AAE / ECON / FOREST 531 (Natural Resource Economics)
Homework #1
Due on Tuesday, September 23, 2008

1. Suppose an individual's utility is expressed as $U = x^{1/2} y^{1/2}$ and their constraint is represented by the following: $2x + 3y = B = 100$.
    a. What level of x and y should the individual consume to maximize utility? What is utility at the optimal level of consumption?
    b. What is the shadow price of relaxing the constraint – that is, having B=101 rather than B=100?
    c. What level of x and y should the individual consume to maximize utility when the constraint is represented by $2x + 3y = B = 101$? What is utility at the optimal level of consumption?

2. Consider a resource-based economy which can allocate labor (L) to harvest timber (T) or fish (F). Assume the economy faces constant world prices for timber and fish, denoted P_T and P_F, respectively. Labor is constrained by the following equation: $L = T^2 + F^2/4$. Further, suppose P_T=\$500/ton, P_F=\$1000/ton, and L = 1700 available hours.
    a. How should labor be allocated to timber and fish production to maximize the <u>one-period</u> value (V) of resource production? (Note: $V = P_T T + P_F F$).
    b. What is the marginal value (shadow price) of an additional unit of labor?

3. Consider the allocation of a depletable resource over two periods. There are $\bar{Q} = 4$ units of the stock available. The total benefits derived from using the resource are defined as $TB_t = 20q_t - (1/2)q_t^2$, and the total cost of extracting the resource is defined as $TC_t = 5q_t$.
    a. What are the values of q_0 and q_1 that maximize the net benefits of using the resource if the discount rate is 10%? What if the discount rate is 5%?
    b. Now suppose there are $\bar{Q} = 30$ units of the stock available. Do the answers to part (a) change? What is your intuition?
    c. Go back to the original assumption of $\bar{Q} = 4$ and a 10% discount rate. What is the price that will be charged in each period? What is the shadow price of the resource?
    d. Again assuming that $\bar{Q} = 4$ with a 10% discount rate, what is the consumer and producer surplus for each period?

4. Consider the optimal management of a renewable resource where the net benefits of using the resource in time t are defined as $\pi(X_t, Y_t) = 3X_t + 3Y_t$, and the resource has a natural rate of growth described by $F(X_t) = 2X_t - 0.5X_t^2$.
    a. Set up the dynamic Langrangian for this problem and solve for the first-order conditions.
    b. What is the steady state optimum (x*, y*, λ*) if the discount rate (δ) is 0.05? (Hint: use the first-order conditions).
    c. What is the steady state optimum (x*, y*, λ*) if the discount rate (δ) is 0.1?

d. Interpret the difference between (b) and (c).
e. Using the fundamental equation of renewable resources (eq. 1.16), derive an expression for the resource's internal rate of return as a function of x*.