AAE / ECON / FOREST 531 (Natural Resource Economics)
Homework #4
Solutions

1. **Biodiversity conservation.** Consider a competitive land market defined by an inverse demand function for development ($P = P_d - m_d q$) and a supply function of land ($P = P_s + m_s q$), where P is price, q is quantity of land (acres), and P_d, P_s, m_d, and m_s are positive parameters. There are 100 total acres available in this market and all undeveloped land is considered open land. Suppose a conservation group enters the land market to purchase land for biodiversity reserves, creating a new inverse demand function of $P' = P_d' - m_d q$. Suppose the parameter α represents the ecological value of open land as a fraction of reserves, where α=0.8.
   a. What is the ecological value of this land market in the absence of the conservation group?

   First, solve for equilibrium: $P_d - m_d q = P_s + m_s q \Rightarrow q_1 = (P_d - P_s)/(m_d + m_s)$. *If A=100, the ecological value of this land market* $0.8*(100-q_1)$.

   b. Suppose the following numerical values for all parameters: $P_d=10$, $P_d'=12$, $P_s=1$, $m_d=m_s=0.1$. If the conservation group enters the land market, how much land will they buy (q_c) and what is the total change in conservation (ΔC)?

   First, solve for equilibrium without conservation: $10-0.1q=1+0.1q \Rightarrow q_1=45, p_1=5.5$. *Second, solve for equilibrium with conservation:* $12-0.1q=1+0.1q \Rightarrow q_2=55, p_2=6.5$. *Third, solve for* q_d, *or the total amount of developed land:* $q_d=(10-6.5)/0.1 = 35$ *acres. So, the conservation group buys* $q_c=20$ *acres, and the change in conservation equals* $(1-\alpha)q_c + \alpha(q_1-q_d) = 12$.

   c. Suppose the following numerical values for all parameters: $P_d=12.25$, $P_d'=15.25$, $P_s=1$, $m_d=0.15$, and $m_s=0.1$. If the conservation group enters the land market, how much land will they buy (q_c) and what is the total change in conservation (ΔC)?

   First, solve for equilibrium without conservation: $12.25-0.15q=1+0.1q \Rightarrow q_1=45, p_1=5.5$. *Second, solve for equilibrium with conservation:* $15.25-0.15q=1+0.1q \Rightarrow q_2=57, p_2=6.7$. *Third, solve for* q_d, *or the total amount of developed land:* $q_d=(12.25-6.7)/0.15 = 37$ *acres. So, the conservation group buys* $q_c=20$ *acres, and the change in conservation equals* $(1-\alpha)q_c + \alpha(q_1-q_d) = 10.4$.

   d. Compare the answers to parts b and c and intuitively explain any differences.

   The conservation group buys the same amount of land (20 acres) in both scenarios. However, in part c, the change in conservation is less than in part b because the reserve purchase comes more at the expense of open land than developed land. This is due to the fact that developer's demand is more inelastic in part c.

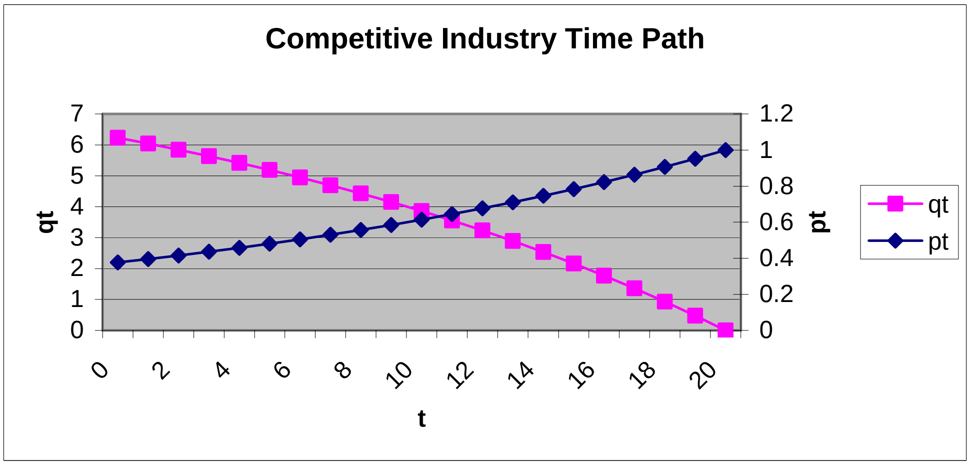
   e. How do your answers to parts b and c change if α increases to 0.95? Explain.

   If α=0.95, ΔC lowers to 10.5 in part b and 8.6 in part c. Higher α implies higher ecological value of open land, and thus purchasing reserves will have less impact.

2. **Competitive vs. Monopolistic Extraction**. Suppose the price of oil is given by $p_t=a-bq_t$, where q_t is the rate of oil extraction in period t, a=1 and b=0.1. The initial oil reserve

(R_0) equals 75, the remaining oil reserves change according to $R_{t+1}=R_t-q_t$, and there are no extraction costs. Further, assume a 5% discount rate.

a. Suppose the oil industry is competitive, and each firm is interested in maximizing the present value of net revenue over the horizon t=0, 1, ….., T_C. Using Hotelling's rule, plot the optimal time path of extraction (q_t) and price (p_t) for the life-cycle of the resource using Excel.



b. Suppose that a monopoly controls oil extraction and is interested in maximizing the present value of net revenue over the horizon t=0, 1, ……, T_M.

i. Starting from the monopolist's optimization problem, derive Hotelling's rule for monopolists: $MR_t=(1+\delta)MR_{t-1}$, where MR_t is the monopolist's marginal revenue in time t.

If the monopolist faces inverse demand of $p_t=a-bq_t$, then their profit function is described by $\pi_t = p_t q_t = aq_t - bq_t^2$ *and their optimization problem is:*

$$Max \quad \sum_{t=0}^{T} \rho^t \pi_t \quad s.t. \quad R_{t+1} = R_t - q_t$$

$$L = \sum_{t=0}^{T} \rho^t \left\{ \pi_t + \rho\lambda_{t+1} \left[R_t - q_t - R_{t+1} \right] \right\}$$

with the following first-order conditions:

$$(I)\frac{\partial L}{\partial q_t} = \rho^t MR_t - \rho^{t+1}\lambda_{t+1} = 0$$

$$(II)\frac{\partial L}{\partial R_t} = \rho^{t+1}\lambda_{t+1} - \rho^t \lambda_t = 0$$

$$(III)\frac{\partial L}{\partial \rho\lambda_{t+1}} = R_t - q_t - R_{t+1} = 0$$

where MR_t is the derivative of π_t with respect to q_t. Using (II), we see that $\rho\lambda_{t+1} = \lambda_t$, *and using (I) we see that* $MR_t = \rho\lambda_{t+1}$, *which implies that*

$MR_{t-1} = \rho\lambda_t$. *Therefore,* $MR_{t-1} = \rho MR_t$ *and* $MR_{t-1}(1+\delta) = MR_t$. *In this problem,* MR_t*=a-2b*q_t*.*

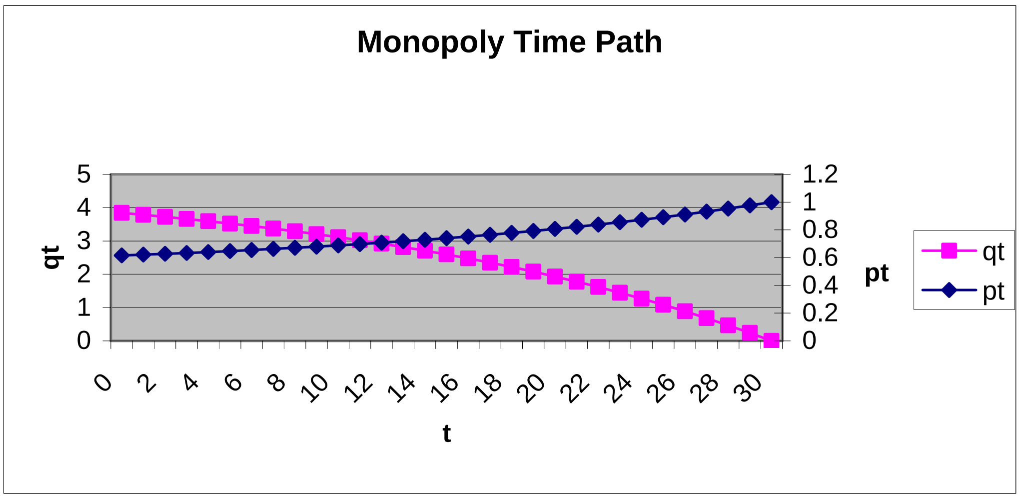
ii. Using Hotelling's rule for monopolists, plot the optimal time path of extraction (q_t) and price (p_t) for the life-cycle of the oil resource using Excel.

Since $MR_t = a - 2bq_t$, *Hotelling's rule for monopolists implies the following:*

$$a - 2bq_{t-1} = \rho(a - 2bq_t)$$

$$\Rightarrow q_{t-1} = (1/2b)\left[a(1-\rho) + \rho 2bq_t\right]$$

This equation provides the basis for solving for the price path (see Excel).



c. Compare the optimal time path of extraction and price for the competitive industry and the monopolist. Is T_C><T_M? Provide intuition.

Since the monopolist is a price-setter, the maximization of monopoly profits always leads to a higher price than the competitive market. Therefore, since demand is downward-sloping, extraction is lower in each period for the monopolist than for the competitive industry and the depletion time is later. Hence, the monopolist is a conservationist.

3. **Landfill Management.** Consider the problem of a landfill manager acting on behalf of society. The initial amount of space available in the landfill in year 0 is S_0. The manager must decide how much fill X_t to put in the landfill in each year until the landfill is filled (it is filled when S=0). Landfill space evolves over time according to $S_{t+1} = S_t - X_t$. The inverse demand for landfill dumping is given by $P_t = a - bX_t$, where P_t is the price charged per unit garbage. The demand is downward sloping because it is possible to

recycle and reuse waste, and because people can change their use of materials as the cost of landfilling increases; for instance, they can switch their use of diapers from disposables to cloth. With this demand curve, the benefit of landfill dumping at any point in time is $aX_t - (b/2)X_t^2$.

a. Set up the dynamic optimization problem of the landfill manager and derive the first-order conditions.

$$Max \quad \sum_{t=0}^{T} \rho^t \left[aX_t - (B/2)X_t^2 \right] \quad s.t. \quad S_{t+1} = S_t - X_t$$

$$L = \sum_{t=0}^{T} \rho^t \left[aX_t - (B/2)X_t^2 + \rho\lambda_{t+1}\left(S_t - X_t - S_{t+1}\right)\right]$$

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$$(i)\frac{\partial L}{\partial X_t} = \rho^t \left[a - bX_t - \rho\lambda_{t+1}\right] = 0$$

$$(ii)\frac{\partial L}{\partial S_t} = \rho^t \left(\rho\lambda_{t+1} - \lambda_t\right) = 0$$

$$(iii)\frac{\partial L}{\partial \rho\lambda_{t+1}} = S_t - X_t - S_{t+1} = 0$$

b. What should be the price charged per unit garbage at each point in time? Provide the economic intuition behind this result.

Using (ii), we see that $\rho\lambda_{t+1}=\lambda_t$. Using (i) and (ii), we see that $P_t= a-bX_t=\rho\lambda_{t+1}= \lambda_t$. Since $P_{t-1}= \rho\lambda_t$, then $P_{t-1}= \rho P_t$. Re-arranging, we get Hotelling's Rule:

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \delta$$

So, the price charged per unit garbage should increase at the rate of discount until the dump is full. When the dump is full, then $P_T=a$, and therefore $P_t=a(1+\delta)^{t-T}$.

c. Would residents dump the optimal amount of garbage if they were not charged a per unit landfill fee? The absence of a landfill fee applies, for instance, in communities that have curbside pickup, and pay for garbage collection through property taxes. Formally state (give a mathematical expression) the amount of garbage that would be dumped in each period in the absence of a landfill fee.

With no garbage price, residents would dump until the marginal net benefits from doing so equal zero: $a-bX_t=0$. This implies the following constant dump rate:

$$X_t = a/b$$

Notice that the optimal dumping rate is lower as residents observe the marginal user cost of dumping:

$$a - bX_t = \rho\lambda_{t+1}$$

$$=> X_t = \frac{a - \rho\lambda_{t+1}}{b}$$

So, residents treat dumps similar to an open access resource when there is no dumping fee. Notice that X_t will optimally decline over time since $\lambda_{t+1} > \lambda_t$ for all t.