

AAE / ECON / FOREST 531 (Natural Resource Economics)
Homework #2
Solutions

1. Consider a fishery where $F(X_t) = rX_t \ln(K/X_t)$, $Y_t = qX_t E_t$, and $C_t = cE_t$, and where r , K , q , and c are positive parameters.
- a. What are the analytic expressions defining the open access equilibrium ($X_\infty, E_\infty, Y_\infty$)? (Recall the calculus rule: $\exp(\ln(G)) = G$).

The open access condition is $\pi = pqX_t E_t - cE_t = 0$. However, this is one equation and two unknowns (X, E), which is not solvable. So, by recognizing that $F(X) = Y(X, E)$ in steady state, we now have two equations and two unknowns. So the first step is to notice that E cancels out of the open-access condition and we get the following:

$$\pi = pqX - c = 0 \Rightarrow X_\infty = \frac{c}{pq}$$

Now, substituting X^ into the equation that relates growth to harvest, we get:*

$$\begin{aligned} F(X) &= rX \ln(K/X) = qXE = Y \\ \Rightarrow r \ln(K/X) &= qE \\ \Rightarrow E_\infty &= (r/q) \ln(K/X_\infty) \\ \Rightarrow E_\infty &= (r/q) \ln(pqK/c) \end{aligned}$$

Lastly, substituting expressions for E_∞ and X_∞ into the fishery production function, we get:

$$Y_\infty = qX_\infty E_\infty = q \left(\frac{c}{pq} \right) (r/q) \ln \left(\frac{pqK}{c} \right) = \frac{rc}{pq} \ln \left(\frac{pqK}{c} \right)$$

Suppose the fishery were to be managed so as to maximize the present value of net revenue given a positive discount rate of δ .

- b. What is the expression for $F'(X)$? (Recall the calculus rule: $D_x \ln u = (1/u) D_x u$, where $u=f(x)$)

Using the chain rule:

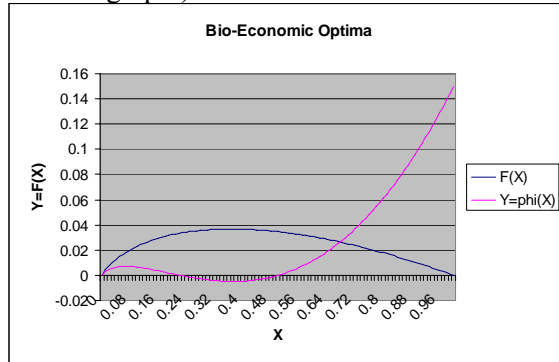
$$F'(X) = r \ln(K/X) + rX(X/K)(-K/X^2) = r[\ln(K/X) - 1]$$

- c. What is the fundamental equation of renewable resources for this problem? Re-arrange this equation such that Y is written as a function of X and model parameters ($Y = \phi(X)$).

Recall that $\partial \pi / \partial X / \partial \pi / \partial Y$ was derived in class for these functions (see 9/23), so:

$$\begin{aligned} r[\ln(K/X) - 1] + \frac{cY}{X(pqX - c)} &= \delta \\ \Rightarrow Y = \phi(X) &= \frac{X(pqX - c)[\delta - r[\ln(K/X) - 1]]}{c} \end{aligned}$$

- d. If $\delta=0.05$, use Excel to graph $Y = \phi(X)$ and $F(X)$ for the following parameter values: $r=0.1$, $K=1$, $q=0.01$, $p=200$, and $c=1$. (Hint: start with a column of X 's from $X=0$ to $X=1$ in intervals of 0.01; then write formulas for $Y = \phi(X)$ and $F(X)$ in neighboring columns and graph.)



- e. Using your graph from part d, what is the optimal steady state (X^*, Y^*) ? Compare to the open access equilibrium (X_∞, Y_∞) calculated from part a with the same parameter values.

Looking at the graph, $X^=0.67$, $Y^*=0.025$, $E^*=3.74$ (these are all approximate). Compared to open access, the bio-economic optimum consists of a higher stock, lower harvest, and lower effort level in steady-state than the comparable open access equilibrium values.*

- f. Given the optimal steady state calculated in part e, is the marginal stock effect greater than, less than, or equal to the discount rate? Explain.

In the graph, the bio-economic optimum is to the right of the point where $F'(X)=0$, so $F'(X)<0$ and therefore, the marginal stock effect must be greater than the discount rate in

order for FERR to hold ($F'(X) + \frac{\partial \pi / \partial X}{\partial \pi / \partial Y} = \delta$).

2. Consider the optimal management of the renewable resource from homework #1, question #4. The net benefits of using the resource in time t are defined as $\pi(X_t, Y_t) = 3X_t + 3Y_t$, and the resource has a natural rate of growth described by $F(X_t) = 2X_t - 0.5X_t^2$. Suppose $X_0=0.05$. Use Excel's Solver to determine the optimal approach to steady state over a ten year period ($t=0,1,\dots,9$) when the discount rate (δ) is 0.05. (Hint: start with an initial guess of $Y_t = 0.01$ for each t). Graph the approach to steady state.¹

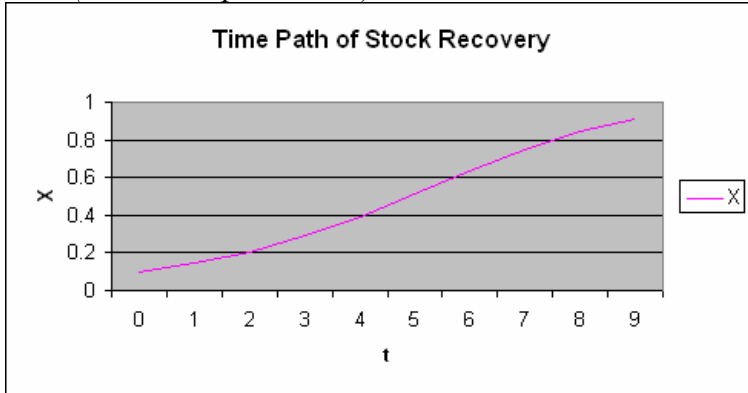
The optimal approach is a harvest moratorium until $t=4$, at which point harvest increases to the steady state value of 1.5487 while the stock increases to its steady state value of 2.95. These values are the same as predicted analytically in homework #1. See Excel spreadsheet for formulas and Solver setup. Note that the formula in cell D18 is the

¹ Note: I won't collect it, but you should feel free to play with different parameter values and use Solver to determine the approach to steady state under different assumptions. Before running solver, think intuitively about what you expect to happen to the dynamics of the solution, and then check your intuition after running solver.

present value of an infinite stream of net benefits from the steady state optimum discounted back to time $t=0$: $\rho^t \frac{\pi(X_{T+1}, F(X_{T+1}))}{\delta}$, where this formula explicitly recognizes the steady state condition that $Y^* = F(X^*)$.

3. E1.1 and E1.2 in Conrad (pages 16 & 17).

E1.1 (see Excel spreadsheet)



E1.2 (see Excel spreadsheet for formulas)

PVNB = 0.8122

4. E2.1 in Conrad (page 31).

See Excel spreadsheet for formulas and Solver setup.

5. E2.2 in Conrad (page 31).

See Excel spreadsheet for formulas. The optimal harvest schedule when $\lambda_{10}=0$ is to drive the fishery to extinction! This happens because the shadow price of the remaining stock at $t=10$ is zero! The optimal harvest schedule when $\lambda_{10}=0.075$ ensures that there will be some stock left at $t=10$ because the shadow price of the remaining stock is positive.