AAE / ECON / FOREST 531 (Natural Resource Economics)
Homework #5
Due in Class on Thursday, December 4, 2008

1. **Renewable Substitutes.** This problem is a continuation of question #2 on Homework #4 for a competitive industry. Suppose the price of oil is given by $p_t=a-bq_t$, where q_t is the rate of oil extraction in period t, a=1 and b=0.1. The initial oil reserve (R_0) equals 75, the remaining oil reserves change according to $R_{t+1}=R_t-q_t$, and there are no extraction costs. Further, assume a 5% discount rate. Now, suppose a scientist develops a breakthrough using hydrogen as a substitute for oil, where hydrogen can be sold at a constant price of 0.8.
    a. If the breakthrough occurs at t=5, plot the optimal time path of extraction (q_t) and price (p_t) for the life-cycle of the oil reserve.
    b. If the breakthrough occurs at t=13, plot the optimal time path of extraction (q_t) and price (p_t) for the life-cycle of the oil reserve.

2. **Investment in cost-reducing technology.** A primary response to resource scarcity is investment in cost-reducing technology. Suppose a firm is interested in maximizing profits from extracting a non-renewable resource over two periods (t=0,1), where their profits in time t are written as $\pi_t = P_t q_t - cq_t^2 / R_0$, where P_t is the price of the resource, q_t is the amount extracted, c is a positive cost parameter, and R_0 is the initial stock. Suppose the firm is interested in investing in research in period 0 that will potentially lower their cost parameter in t=1 from c to $(c-\alpha I_0)$, where $0 < \alpha < 1$ and I_0 is the level of investment in period 0. Suppose further, that they are uncertain of the outcome from investment, and they believe two states are possible in t=1. In state 0, the research investment fails and the cost parameter stays at c. In state 1, the research investment succeeds and the cost parameter in t=1 falls to $(c - \alpha I_0)$. The probability of state 0 is ε, while the probability of state 1 is $1- \varepsilon$.
    a. Derive an expression for the firm's optimal extraction choice in t=1 for both possible states (S=0, 1).
    b. Using your results from part a, what expression determines the firm's optimal profits in t=1?
    c. Suppose the investment level I_0 in period 0 will cost the firm $w_0 I_0^2$, where w_0 is a positive parameter. Using your results from part a and b, what expression determines the firm's optimal profits in t=0?
    d. Suppose the following parameter values: $R_0=100$, $P_0=1$, $P_1=1.1$, c=2, w=5, $\alpha=0.1$, $\varepsilon=0.5$, and a discount rate (δ) of 0.05. Using Solver, maximize your expression from part c by choosing q_0 and I_0. Use starting values of $q_0=20$ and $I_0=0.04$. What is q_0^* and I_0^*?
    e. Using Solver, conduct comparative statics to see how investment changes as a result of a change in each of the eight parameters.

3. **Water resources.** Consider the problem of a water district managing the allocation of irrigation water along the length of a canal. The district wishes to maximize the value of water sent down the canal. The initial stock of water is X_0, and the length of the canal is C. Irrigation ditches lie perpendicular to the canal at intervals c=0, 1, 2,…., C. At each

point along a ditch, net returns to agriculture are defined as $R_c=aY_c-(b/2)Y_c^2$, where Y_c indicates the total amount of water allocated to ditch c, and a and b are positive parameters. As water moves downstream, it seeps out of the canal. Define the seepage at any given ditch c as dX_c, where d represents the proportion of the stock lost to seepage at ditch c. In summary, water moving down the canal is reduced at each ditch by the amount of seepage and by the amount of water removed for agriculture.

a. State the district's optimization problem.
b. Derive the first-order conditions.
c. Suppose d=0. Are water withdrawals increasing, decreasing, or constant along the canal? Provide economic intuition.
d. Suppose $0<d<1$. Are water withdrawals increasing, decreasing, or constant along the canal? Provide economic intuition.