

AAE374 Discussion Section  
Final Review

**1. Production Functions:**

Cobb-Douglas production function  $Y = F(K, L) = K^\alpha L^{1-\alpha}$  where  $\alpha$  is capital's share of income, which is assumed to have a value between 0 and 1.

Output per-capita:  $y = \frac{Y}{L} = \left(\frac{1}{L}K\right)^\alpha \left(\frac{1}{L}L\right)^{1-\alpha} = \left(\frac{K}{L}\right)^\alpha = k^\alpha$ .

a. Returns to Scale

Constant Returns to Scale: Increase all inputs by the same factor; output will increase by the same factor (e.g., double the inputs => output is doubled).

$$F(zK, zL) = (zK)^\alpha (zL)^{1-\alpha} = zK^\alpha L^{1-\alpha} = zF(K, L) = zY \text{ (CRS)}.$$

A more general form is  $Y = F(K, L) = K^\alpha L^\beta$  and CRS if  $\alpha + \beta = 1$ , DRS if  $\alpha + \beta < 1$  and IRS if  $\alpha + \beta > 1$ .

b. Marginal Returns

The marginal product of a particular input is the extra output produced when one more unit of the input is used in production, maintaining every other input at their initial levels. Diminishing marginal product mean that as you increase only one input (leaving all other inputs at the same level) output will increase but at a decreasing rate.

**2. Trade Theory:**

a. Heckscher-Ohlin 2-2-2 (PS1)

(A1) Factor endowments: differences in relative factor endowments

(A2) Identical technologies across countries

(A3) Factor intensities

(A4) Homogenous consumer tastes

Differences in relative prices as a basis for trade

Heckscher-Ohlin Theorem: country with abundant capital will be able to produce relatively more of the capital intensive product

Factor Price Equalization Theorem: As trade takes place between two countries, prices adjust until relative prices are equalized

**Growth Policy: Free trade (no government intervention) seems to be the best strategy for growth.**

### 3. Economic Growth

#### a. Solow & Absolute Convergence (PS2)

Consider a group of economies with a production function specified as in the Solow Model.

$$Y = K^\alpha L^{1-\alpha}$$

$$y = k^\alpha$$

We know that the growth rate of the stock of capital per capita will be:

$$\Delta k = sk^\alpha - (\delta + n)k$$

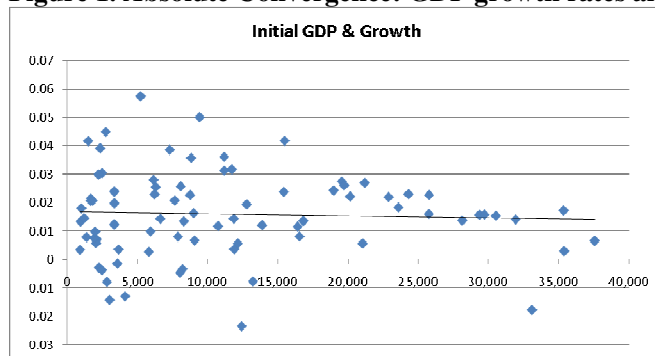
Suppose these countries have access to the same technology, and have the same savings rate, depreciation rate, and population growth.

**Growth policy: Even if the countries start with a different stock of capital per capita, in the long run they will converge to the same equilibrium: zero growth in steady state and same capital labor ratio. South will catch up with the North.**

This equilibrium (steady state) will be characterized by the same capital – labor ratio, output per capita, and growth rate. Recall that in steady state the growth rate of all variables (in per capita terms) is equal to zero.

Then, poor countries will be growing at a higher rate than the rich countries. Why? As  $k_1 < k_2$ , then  $f'(k_1) > f'(k_2)$ ; the marginal product of per capita capital is higher in the poor nations than in the rich ones, hence poor countries will accumulate more capital and grow at a faster rate than rich countries.

**Figure 1. Absolute Convergence: GDP growth rates and initial GDP (1965)**



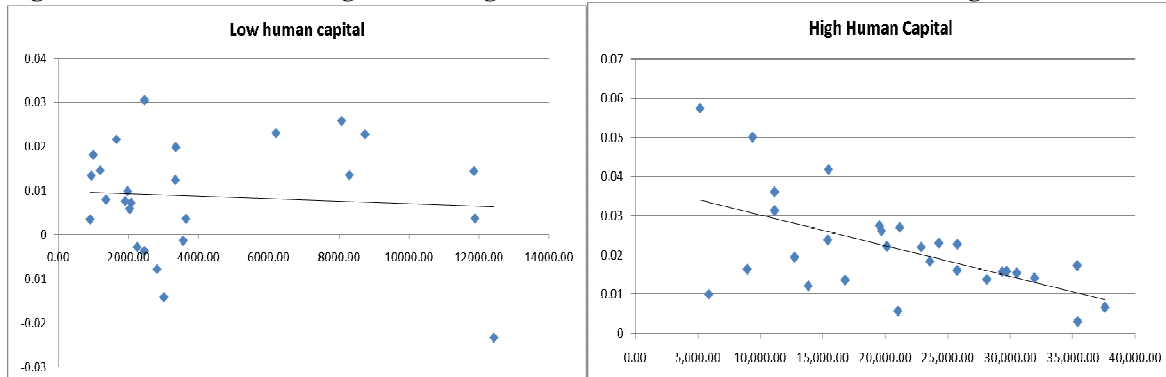
#### b. Savings, Human Capital & Conditional Convergence (PS3)

Suppose countries have the same technological possibilities, depreciation rate and population growth rates, but exhibit differences in savings rate and initial stock of capital per capita. Then, under conditional convergence, countries will still converge to the same growth rate in the long run (growth of zero in steady state), but not necessarily to the same capital-labor ratio.

The reason for this is the differences in savings. Thus, conditional convergence hypothesis asserts that countries can differ in their steady-state ratios (e.g.  $k_1^*$  vs.  $k_2^*$ ) and thus in consumption per capita, but as long as they have the same population growth and depreciation rate eventually they will grow at the same rate in steady state ( $g^*=0$ ).

Human capital is another factor that supports the conditional convergence hypothesis. We saw from the models in class that countries with higher human capital will have different steady states of output and capital per capita.

**Figure 2. Conditional Convergence: GDP growth rates and initial GDP in low and high education countries**



#### 4. Technology, Externalities & Scale Economies

Consider the production of bananas and computers

$$\text{Bananas: } Y_b(t) = A_b [K_b(t)]^\alpha [L_b(t)]^{1-\alpha}$$

$$\text{Computers: } Y_c(t) = A_c(t) [K_c(t)]^\alpha [L_c(t)]^{1-\alpha}$$

Growth is a story about accumulation of technology, spillovers, externalities, and coordination.

**Growth Policy: governments can play a role by providing incentives for technological change, and solving coordination problems**

#### 5. Trade Theory + Growth + Economies of Scale (PS4)

##### a. No IRS:

Convergence from both from convergence of capital/worker (Solow) which over time also causes convergence in H-O because as capital/worker converges the basis for comparative advantage shrinks, as seen in the % of capital in the computer sector. Convergence in capital/labor ratio, patterns of specialization, and welfare.

**Growth Policy: Free market rules! (again?)**

##### b. IRS:

Trade leads to specialization, but as the north begins to invest more and more in the computer sector, it will reach the threshold for economies of scale. When this happens, the North's

comparative advantage in computers increases (H-O) the relatively slower growth in the South keeps the South from “catching-up” in computer technology, so over time their comparative advantage goes away from computers (less % of Capital in computer sector graph).

Thus, trade worsens the “core-periphery” structure, as in terms of specialization, the South eventually “deindustrializes” its computer sector. South becomes relatively worse off while North becomes relatively better off. Differences in income/capita growth rate, income/capita levels and utility levels persist and get bigger.

**Growth Policy: Free market does not rule!**

c. Autarchy:

The South would have faster income growth, higher utility and income, and would specialize less in bananas than in autarchy.

Some costs in the first periods as it can not take advantage of its comparative advantage, but over time it will invest in the computer sector and become a computer “super star”.

**Growth Policy: does autarchy rule?**

d. Coordination failure?

No single capitalist alone in the South would find it rational or profitable to invest a large amount in a computer factory, but if everyone got together and did so the country would be better off.

e. Tariffs

Over the first five years a tariff of 10% causes the country to produce a higher proportion of manufactured products (computers), a lower production of agricultural output (bananas). Hence it limits the scope for specialization.

Over the long run the tariff will make the country better off than either free trade or autarchy. This is because it protects the infant computer sector in the South and leads to a higher investment in computers than it would have with either free trade or autarchy. In such case, the South will accumulate capital in the computer sector and will eventually reach the threshold where scale economies kick in, and thus would enjoy from higher productivity.

**Growth Policy: do tariffs rule? Now I'm super confused.**