AAE / ECON / Forest 531 (Natural Resource Economics)

Final Exam. Suggested Solutions.

Please answer the following five questions, worth a total of 30 points. Question 1 is worth 10 points; questions 2 through 5 are worth 5 points each. Good luck.

1. **Mathematics / Derivation (10 points)**: Stream restoration typically involves planting trees in the riparian zone for shade and installing streambed habitat structures. These efforts can increase fish populations by reducing stream temperature and improving the quality of fish habitat. Consider the problem of a natural resource manager attempting to allocate conservation efforts *C* along a stream of length *S*. Benefits are derived from the recreational use of the stream at intervals *s*=1, 2, ….., *S*, where *s*=1 defines the headwaters of the stream. At each interval *s* along the stream, the net benefits of recreation are a function of conservation effort (C_s) and stream temperature (T_s). This relationship can be formally defined as $B_s = C_s - 0.25C_s^2 - 0.1T_s$. The stream temperature naturally increases as water gets further from the headwaters, and is affected by conservation efforts through the following relationship: $T_{s+1} = T_s(1+0.05) - C_s$. Suppose the resource manager is attempting to maximize the net benefits of restoration along the entire stream, and the shadow price of temperature at the stream's outlet is defined by $\lambda_{S+1} = 0$. Formally state the manger's optimization problem. If *S*=3, what is the optimal allocation of conservation effort C_s^* along each point *s* of the stream? Show your work, and explain the intuition regarding the allocation of conservation effort and the shadow price of temperature along the stream.

   The resource manager's problem can be stated as:

$$Max \quad \sum_{s=0}^{S} C_s - 0.25 \cdot C_s^2 - 0.1 \cdot T_s \quad s.t. \quad T_{s+1} = T_s(1+0.05) - C_s$$

   with the following Lagrangian:

$$L = \sum_{s=0}^{S} [C_s - 0.25 \cdot C_s^2 - 0.1 \cdot T_s] + \lambda_{s+1}[T_s(1+0.05) - C_s - T_{s+1}]$$

   and the following FOC:

$$(i)\, \partial L / \partial C_s = 1 - 0.5 \cdot C_s - \lambda_{s+1} = 0$$
$$(ii)\, \partial L / \partial T_s = -0.1 + \lambda_{s+1}(1+0.05) - \lambda_s = 0$$
$$(iii)\, \partial L / \partial \lambda_{s+1} = T_s(1+0.05) - C_s - T_{s+1} = 0$$

   We can use the fact that S=3, $\lambda_{3+1} = 0$*, and equation (ii) to solve for the shadow price at each point of the stream. Equation (ii) can be re-arranged to yield* $\lambda_s = \lambda_{s+1}(1+0.05) - 0.1$*, which then can be used to get the following time path for the shadow price:*

$$\lambda_4 = 0; \lambda_3 = -0.1; \lambda_2 = -0.205; \lambda_1 = -0.315$$

   Further, since equation (i) can be re-arranged to yield $C_s = 2(1 - \lambda_{s+1})$*, which can then be combined with the shadow price time path to yield the following:*

$$C_3^* = 2; C_2^* = 2.2; C_1^* = 2.41$$

   The shadow price is always negative because higher temperatures reduce the net benefits of recreation – the shadow price is most negative at the headwaters, as high temperatures there imply high temperatures downstream. Since conservation efforts upstream reduce temperature at each point downstream, conservation effort is declining as you move further downstream.

Short Answer Section (2-5 sentences; 5 points each)

2. In class, we examined a simple version of the problem of optimal extraction of a nonrenewable resource. The key result was that prices should rise at the rate of interest. In his classic article on this subject, Harold Hotelling argued that a competitive market—that is, many price-taking mine owners—would produce the same result. Explain.

   Suppose $\rho P_{t+1} > P_t$ *. In this case, a mine owner has an incentive to wait until t+1 to extract because their investment earns more in the ground. If all mine owners behave this way,* P_t *will rise due to reduced supply. Now suppose* $\rho P_{t+1} < P_t$ *. In this case, a mine owner has an incentive to extract everything in t because alternative investments have higher returns. If all mine owners behave this way,* P_t *will fall due to decreased supply. Therefore, the competitive equilibrium is* $\rho P_{t+1} = P_t$ *, or Hotelling's rule.*

3. Uncertainty over future parameters can influence the efficient management of environmental resources.

   a. What are the implications of random shocks (e.g. drought, fires, etc.) for the concept of steady-state as a sustainable development principle? Explain.

      Random shocks suggest that a deterministic steady-state may not be practical as shocks will cause the system to deviate. Therefore, alternative definitions – like stochastically sustainable development – can be applied by using stochastic simulation to examine conditions under which the resource stock size always remains above some pre-specified minimum for a wide range of feasible shocks.

   b. What are the implications of uncertain returns to investment on the efficient time path of reducing CO2 emissions to control climate change? Explain.

      Uncertain returns to investment imply an uncertain discount rate, as the discount rate is motivated by the opportunity cost of foregone investment. Typically, accounting for such discount rate uncertainty with simulation (ala Newell and Pizer) implies lower estimates of the present value of long term marginal pollution damages relative to using a constant discount rate – thus yielding a more aggressive efficient time-path for reducing CO2 emissions (i.e. more abatement in early periods).

4. When extracting groundwater, landowners typically only pay for pumping costs. Two regions have different groundwater resources:

c. Region A: the groundwater aquifer is characterized by a large lake under the land (i.e. a bathtub). A farmer drawing water from one end of the underground lake immediately lowers the level of water throughout the lake, causing the cost of groundwater pumping to increase for all farmers with land overlying the lake.
d. Region B: the groundwater aquifer is a series of underground "built-in pools", with each pool abutting other pools in the neighborhood, but hydrologically disconnected from them (that is, water does not flow between the pools). In this region, the aquifer is affected only by the activities on the land directly over it. So, for instance, if a farmer draws groundwater for irrigation, the water level beneath his/her land falls, but the water level beneath the land of neighboring farmers is unaffected.

Will initial extraction rates be higher in Region A or Region B? What happens to the groundwater extraction path in the two regions if the discount rate increases? Explain.

Initial extraction rates will be higher in region A as the groundwater is open access and users won't observe the shadow price of water – and so won't account for the impacts of their use on future pumping costs. In contrast, the shadow price of water is observed by users in region B since reducing a groundwater pool only impacts the user that is pumping water. If the discount rate increases, there will be no effect on extraction in region A since the future is not accounted for because the shadow price of water is not observed by users. In contrast, a higher discount rate will increase the rate of extraction in region B as the present value of the shadow price is lower, so users will optimally extract water faster when marginal net benefits of using water are downward sloping.

5. True, False, Uncertain. Explain your answer in one to two sentences each.

a. A decrease in the back-stop price of a non-renewable resource leads to higher current extraction rates because the marginal user cost of extraction is higher.

False. While a decrease in the back-stop price does in fact lead to higher extraction rates, this result occurs because the marginal user cost of extraction is lower – the future value of the non-renewable is lower with a lower back-stop price.

b. If a resource is becoming physically scarce and its price does not increase, it is becoming less scarce economically.

Uncertain. The marginal user cost (or shadow price of the stock) is an indicator of economic scarcity, and is equal to price minus marginal extraction costs when a resource is being optimally extracted. Even if price does not increase, a resource might be increasingly scarce if the marginal extraction costs are declining faster than price.