representing increasing marginal costs of processing intermediate inputs into outputs. As the production of secondary or final commodities expands, the costs of transformation of additional units increase. Thus, even though a given region may have lower costs of transformation at a base level, it may not be worth expanding its production above a certain level, since its costs of transformation will increase and eventually become prohibitive.

Next section presents the formulation of the spatial equilibrium framework with intermediate products, under the assumption of positively sloped cost of transformation functions. Then, the needs and sources of data that are necessary to implement the model are discussed. In the following sections, the model is implemented and validated to the case of the animal products, grains and oilseeds subsectors in the MERCOSUR. The last section presents the conclusions of the paper.

The model

The spatial equilibrium model with intermediate products developed in this paper is static and involves partial equilibrium. It assumes perfect competition and homogeneous products. It also considers that there are no structural changes in supply and demand in the transition from a starting position to the new equilibrium, that is, prices and quantities are determined along supply and demand functions which remain unchanged in the basic model. In order to formulate the mathematical programming model, it is necessary to assume that the integrability condition[1] of supply and demand functions is satisfied.

The model presented in this paper uses a quantity formulation (primal), in which the decision variables are quantities (production, consumption, trade flows). The Lagrange multipliers are interpreted as shadow prices. Alternatively, it can be presented in an equivalent price formulation (dual), in which the decision variables are prices, and the Lagrange multipliers are interpreted as shadow quantities. The dual formulation may become

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[1] Takayama (1994) distinguishes mathematical integrability from economic integrability. Mathematical integrability refers to the condition in which the matrix of first derivatives is symmetric. In the case of supply functions, the classical assumptions of the theory of production yield the symmetry condition. However, in the case of demand functions, symmetry is predicted for Hicksian functions, which are employed in this study. Marshallian demand functions need not be symmetric, unless the income effect is identical across all commodities. On the other hand, economic integrability refers to the condition in which the matrix of first derivatives is positive semi-definite for supply functions, and negative semi-definite for demand functions.

more appropriate in the presence of non-linear constraints on prices.

In what follows, the primal framework is developed with production occurring in a two-stage process. However, it can be generalized for any number of stages of production. Thus, consider the allocation of a set of K primary commodities and a set of N final commodities among J spatially separated regions. The K primary commodities can be produced and processed into final commodities in each of the J regions. The N final commodities can be consumed in each of the J regions. All the primary and final commodities can be traded among regions.

The production of final commodities involves two kinds of inputs: the primary commodities and other inputs. Chavas, Cox and Jesse (1993) present a restricted cost function, measuring the cost of optimal use of other inputs conditional on the primary inputs and final outputs:

$$CT_i(x_i, y_i) = Min_{v_i} \{r_i' v_i : (v_i, x_i, y_i) \in F_i\} \tag{1}$$

where x_i denotes the vector of primary inputs used in the production of y_i ; y_i denotes the vector of final outputs; v_i denotes the vector of other inputs (besides x_i); r_i denotes the vector of market prices for the inputs v_i ; and F_i denotes the production possibility set, establishing the technological relationship between inputs v_i and x_i and feasible outputs y_i in each region *i*. The function $CT_i(x_i , y_i)$ is, then, a measure of the costs of transformation of primary into final commodities in region *i*. It is assumed to be a decreasing function of x_i , and an increasing function of y_i . It is also assumed that primary inputs x_i and other inputs v_i are weakly separable.

The notation to be employed in the formulation of the spatial equilibrium model with intermediate products is described next:

$PPRD_{k,i}$ = production of the k-th primary commodity in region i, for k = 1,...,K and i = 1,...,J;

$FPRD_{n,i}$ = production of the n-th final commodity in region i, for n = 1,...,N and i = 1,...,J;

$CONS_{n,i}$ = consumption of the n-th final commodity in region i, for n = 1,...,N and i = 1,...,J;

$XP_{k,i,j}$ = exports of the k-th primary commodity from region i to region j, for k = 1,...,K, i = 1,...,J and j = 1,...,J;

$XF_{n,i,j}$ = exports of the n-th final commodity from region i to region j, for n = 1,...,N, i = 1,...,J and j = 1,...,J;

$TP_{k,i,j}$ = unit cost of transportation of the k-th primary commodity from region i to region j, determined exogenously, for k = 1,...,K, i = 1,...,J and j = 1,...,J;

$TF_{n,i,j}$ = unit cost of transportation of the n-th final commodity from region i to region j, determined exogenously, for n = 1,...,N, i = 1,...,J and j = 1,...,J;

$P^s_{k,i}$ = price-dependent supply function for the k-th primary commodity in region i, for k = 1,...,K and i = 1,...,J;

$P^d_{n,i}$ = price-dependent Hicksian demand function for the n-th commodity in region i, for n = 1,...,N and i = 1,...,J;

$CT_{n,i}$ = cost of transformation function for the n-th final commodity in region i, for n = 1,...,N and i = 1,...,J.

Note that $P^s_{k,i}$, $P^d_{n,i}$ and $CT_{n,i}$ are all functions of quantities, and include own-price and cross-price effects.

Using the primal approach, Samuelson (1952) shows that market equilibrium is achieved through the maximization of a net social payoff (NSP) function, given by the sum of producer surplus and consumer surplus. In a multi-commodity, multi-region dimension, an aggregate net social payoff function is obtained by summing the NSP functions across commodities and across regions, and subtracting the costs of transportation of commodities from one region to another. In the presence of intermediate products, it is also necessary to subtract the costs of transformation in each stage of production.

The maximization of the aggregate net social payoff function is subject to two sets of constraints: the trade flows and the non-negativity constraints. The technological relationships in the transformation process are included in the trade flow constraints. Each region cannot use domestically and export more than the amount it is producing, and cannot consume (either as intermediate or final product) more than the amount it is producing and importing from all other regions. Also, no region can produce, consume or trade negative quantities.

In this context, the maximization problem can be stated as follows:

Maximize

$$\sum_i \left\{ \sum_n \int_0^{CONS_{n,i}} P_{n,i}^d (t) dt - \sum_k \int_0^{PPRD_{k,i}} P_{k,i}^s (u) du \right.$$

$$\left. - \sum_n \int_0^{FPRD_{n,i}} CT_{n,i} (w) dw \right\}$$

$$-\sum_k \sum_i \sum_j TP_{k,i,j} \cdot XP_{k,i,j} \ -\sum_n \sum_i \sum_j TF_{n,i,j} \cdot XF_{n,i,j}$$

subject to:

$$PPRD_{k,i} \geq \sum_j XP_{k,i,j}$$

$$f_i\left(\sum_j XP_{k,j,i}, \forall k; FPRD_{n,i}, \forall n\right) \leq 0$$

$$FPRD_{n,i} \geq \sum_j XF_{n,i,j}$$

$$\sum_j XF_{n,j,i} \geq CONS_{n,i}$$

$$PPRD_{k,i} \geq 0,\ FPRD_{n,i} \geq 0, CONS_{n,i} \geq 0,\ XP_{k,i,j} \geq 0,\ XF_{n,i,j} \geq 0$$

$$\forall k, \forall n, \forall i, \forall j. \tag{2}$$

Note here that $f_i(\bullet)$ is an implicit transformation function representing the technological relationship established by the production possibility set F_i . It is a general characterization of technology, allowing for variable returns to scale (VRTS) and less restrictive Allen elasticities of substitution. In this case, the constraints could become non-linear. In such situation, one has to consider the Arrow-Hurwicz-Uzawa (A-H-U) theorem (Arrow, Hurwicz and Uzawa, 1961; Takayama, 1985, 1994) for the Kuhn-Tucker conditions to correspond to the optimal solution. At least one of the A-H-U conditions has to be satisfied, implying the specification of “well-behaved” technologies.

In this study, however, the assumption of Leontief technologies is made for its tractability in empirical work. It is a special case (a subset) of the production possibility set defined above, and it implies that the transformation of primary into final commodities is characterized by fixed coefficients. Thus, the constraints involving technological relationships can be written as:

$$\sum_j XP_{k,j,i} \geq \sum_n \alpha_{k,n,i} \cdot FPRD_{n,i} \tag{3}$$

where $\alpha_{k,n,i}$ is the amount of the k-th primary commodity used as input to produce one unit of the n-th final commodity in region i, for $k = 1, ..., K$, $n = 1, ..., N$ and $i = 1, ..., J$. Note that under the assumption of Leontief technologies, these constraints become linear.

After substituting constraints (3) in the maximization problem (2), the associated Lagrangean function is:

$$\begin{aligned} L = \sum_i \Big\{ & \sum_n \int_0^{CONS_{n,i}} P_{n,i}^d(t)dt - \sum_k \int_0^{PPRD_{k,i}} P_{k,i}^s(u)du \\ & - \sum_n \int_0^{FPRD_{n,i}} CT_{n,i}(v)dv \Big\} \\ & - \sum_k \sum_i \sum_j TP_{k,i,j} \cdot XP_{k,i,j} - \sum_n \sum_i \sum_j TF_{n,i,j} \cdot XF_{n,i,j} \\ & + \sum_i \sum_k \lambda_{k,i} \cdot \Big[PPRD_{k,i} - \sum_j XP_{k,i,j} \Big] \\ & + \sum_i \sum_k \mu_{k,i} \cdot \Big[\sum_j XP_{k,j,i} - \sum_n \alpha_{k,n,i} \cdot FPRD_{n,i} \Big] \\ & + \sum_i \sum_n \eta_{n,i} \cdot \Big[FPRD_{n,i} - \sum_j XF_{n,i,j} \Big] \\ & + \sum_i \sum_n \tau_{n,i} \cdot \Big[\sum_j XF_{n,j,i} - CONS_{n,i} \Big] \end{aligned} \tag{4}$$

where $\lambda_{k,i}$, $\mu_{k,i}$, $\eta_{n,i}$ and $\tau_{n,i}$ are the corresponding Lagrange multipliers.

The Kuhn-Tucker conditions associated with the problem above are both necessary and sufficient conditions for an optimal global solution, under the assumptions of differentiability and concavity of the objective function, and in the presence of linear constraints (Sposito, 1975; Takayama, 1985, 1994). These Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial CONS_{n,i}} = P^{d}_{n,i} - \tau_{n,i} \leq 0 \quad ;$$

$$CONS_{n,i} \geq 0 \quad ; \qquad \frac{\partial L}{\partial CONS_{n,i}} \cdot CONS_{n,i} = 0 \tag{5.a}$$

$$\frac{\partial L}{\partial PPRD_{k,i}} = - P^{s}_{k,i} + \lambda_{k,i} \leq 0 \quad ;$$

$$PPRD_{k,i} \geq 0 \quad ; \qquad \frac{\partial L}{\partial PPRD_{k,i}} \cdot PPRD_{k,i} = 0 \tag{5.b}$$

$$\frac{\partial L}{\partial FPRD_{n,i}} = - CT_{n,i} - \sum_{k} \alpha_{k,n,i} \cdot \mu_{k,i} + \eta_{n,i} \leq 0 \quad ;$$

$$FPRD_{n,i} \geq 0 \quad ; \qquad \frac{\partial L}{\partial FPRD_{n,i}} \cdot FPRD_{n,i} = 0 \tag{5.c}$$

$$\frac{\partial L}{\partial XP_{k,i,j}} = -TP_{k,i,j} - \lambda_{k,i} + \mu_{k,j} \leq 0 \quad ;$$

$$XP_{k,i,j} \geq 0 \quad ; \quad \frac{\partial L}{\partial XP_{k,i,j}} \cdot XP_{k,i,j} = 0 \tag{5.d}$$

$$\frac{\partial L}{\partial XF_{n,i,j}} = -TF_{n,i,j} - \eta_{n,i} + \tau_{n,j} \leq 0 \quad ;$$

$$XF_{n,i,j} \geq 0 \quad ; \quad \frac{\partial L}{\partial XF_{n,i,j}} \cdot XF_{n,i,j} = 0 \qquad (5.e)$$

$$\frac{\partial L}{\partial \lambda_{k,i}} = PPRD_{k,i} - \sum_j XP_{k,i,j} \geq 0 \quad ;$$

$$\lambda_{k,i} \geq 0 \quad ; \quad \frac{\partial L}{\partial \lambda_{k,i}} \cdot \lambda_{k,i} = 0 \qquad (5.f)$$

$$\frac{\partial L}{\partial \mu_{k,i}} = \sum_j XP_{k,j,i} - \sum_n \alpha_{k,n,i} \cdot FPRD_{n,i} \geq 0 \quad ;$$

$$\mu_{k,i} \geq 0 \quad ; \quad \frac{\partial L}{\partial \mu_{k,i}} \cdot \mu_{k,i} = 0 \qquad (5.g)$$

$$\frac{\partial L}{\partial \eta_{n,i}} = FPRD_{n,i} - \sum_j XF_{n,i,j} \geq 0 \quad ;$$

$$\eta_{n,i} \geq 0 \quad ; \quad \frac{\partial L}{\partial \eta_{n,i}} \cdot \eta_{n,i} = 0 \qquad (5.h)$$

$$\frac{\partial L}{\partial \tau_{n,i}} = \sum_j XF_{n,j,i} - CONS_{n,i} \geq 0 \quad ;$$

$$\tau_{n,i} \geq 0 \quad ; \quad \frac{\partial L}{\partial \tau_{n,i}} \cdot \tau_{n,i} = 0 \qquad (5.i)$$

The Kuhn-Tucker conditions (5) imply the following statements:

$$CONS_{n,i} > 0 \Rightarrow P^d_{n,i} = \tau_{n,i} \qquad (6.a)$$

$$PPRD_{k,i} > 0 \Rightarrow P^s_{k,i} = \lambda_{k,i} \qquad (6.b)$$

$$FPRD_{n,i} > 0 \;\Rightarrow\; CT_{n,i} + \sum_k \alpha_{k,n,i} \cdot \mu_{k,i} = \eta_{n,i} \tag{6.c}$$

$$XP_{k,i,j} > 0 \;\Rightarrow TP_{k,i,j} = \mu_{k,j} - \lambda_{k,i} \tag{6.d}$$

$$XF_{n,i,j} > 0 \;\Rightarrow TF_{n,i,j} = \tau_{n,j} - \eta_{n,i} \tag{6.e}$$

$$\lambda_{k,i} > 0 \;\Rightarrow\; PPRD_{k,i} = \sum_j XP_{k,i,j} \tag{6.f}$$

$$\mu_{k,i} > 0 \;\Rightarrow\; \sum_j XP_{k,j,i} = \sum_n \alpha_{k,n,i} \cdot FPRD_{n,i} \tag{6.g}$$

$$\eta_{n,i} > 0 \;\Rightarrow\; FPRD_{n,i} = \sum_j XF_{n,i,j} \tag{6.h}$$

$$\tau_{n,i} > 0 \;\Rightarrow\; \sum_j XF_{n,j,i} = CONS_{n,i} \tag{6.i}$$

The Lagrange multipliers are interpreted as shadow prices in competitive equilibrium. Statement (6.a) indicates that the demand price for the *n*-th final commodity in region *i* equals its market consumer price, whenever the consumption of such commodity is positive. If there is no consumption, then the Kuhn-Tucker conditions imply that the demand price is less than or equal to the market consumer price. Accordingly, statement (6.b) indicates that whenever the production of the *k*-th primary commodity in region *i* is positive, its supply price is equal to the market producer price. Now, if there is no production of that commodity in that region, then the supply price is greater than or equal to the market producer price. Statement (6.c) implies that if there is production of the *n*-th final commodity in region *i*, then the market producer price is equal to the costs of transformation plus the costs of intermediate inputs (primary commodities). If there is no production of some final commodity in a region, the costs of production (costs of transformation plus costs of intermediate products) for that commodity are greater than or equal to the market producer price in that region. These statements imply a zero profit condition for the processing sector, under CRTS.

The following statements (6.d) and (6.e) imply a zero profit condition for the transportation sector. The price differential between regions is less than or equal to the unit cost of transportation, for all primary and final commodities. Whenever trade of a given commodity takes place, the difference between its market consumer price in the importing region and its market producer price in the exporting region is exactly equal to the unit cost of transportation of that commodity between the two regions. Under the assumption of zero transportation costs within each region ($TP_{k,i,i} = TF_{n,i,i} = 0$), the market producer prices are identical to the market consumer prices within each region, for all commodities.

Lastly, the remaining statements (6.f) to (6.i) reproduce the linear constraints of the optimization problem, implying the market clearing conditions. In each region, production has to be greater than or equal to the domestic use plus exports to other regions, and consumption (either as an intermediate input or as a final output) has to be less than or equal to domestic production plus imports from other regions. When the Lagrange multipliers (shadow prices) are positive, the conditions above hold with equality. On the other hand, if markets do not clear, then the shadow prices are equal to zero.

All these statements taken together characterize an optimal solution for the primal spatial equilibrium problem with intermediate products in competitive equilibrium. This framework can be extended to incorporate other aspects of the theory of international trade. More specifically, some limitations on the trade flows, such as the presence of tariffs, quotas, or other governmental programs, can be included to bring a closer representation of reality.

Data needs and sources

The animal products, grains and oilseeds subsectors of agriculture are amongst the most relevant subsectors in the four MERCOSUR countries. In this paper, 11 commodities are selected for analysis: beef, pork, poultry, fluid milk, cheese, dry milk, wheat, coarse grains[2], rice, soybeans and soybean meal. Other commodities are excluded because they are not traded by the MERCOSUR countries, or because their links with the animal products, grains and oilseeds subsectors are weak.

The implementation of the model comprises the characterization of a sequence of four stages of production. In the first stage, there occurs the production of primary products: beef [3], wheat, coarse grains, rice and soybeans. In the second stage, soybeans can be processed into soybean meal. In the third stage, secondary products (wheat, coarse grains, soybeans and soybean meal) can be used as inputs in the production of pork, poultry and fluid milk. Finally, in the fourth stage, fluid milk can be processed into cheese and dry milk. Note that it is possible that one product keeps its form from one stage to another. In the last stage, all products are destined to final consumption.

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[2] Coarse grains include barley, maize, rye, oats, millets and sorghum.

[3] The production of beef in the MERCOSUR countries is characterized as being extensive, mainly on native or improved pastures. No other primary products serve as inputs for the production of beef; it is, therefore, possible to consider beef as a primary product.

To implement the spatial equilibrium model with intermediate products, developed in the previous section, it is necessary to specify supply functions for the primary commodities; cost of transformation functions for the secondary, terciary and final commodities; and demand functions for the final commodities. Most of the empirical research on spatial equilibrium models has been made under the assumption of linear functions, following the programming framework developed by Takayama and Judge (1964a). With linear supply, cost of transformation and demand functions, the objective function of the maximization problem becomes a quadratic expression. In the presence of linear constraints, the problem can be solved by quadratic programming.

In this paper, supply, cost of transformation and demand functions are also assumed to be linear. They are derived from own-price and cross-price elasticities, and from quantities and prices observed in a base period. Border prices are employed, as defined implicitly by the value of trade divided by the amount traded. The base period considered is a five-year average (1989-1993), in order to avoid any atypical situation caused by bad weather, exceptional domestic policies, or other conditions affecting production, consumption and trade only in the short-run. Five regions are taken into account: the four MERCOSUR countries, and a region designated as the rest of the world (ROW). It is important to include the ROW in the implementation of the model, to consider the international trade linkages between the MERCOSUR and the ROW.

The sources of data for quantities produced and consumed, and for prices are the FAO production yearbooks and FAO trade yearbooks. Price elasticities are obtained from USDA

(Sullivan et al., 1992). In addition, other parameters are necessary: unit costs of transportation and the amounts of intermediate products used as inputs to produce one unit of secondary, terciary or final products. These are obtained from a study of costs of production and competitiveness of agricultural products in the MERCOSUR (IEPE, 1992; MGAyP, 1992).

Spatial equilibrium in the animal products, grains and oilseeds subsectors

The model is formulated as a quadratic programming problem. The optimal solution is obtained through the use of the General Algebraic Modeling System (GAMS), and satisfies the Kuhn-Tucker conditions associated with the problem. The optimal levels of production, consumption and trade flows are presented in Tables 3, 4 and 5 respectively.

As a bloc, the MERCOSUR is a major producer and net exporter of almost all selected commodities. The region appears as a net importer of dry milk only. Because of the sizes of their economies, most of the production and consumption within the regional bloc take place in Argentina and Brazil.

The production of beef is relatively important in the four countries. In the optimal solution, Argentina, Paraguay and Uruguay are net exporters, while Brazil produces only the amount to satisfy its domestic consumption. The exports of beef to the ROW sum to 298 thousand MT. On the other hand, Brazil is the only major producer and exporter of pork and poultry. Alone it exports 153 thousand MT of pork, and 401 thousand MT of poultry to the ROW. Argentina is a net importer of poultry, importing 31 thousand MT from Brazil.

The optimal results indicate that fluid milk is a non-traded commodity, because its unit transportation costs are high relative to its price. Each country produces the amount necessary to satisfy its domestic consumption of fluid milk plus the amount utilized as intermediate product in the production of cheese and dry milk. However, the trade flows of other dairy products are substantial. Argentina and Uruguay are net exporters of cheese and dry milk, whereas Brazil and Paraguay are net importers. Argentina exports 31 thousand MT of cheese to the ROW, 10 thousand MT of dry milk to Brazil, and 2 thousand MT of dry milk to Paraguay. Uruguay exports 5 thousand MT of cheese and 8 thousand MT of dry milk to Brazil, and 5 thousand MT of cheese to the ROW. In addition, the ROW exports 33 thousand MT of dry milk to Brazil.

The bloc is also a net exporter of grains and oilseeds. Still, there are good opportunities for intraregional trade. Argentina and Paraguay export all remaining commodities. Brazil exports soybeans and soybean meal, but imports wheat, coarse grains and rice. Uruguay exports wheat, rice and soybeans, but imports coarse grains and soybean meal. In the optimal solution, all Brazilian and Uruguayan imports of these commodities come from within the bloc. Exports from the bloc to the ROW assume more importance for soybeans and soybean meal; together the MERCOSUR countries export 7,276 thousand MT of soybeans and 14,745 thousand MT of soybean meal to the ROW.

Table 6 shows the optimal prices obtained for the problem. As required by the formulation of the problem, the price differential between each pair of regions is less than or equal to the unit transportation costs used in the implementation of the model. Whenever trade takes place, the price differential is exactly equal to the unit transportation costs. The optimal prices in the MERCOSUR countries are lower than the prices in the ROW, except for dry milk (remember that the MERCOSUR is a net importer of dry milk only). Thus, the imposition of common external tariffs is effective just in the case of dry milk, among the selected commodities. The effect of a higher tariff on imports from the ROW is an increase in the price of the imported commodity, causing a reduction in imports from the ROW and increasing intraregional trade.

Here it is important to observe that the framework uses supply and demand functions, and endogenously computes excess supply and excess demand functions for each region. These resulting excess demand functions for the ROW are almost horizontal (highly elastic), because the ROW accounts for very large shares of the world markets. It follows that variations in the trade flows from the MERCOSUR to the ROW will not affect world prices significantly. Only in the case of soybeans and soybean meal, the MERCOSUR accounts for substantial shares of the world markets. Even though they are still elastic, the excess demand elasticities for the ROW are of much smaller magnitudes (in absolute values). Variations in the trade flows may affect the world prices of these commodities. Thus, the formation of the regional bloc can bring gains to the MERCOSUR countries, due to the enlargement of the size of the markets and improved bargaining power in the world markets.

Validation of the model

Validation indicates the adequacy of the coefficients and structure of the model. The model is validated by verifying how well the solution to the problem, when specified with base period data, corresponds to the real situation in that base period. Thompson (1981) indicates several reasons why spatial equilibrium models cannot replicate all observed trade flows of agricultural commodities. Among these reasons, there are: products may not be perfectly homogeneous; harvests occur six months out of phase in the Northern and Southern hemispheres; some countries impose sanitary restrictions on imports; and importers may diversify their purchases among several suppliers to spread risk. Hence, small differences between observed and optimal trade flows may be expected without invalidating the model.

Tables 7, 8 and 9 present the percentage changes in the optimal levels of production, consumption and prices relative to the observed data. The absolute values of these percentage changes are small, generally less than 5%. It can be noted that the percentage changes in prices are larger than the percentage changes in quantities, implying that the results on quantities correspond better to the observed data than the results on prices do. This is because movements along inelastic demand functions (generally the case of agricultural products) yield percentage changes in prices higher than percentage changes in quantities.

These fairly low percentage differences relative to the observed data utilized to implement the model, suggest that the model provides a reasonably good representation of the real situation. The incorporation of stages of production with intermediate products in the spatial equilibrium framework improves the validation exercise. Thus, the model is validated for this scenario and can be utilized to provide policy analyses and forecasts, under alternative scenarios.

Conclusions

This paper presents the formulation, implementation and validation of a spatial equilibrium model with intermediate products. The presence of intermediate products is important to account for the technological relationships among commodities in several stages of production. It helps bring a closer representation of the real world and improves the validation of the model. In this study, positively sloped cost of transformation functions are assumed, reflecting increasing marginal costs of transformation, instead of the usual assumption of constant costs of transformation.

The model is implemented with four stages of production, to analyze the optimal allocation and pricing of animal products, grains and oilseeds in the MERCOSUR. The regional bloc is a major producer and net exporter of almost all selected commodities; it is a net importer of dry milk. Intraregional trade is substantial in the case of poultry, dairy products and grains. The model developed here can be extended to other sectors and other regions, or can be implemented to simulate different policy scenarios, verifying the possible effects on the allocation, pricing and aggregate welfare in the MERCOSUR countries. It serves, then, as a foundation for the decision-making process.

The implementation of the model to the Western Hemisphere is among the possible extensions. There has been growing interest in building a framework to analyze the implications of economic integration and the establishment of a Free Trade Area of the Americas (FTAA) by the year 2005. Hufbauer and Schott (1994) assess the unrealized trade potential that could be attained with hemispheric integration, and discuss the strength of existing regional economic ties and the extent of economic distance measured by regional diversity. Lee (1995) also studies some implications and prospects of Western Hemisphere economic integration. The study of the optimal allocation and pricing of agricultural commodities, regional trade and market equilibrium with intermediate products will contribute to this research base.

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Table 1. Basic statistics for Argentina, Brazil, Paraguay and Uruguay - 1990.

	Argentina	Brazil	Paraguay	Uruguay
Population (million)	32.3	150.4	4.3	3.1
Urban Population (%)	86.3	74.9	47.5	85.5
Population Annual Growth Rate (%)	1.3	2.1	3.0	0.6
Total Area (1,000 km^2)	2,767	8,512	407	177
Agricultural Area (1,000 km^2)	1,694	2,442	233	148
GDP (US$ billion)	105.5	473.7	5.3	8.2
Share of Agriculture in GDP (%)	14.5	10.2	27.8	10.9
GNP *per capita* (US$)	2,400	2,680	1,110	2,560
Daily Calorie Supply *per capita*	3,113	2,751	2,757	2,653
Daily Protein Supply *per capita* (grams)	101	62	72	79

Source: World Bank (1993).

Table 2. Value of agricultural exports and imports, and total exports from the MERCOSUR countries, in US$ million - 1992.

	Agricultural Exports	Total Exports	% of Total	Agricultural Imports	Net Agricultural Exports
Argentina	7,122	12,235	58.2	941	6,181
Brazil	8,934	35,793	25.0	2,586	6,348
Paraguay	492	657	74.9	166	326
Uruguay	654	1,703	38.4	220	434

Source: FAO (1993), IMF(1994).

Table 3. Optimal production levels, in 1000 MT.

	Argentina	Brazil	Paraguay	Uruguay	ROW
Beef	2,687	2,950	160	356	45,055
Pork	187	1,249	143	20	69,486
Poultry	470	2,775	23	30	38,954
Fluid Milk	6,673	15,301	233	1,137	443,665
Cheese	291	212	1	19	13,518
Dry Milk	120	146	-	11	5,701
Wheat	10,315	3,404	317	372	548,752
Coarse Grains	10,696	25,174	722	423	794,046
Rice	455	9,742	64	570	511,141
Soybeans	10,662	19,908	1,539	30	76,174
Soybean Meal	5,941	12,159	309	9	54,828

Table 4. Optimal consumption levels, in 1000 MT.

	Argentina	Brazil	Paraguay	Uruguay	ROW
Beef	2,541	2,950	108	256	45,353
Pork	185	1,096	141	20	69,643
Poultry	501	2,344	23	30	39,355
Fluid Milk:					
-interm. product	4,193	3,859	10	323	204,292
-final product	2,480	11,441	223	815	239,373
Cheese	260	217	1	9	13,554
Dry Milk	107	197	2	3	5,668
Wheat:					
-interm. product	91	-	-	-	88,214
-final product	4,215	6,982	293	350	463,015
Coarse Grains:					
-interm. product	3,171	14,605	627	364	399,972
-final product	3,101	14,893	16	103	394,209
Rice	341	10,091	61	246	511,233
Soybeans:					
-interm. product	7,558	16,165	406	17	74,773
-final product	7	701	7	1	8,677
Soybean Meal:					
-interm. product	265	1,884	96	6	52,576
-final product	18	1,363	35	5	16,997

Note: where it is not specified, the levels above refer to final consumption.

Table 5. Optimal trade flows, in 1000 MT.

Exports to:	Argentina	Brazil	Paraguay	Uruguay	ROW
Exports from:					
Beef:					
Argentina	2,541	-	-	-	145
Brazil	-	2,950	-	-	-
Paraguay	-	-	108	-	53
Uruguay	-	-	-	256	100
ROW	-	-	-	-	45,055
Pork:					
Argentina	185	-	-	-	3
Brazil	-	1,096	-	-	153
Paraguay	-	-	141	-	1
Uruguay	-	-	-	20	-
ROW	-	-	-	-	69,486
Poultry:					
Argentina	470	-	-	-	-
Brazil	31	2,344	-	-	401
Paraguay	-	-	23	-	-
Uruguay	-	-	-	30	-
ROW	-	-	-	-	38,954
Fluid Milk:					
Argentina	6,673	-	-	-	-
Brazil	-	15,301	-	-	-
Paraguay	-	-	233	-	-
Uruguay	-	-	-	1,137	-
ROW	-	-	-	-	443,665
Cheese:					
Argentina	260	-	-	-	31
Brazil	-	212	-	-	-
Paraguay	-	-	1	-	-
Uruguay	-	5	-	9	5
ROW	-	-	-	-	13,518
Dry Milk:					
Argentina	107	10	2	-	-

Exports to:	Argentina	Brazil	Paraguay	Uruguay	ROW
Brazil	-	146	-	-	-
Paraguay	-	-	-	-	-
Uruguay	-	8	-	3	-
ROW	-	33	-	-	5,668
<u>Wheat:</u>					
Argentina	4,306	3,532	-	-	2,477
Brazil	-	3,404	-	-	-
Paraguay	-	24	293	-	-
Uruguay	-	22	-	350	-
ROW	-	-	-	-	548,752
<u>Coarse Grains:</u>					
Argentina	6,272	4,244	-	44	135
Brazil	-	25,174	-	-	-
Paraguay	-	79	643	-	-
Uruguay	-	-	-	423	-
ROW	-	-	-	-	794,046
<u>Rice:</u>					
Argentina	341	22	-	-	92
Brazil	-	9,742	-	-	-
Paraguay	-	3	61	-	-
Uruguay	-	324	-	246	-
ROW	-	-	-	-	511,141
<u>Soybeans:</u>					
Argentina	7,565	-	-	-	3,097
Brazil	-	16,866	-	-	3,042
Paraguay	--	-	413	-	1,126
Uruguay	-	-	-	19	11
ROW	-	-	-	-	76,174
<u>Soybean Meal:</u>					
Argentina	283	-	-	3	5,655
Brazil	-	3,247	-	-	8,912
Paraguay	-	-	131	-	178
Uruguay	-	-	-	9	-
ROW	-	-	-	-	54,828

Table 6. Optimal prices, in US$/MT.

	Argentina	Brazil	Paraguay	Uruguay	ROW
Beef	2,844	2,884	2,844	2,844	2,967
Pork	2,525	2,525	2,525	2,550	2,648
Poultry	1,568	1,478	1,533	1,541	1,600
Fluid Milk	452	452	448	429	469
Cheese	3,679	3,764	3,703	3,679	3,801
Dry Milk	2,125	2,215	2,188	2,130	2,093
Wheat	131	148	136	132	153
Coarse Grains	112	129	118	117	135
Rice	333	349	338	333	355
Soybeans	219	219	219	219	241
Soybean Meal	207	207	207	212	230

Table 7. Percentage changes in production levels relative to observed data, in %.

	Argentina	Brazil	Paraguay	Uruguay	ROW
Beef	1.0	1.4	0.3	2.1	-0.2
Pork	5.3	6.5	0.6	3.8	-0.2
Poultry	0.9	3.4	0.9	2.9	-0.2
Fluid Milk	1.3	0.4	0.5	1.2	-0.1
Cheese	2.5	1.5	2.2	5.3	-0.2
Dry Milk	-1.1	1.2	0.1	1.7	-
Wheat	2.9	2.8	-2.2	-2.6	-0.2
Coarse Grains	-5.1	-7.1	0.1	-5.0	0.3
Rice	-3.0	1.2	1.2	-0.8	-0.1
Soybeans	-1.8	-1.3	-2.3	0.5	0.3
Soybean Meal	0.1	0.4	0.4	1.1	-0.3
Average of absolute values	2.3	2.5	1.0	2.4	0.2

Table 8. Percentage changes in final consumption levels relative to observed data, in %.

	Argentina	Brazil	Paraguay	Uruguay	ROW
Beef	-0.9	-0.5	-0.2	-3.0	0.2
Pork	-2.3	-5.5	-0.4	-	0.1
Poultry	3.4	0.6	0.5	10.9	0.2
Fluid Milk	-0.1	-	0.2	0.2	-
Cheese	-5.6	-2.6	-7.2	-7.5	0.3
Dry Milk	-0.8	-4.4	-4.0	-6.7	0.1
Wheat	-0.7	0.9	1.2	2.9	0.1
Coarse Grains	3.4	6.9	1.2	6.9	-0.2
Rice	1.7	-0.9	-1.4	0.1	0.1
Soybeans	0.8	1.1	0.9	2.1	-0.3
Soybean Meal	-10.8	-7.9	0.8	1.3	0.3
Average of absolute values	2.8	2.8	1.6	3.8	0.2

Table 9. Percentage changes in prices relative to observed data, in %.

	Argentina	Brazil	Paraguay	Uruguay	ROW
Beef	2.1	2.1	0.7	4.6	-0.4
Pork	6.0	6.9	0.7	1.7	-0.2
Poultry	0.3	0.9	1.1	-1.4	-0.3
Fluid Milk	2.0	0.1	1.1	0.9	-
Cheese	8.6	3.5	14.6	12.9	-0.6
Dry Milk	0.9	5.1	4.7	7.9	-0.2
Wheat	3.1	-1.0	-6.1	-16.5	-0.3
Coarse Grains	-7.2	-14.6	-1.2	-22.3	0.6
Rice	-3.9	1.5	0.3	-3.9	-0.3
Soybeans	-2.8	-4.1	-4.1	-8.5	0.9
Soybean Meal	-3.3	-4.2	-4.2	-7.8	-0.2
Average of absolute values	3.7	4.0	3.5	8.0	0.4