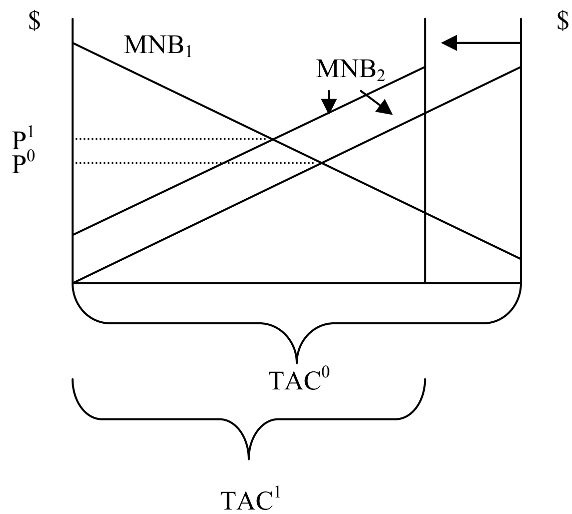
AAE / ECON / FOREST 531 (Natural Resource Economics)
Homework #3
Suggested Solutions

1. Suppose a fishery is managed in steady state with an Individual Transferable Quota (ITQ) system.
   a. What is the relationship between the equilibrium quota price and the optimal shadow price of the stock? Explain.

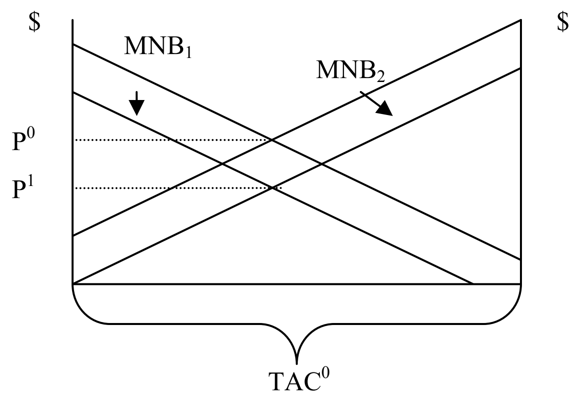
   If the fishery is managed for a bioeconomic optimal stock and harvest, then the first-order conditions derived from the relevant dynamic optimization problem suggest the marginal net benefit of the last unit of harvest should equal the optimal shadow price of the stock. With an ITQ system, the equilibrium quota price equals the marginal net benefit of the last unit of harvest, so the quota price should reflect the optimal shadow price – assuming the fishery is managed at its steady state bioeconomic optimum.

   b. Graphically illustrate the effect of the following factors on the equilibrium quota price: i) a <u>decrease</u> in the government-set total allowable catch (TAC), and ii) an <u>increase</u> in fuel prices for the boats in the fleet. Explain.

   i) A decrease in the TAC from TAC^0 to TAC^1 will <u>increase</u> the quota price from P^0 to P^1.



   ii) An increase in fuel prices will increase the MC of fishing, thus lower the MNB of fishing for all boats and <u>reduce</u> the quota price from P0 to P1.



c. Using the biological and economic functional forms used in class on 9/23 and 9/30, how should the fishery manager adjust the steady-state stock size in response to an increase in the discount rate if their objective is to maximize fishery profits? Explain.

The fundamental equation of renewable resources must hold if fishery profits are maximized. If the discount rate increases, then the left-hand side of FERR - $F'(X)+\frac{\partial\pi/\partial X}{\partial\pi/\partial Y}$ *- must increase to maintain the equality implied by FERR. Since both terms on the left-hand side of FERR are decreasing in X, then the manager should <u>lower</u> the steady-state stock size to increase the internal rate of return of the fishery.*

2. You manage a forest products company with land recently planted (t=0) with maple. The merchantable volume of timber at $t\geq 0$ is given by $Q(t)=at+bt^2-ct^3$, where a=10, b=5, and c=0.02.
   a. What is the maximum volume and when does it occur?

      Maximum timber volume
      $Q(t)=10t+5t^2-0.02t^3 \Rightarrow Q'(t)=10+10t-0.06t^2=0 \Rightarrow t=167.67$ *(using quadratic formula).*

   b. What rotation length maximizes mean annual increment [Q(T)/T], and what is the associated volume?

$$Max \quad Q(T)/T$$

$$F.O.C. \quad \frac{Q'(T)T-Q(T)}{T^2}=0$$

$$\Rightarrow \frac{10T+10T^2-0.06T^3-10T-5T^2+0.02T^3}{T^2}=0$$

$$\Rightarrow 5T-0.04T^2=0 \Rightarrow 5-0.04T=0$$

$$\Rightarrow T=125$$

   c. Suppose the net price per unit volume is p=1 and the discount rate is δ=0.05, what is the optimal single rotation T_S; volume at harvest; and present value at t=0? (Hint: use Solver) What is the marginal cost of waiting at T_S?

Using Solver (see Excel sheet), $T_S=35.65$, $Q(T_S)=5804.85$, *PV=\$976.49, MC=\$290.43*

d. If the cost of replanting is c=150, what is the optimal Faustmann rotation T^*; volume at harvest; and present value at t=0? (Hint: use Solver) What is the marginal cost of waiting at T^*?

Using Solver (see Excel sheet), T=28.58, Q(T*)=3904, PV=$1182, MC=$247*
The Faustmann rotation is shorter than the optimal single rotation, reflecting the additional marginal cost of waiting associated with delaying all future harvests.

e. If the price increases to p=2, what are the new values for T_S and T^*? (Hint: use Solver) Interpret the change from when P=1.

Optimal T_S and T when p=2.*
Using Solver (see Excel sheet), T=27.69 and T_S=35.65. The Faustmann rotation is shorter with higher prices, reflecting the increased marginal cost of waiting associated with higher prices. However, the optimal single rotation is the same because timber prices cancel out of the first-order condition, since a constant timber price equally affects both the marginal value and marginal cost of waiting.*

f. If the discount rate increases from δ=0.05 to δ=0.1, what are the new values for T_S and T^*? (Hint: use Solver) Interpret the change from when δ=0.05.

Optimal T_S and T when δ=0.1.*
Using Solver (see Excel sheet), T=15.11 and T_S=18.23. Both rotations are shorter with a higher discount rate, reflecting the increased marginal cost of waiting associated with higher prices.*

3. E4.3 in Conrad (page 76). Hint: For parts (d) and (e), set up the equation G(T) that you derived in part (c) in Excel with an initial guess of T=150. Use Solver to pick a value of T to drive this equation to zero (in the Solver box, choose "Value of: 0" rather than "Max" under the "Equal to:" box).

a. *F.O.C.:*
$$PQ'(T) + A(T) = \delta PQ(T)$$
b. *Marginal value of waiting is the left-hand side of the equation in (a), while the marginal cost of waiting is the right-hand side of the equation in (a).*
c. *Given,* $Q(T) = e^{a-b/T}$ *and* $A(t) = vt - wt^2$, *we can calculate* $Q'(T) = (b/T^2)e^{a-b/T}$.
The equation G(T)=0 will be of the form: $G(T) = PQ'(T) + A(T) - \delta PQ(T) = 0$ *using the function derived in part (a). So, plugging in the functional forms for this problem, we get:*
$$G(T) = (Pb/T^2)e^{a-b/T} + vT - wT^2 - \delta Pe^{a-b/T} = 0$$
$$\Rightarrow G(T) = Pe^{a-b/T}(b - \delta T^2) + vT^3 - wT^4 = 0 \qquad (i)$$
d. *Rotation that maximizes π:*
Using Solver to drive (i) to zero, we get T_A=100.
e. *Amenity rotation vs. Commercial rotation*
If A(T)=0, then v=w=0 and using Solver to drive (i) to zero gets T_S=60. So, the optimal single rotation is shorter than the optimal amenity rotation because the marginal value of waiting is lower if there is no amenity value.

4. E7.1 in Conrad (page 164).

The payoffs can be described as:

t=0	t=1	t=2
T_0	N_1	N_2
	T_1 (w/ prob. π_1)	T_2 (w/ prob. π_2)
A_0	A_1 (w/ prob. 1- π_1)	A_2 (w/ prob. 1- π_2)

a) *The expression for option value is:*
$OV = \rho[\pi_1 T_1 + (1-\pi_1)A_1] + \rho^2[\pi_1 N_2 + (1-\pi_1)\{\pi_2 T_2 + (1-\pi_2)A_2\}]$. *The second term in the expression reflects that fact that if state 1 happens in t=1—at probability π_1—then the parcel will be developed and receive agricultural revenues in period 2 (N_2). And, if state 2 happens in t=1, then the parcel will be preserved and the optimal development decision will occur in time t=2 with probability 1-π_1.*

b) *Optimal first-period decision if δ=0.05 :*
With the given parameter values,
$P = A_0 + OV = 112.76$ *(the value of preserving the parcel in t=0)*
$D = T_0 + \rho N_1 + \rho^2 N_2 = 109.3$ *(the value of cutting the parcel in t=0)*
So, since P>D, it is optimal to preserve the parcel in t=0. Note: the attached Excel spreadsheet has the formulas programmed.

c) *Optimal first-period decision if δ=0.1:*
With the given parameter values, P=106.4 and D=108.7. So, since D>P, it is optimal to cut the parcel in t=0. As shown in the book, an increase in the discount rate reduces the option value of preserving by more than D, thus tipping the scales in favor of cutting.

d) *Probability that the forest will not be cut:*
Here, I'm assuming that δ=0.05, which implies that the forest will not be cut in period 0 with probability 1. Consider the probability that the parcel is forested in each of the three periods:
Prob(F in t=1) = 1
Prob(F in t=2) =1-π_1= 0.5
*Prob(F in t=3) = Prob(F in t=2)*Prob(not cutting in t=2)=(1-π_1)*(1-π_2) = 0.5*0.4 = 0.2.*

5. Wisconsin's Managed Forest Law aims to "encourage sustainable forestry on private lands by providing property tax incentives to landowners" and is described more fully at: http://dnr.wi.gov/forestry/ftax/MFL.htm. Drawing on concepts discussed in class and in section 4.5 in Conrad, intuitively discuss the potential impacts of the Managed Forest Law on total long-run timber supply generated from Wisconsin. State your assumptions explicitly.

There are many ways to answer this problem. Ultimately, since the answer is by no means obvious, the way to approach this problem is to make assumptions regarding how the managed forest law (MFL) might affect timber price (P), replanting costs (c), and forest rents ($\delta\pi$). Here's one possible—though by no means the only—way to answer.

- *Since the program is voluntary and many forest owners have enrolled in MFL, and supposing that landowners are profit maximizers, then forest owners would only be enrolling if it made them better off (i.e. increased timber price relative to management costs). Further, enrollment in MFL qualifies land to be "certified" by the Forest Stewardship Council under group certification, which allows enrolled landowners to obtain a price premium when selling their wood.*

- *Normally, an increase in timber price relative to management costs will shorten rotations, resulting in lower average annual volume and reduced long-run timber supply.*
- *However, since forests have to adopt "sound management practices" to be enrolled in MFL, then forest rotations may be longer if mandated and enforced by DNR and the group certification program, resulting in higher volume and more timber.*
- *In the long-run, if MFL in fact makes forestry more profitable (that may be a big "if"), then forest rents will increase and draw land into forestry, resulting in an increase in timber supply.*
- *So, if the mandated longer rotations outweigh the tax incentives and increase rotation lengths, then MFL could have a positive impact on timber supply. If longer rotation lengths are not enforced by DNR, then the tax incentives could reduce rotation lengths while increasing forest rents, resulting in an ambiguous timber supply effect.*