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AAE374 Discussion Section[1]
Functions, Graphs and Excel

Functions:

A rule that assigns one & only one output value to every input value.

Ex 1. $y = f(x) = x^{1/2}$

Ex 2. $y = f(x_1, x_2) = x_1 + x_2$

Graphs: (see functions.xls)

- Linear functions

Ex 3. $y = f(x) = 2x + 1$

Slope: represents the relationship between input & output, or $\frac{\Delta y}{\Delta x}$ (Δ means change).

- Non-linear functions

Ex 4. $y = f(x) = -x^2 + 2x$

Ex 5. $y = f(x) = \log(x)$ $\quad$ ($10^{\log x} = x$)

Derivative: In calculus, the derivative is a measure of slope at a particular point.

Two examples in economics

1. Production functions

Production functions describe how inputs such as capital K (machinery) & labor L (workers) combine to produce output Y (consumption goods).

Ex. Cobb-Douglas function $Y = F(K, L) = K^{\alpha} L^{1-\alpha}$ where α is capital's share of income, which is assumed to have a value between 0 and 1.

Constant Returns to Scale: When you multiply all inputs by some factor, output will increase/decrease by the same factor (e.g., double inputs -> double output).

$$F(zK, zL) = (zK)^{\alpha} (zL)^{1-\alpha} = zK^{\alpha} L^{1-\alpha}$$
$$= zF(K, L) = zY \quad \text{(CRS).}$$

Ouput per-capita: $\quad y = \frac{Y}{L} = (\frac{1}{L}K)^{\alpha} (\frac{1}{L}L)^{1-\alpha} = (\frac{K}{L})^{\alpha} = k^{\alpha}$.

A more general form is $Y = F(K, L) = K^{\alpha} L^{\beta}$ and CRS if $\alpha + \beta = 1$, DRS if $\alpha + \beta < 1$ and

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IRS if $\alpha + \beta > 1$.

Marginal Product: The marginal product of a particular input is the extra output produced when one more unit of the input is used in production. The diminishing marginal product means that as you increase only one input, output will increase, but at a decreasing rate.

[Can you come up with an example that justifies diminishing marginal product?]

2. Growth rates

Country A has the following GDP for the corresponding years (GDP is in billions of Country A's currency).

Year	GDP
2000	5
2005	5.5
2010	6
2015	20
2020	8

Calculate the GDP growth (g) between 2 years.

We take the difference between GDP in the 2 years and divide it by the value in the starting year.

$$g = \frac{Y_t - Y_0}{Y_0}$$

Ex. GDP growth from 2000 to 2010 is (6-5)/5=0.2.   20%

Ex. What is the average growth rate per year over this period?

$$Y_t = (1+g)Y_{t-1} = (1+g)^2 Y_{t-2} = \cdots = (1+g)^t Y_0$$

$$g = (\frac{Y_t}{Y_0})^{1/t} - 1$$

Therefore, the average GDP growth per year is $(\frac{6}{5})^{1/10} - 1 \approx 0.0184$.   1.84%

Similarly, the average GDP growth per year between 2000 and 2020 is

$$(\frac{8}{5})^{1/20} - 1 \approx 0.0238$$ 2.38%

Rule of 72: How long does it take for a country's GDP to double? A useful mathematical approximation is the rule of 72.

$$\text{Doubling time } \approx \frac{72}{g}$$

If g is 2%, then the doubling time is about 36 years.

Simulations: (see growth.xls and the Excel handout for more tips of using Excel)

Exercise

1. Chinese GDP per capita in 2000 is 3843 US$. Numerically simulate the GDP growth with the growth rate 7% per year up to 2030.
2. Country A's GDP per capita in 2000 is 38000 US$. Numerically simulate the GDP growth with the growth rate 2% per year up to 2030.
3. Draw graphs and compare the results.