CHAPTER 5
INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY

Answers to Concepts Review and Critical Thinking Questions

1. The four parts are the present value (PV), the future value (FV), the discount rate (*r*), and the life of the investment (*t*).

2. Compounding refers to the growth of a dollar amount through time via reinvestment of interest earned. It is also the process of determining the future value of an investment. Discounting is the process of determining the value today of an amount to be received in the future.

3. Future values grow (assuming a positive rate of return); present values shrink.

4. The future value rises (assuming it's positive); the present value falls.

5. It would appear to be both deceptive and unethical to run such an ad without a disclaimer or explanation.

6. It's a reflection of the time value of money. GMAC gets to use the $500 immediately. If GMAC uses it wisely, it will be worth more than $10,000 in thirty years.

7. Oddly enough, it actually makes it more desirable since GMAC only has the right to pay the full $10,000 before it is due. This is an example of a "call" feature. Such features are discussed at length in a later chapter.

8. The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the $10,000? Thus, our answer does depend on who is making the promise to repay.

9. The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.

10. The price would be higher because, as time passes, the price of the security will tend to rise toward $10,000. This rise is just a reflection of the time value of money. As time passes, the time until receipt of the $10,000 grows shorter, and the present value rises. In 2008, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or GMAC's financial position could deteriorate. Either event would tend to depress the security's price.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The simple interest per year is:

$5,000 × .07 = $350

So after 10 years you will have:

$350 × 10 = $3,500 in interest.

The total balance will be $5,000 + 3,500 = $8,500

With compound interest we use the future value formula:

$FV = PV(1 + r)^t$
$FV = \$5,000(1.07)^{10} = \$9,835.76$

The difference is:

$9,835.76 – 8,500 = $1,335.76

2. To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$

$FV = \$2,250(1.10)^{19}$ = $ 13,760.80
$FV = \$9,310(1.08)^{13}$ = $ 25,319.70
$FV = \$76,355(1.22)^{4}$ = $169,151.87
$FV = \$183,796(1.07)^{8}$ = $315,795.75

3. To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$

$PV = \$15,451 / (1.05)^{6}$ = $11,529.77
$PV = \$51,557 / (1.11)^{9}$ = $20,154.91
$PV = \$886,073 / (1.16)^{23}$ = $29,169.95
$PV = \$550,164 / (1.19)^{18}$ = $24,024.09

4. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for r, we get:

$r = (FV / PV)^{1/t} - 1$

$FV = \$307 = \$265(1 + r)^2$; $r = (\$307 / \$265)^{1/2} - 1 = 7.63\%$
$FV = \$896 = \$360(1 + r)^9$; $r = (\$896 / \$360)^{1/9} - 1 = 10.66\%$
$FV = \$162{,}181 = \$39{,}000(1 + r)^{15}$; $r = (\$162{,}181 / \$39{,}000)^{1/15} - 1 = 9.97\%$
$FV = \$483{,}500 = \$46{,}523(1 + r)^{30}$; $r = (\$483{,}500 / \$46{,}523)^{1/30} - 1 = 8.12\%$

5. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for t, we get:

$t = \ln(FV / PV) / \ln(1 + r)$

$FV = \$1{,}284 = \$625(1.08)^t$; $t = \ln(\$1{,}284/ \$625) / \ln 1.08 = 9.36$ yrs
$FV = \$4{,}341 = \$810(1.07)^t$; $t = \ln(\$4{,}341/ \$810) / \ln 1.07 = 24.81$ yrs
$FV = \$402{,}662 = \$18{,}400(1.21)^t$; $t = \ln(\$402{,}662 / \$18{,}400) / \ln 1.21 = 16.19$ yrs
$FV = \$173{,}439 = \$21{,}500(1.29)^t$; $t = \ln(\$173{,}439 / \$21{,}500) / \ln 1.29 = 8.20$ yrs

6. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for r, we get:

$r = (FV / PV)^{1/t} - 1$
$r = (\$250{,}000 / \$43{,}000)^{1/18} - 1 = 10.27\%$

7. To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for t, we get:

$t = \ln(FV / PV) / \ln(1 + r)$

The length of time to double your money is:

$FV = \$2 = \$1(1.07)^t$
$t = \ln 2 / \ln 1.07 = 10.24$ years

The length of time to quadruple your money is:

$FV = \$4 = \$1(1.07)^t$
$t = \ln 4 / \ln 1.07 = 20.49$ years

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

8. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for r, we get:

$r = (FV / PV)^{1/t} – 1$
$r = (\$28,835 / \$21,608)^{1/5} – 1 = 5.94\%$

9. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for t, we get:

$t = \ln(FV / PV) / \ln(1 + r)$
$t = \ln (\$150,000 / \$40,000) / \ln 1.055 = 24.69$ years

10. To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$
$PV = \$800,000,000 / (1.095)^{20} = \$130,258,959.12$

11. To find the PV of a lump sum, we use:

$$PV = FV / (1 + r)^t$$
$$PV = \$1M / (1.10)^{80} = \$488.19$$

12. To find the FV of a lump sum, we use:

$$FV = PV(1 + r)^t$$
$$FV = \$50(1.05)^{102} = \$7{,}249.01$$

13. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for *r*, we get:

$$r = (FV / PV)^{1/t} - 1$$
$$r = (\$1{,}080{,}000 / \$150)^{1/108} - 1 = 8.57\%$$

To find the FV of the first prize, we use:

$$FV = PV(1 + r)^t$$
$$FV = \$1{,}080{,}000(1.0857)^{37} = \$22{,}642{,}130.85$$

14. To find the PV of a lump sum, we use:

$$PV = FV / (1 + r)^t$$

$$PV = \$350{,}000 / (1.2609)^{65} = \$0.10$$

15. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for *r*, we get:

$$r = (FV / PV)^{1/t} - 1$$
$$r = (\$10{,}311{,}500 / \$12{,}377{,}500)^{1/4} - 1 = -4.46\%$$

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

Intermediate

16. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for r, we get:

$r = (FV / PV)^{1/t} - 1$

a. $FV = \$10{,}000 / (1 + r)^{30} = \$500$
$r = (\$10{,}000 / \$500)^{1/30} - 1 = 10.50\%$

b. $FV = \$6{,}700 / (1 + r)^{22} = \$500$
$r = (\$6{,}700 / \$500)^{1/22} - 1 = 12.52\%$

c. $FV = \$10{,}000 / (1 + r)^{8} = \$6{,}700$
$r = (\$10{,}000 / \$6{,}700)^{1/8} - 1 = 5.13\%$

17. To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$
$PV = \$150{,}000 / (1.11)^{10} = \$52{,}827.67$

18. To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$

$FV = \$2{,}000\ (1.10)^{45} = \$145{,}780.97$

$FV = \$2{,}000\ (1.10)^{35} = \$56{,}204.87$

Better start early!

19. We need to find the FV of a lump sum. However, the money will only be invested for six years, so the number of periods is six.

$FV = PV(1 + r)^t$
$FV = \$30{,}000(1.065)^6 = \$43{,}774.27$

20. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for t, we get:

$t = \ln(FV / PV) / \ln(1 + r)$
$t = \ln(\$120{,}000 / \$10{,}000) / \ln(1.10) = 26.07$

So, the money must be invested for 26.07 years. However, you will not receive the money for another two years. From now, you'll wait:

2 years + 26.07 years = 28.07 years

Calculator Solutions

1.

Enter	10	7%	$5,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$9,835.76

$9,835.76 – 8,500 = $1,335.76

2.

Enter	19	10%	$2,250		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$13,760.80

Enter	13	8%	$9,310		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$25,319.70

Enter	4	22%	$76,355		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$169,151.87

Enter	8	7%	$183,796		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$315,795.75

3.

Enter	6	5%			$15,451
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$11,529.77		

	N	I/Y	PV	PMT	FV
Enter	9	11%			$51,557
Solve for			$20,154.91		

	N	I/Y	PV	PMT	FV
Enter	23	16%			$886,073
Solve for			$29,169.95		

	N	I/Y	PV	PMT	FV
Enter	18	19%			$550,164
Solve for			$24,024.09		

4.

	N	I/Y	PV	PMT	FV
Enter	2		$265		±$307
Solve for		7.63%			

	N	I/Y	PV	PMT	FV
Enter	9		$360		±$896
Solve for		10.66%			

	N	I/Y	PV	PMT	FV
Enter	15		$39,000		±$162,181
Solve for		9.97%			

	N	I/Y	PV	PMT	FV
Enter	30		$46,523		±$483,500
Solve for		8.12%			

5.

	N	I/Y	PV	PMT	FV
Enter		8%	$625		±$1,284
Solve for	9.36				

	N	I/Y	PV	PMT	FV
Enter		7%	$810		±$4,341
Solve for	24.81				

	N	I/Y	PV	PMT	FV
Enter		21%	$18,400		±$402,662
Solve for	16.19				

Enter		29%	$21,500		±$173,439
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	8.20				

6.

Enter	18		$43,000		±$250,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		10.27%			

7.

Enter		7%	$1		±$2
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	10.24				

Enter		7%	$1		±$4
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	20.49				

8.

Enter	5		$21,608		±$28,835
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		5.94%			

9.

Enter		5.5%	$40,000		±$150,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	24.69				

10.

Enter	20	9.5%			$800,000,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$130,258,959.12		

11.

Enter	80	10%			$1,000,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$488.19		

12.

Enter	102	5%	$50		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$7,249.01

13.

Enter	108		±$150		$1,080,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		8.57%			

Enter	37	8.57%	$1,080,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$22,642,130.85

14.

Enter	65	26.09%			$350,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$0.10		

15.

Enter	4		±$12,377,500		$10,311,500
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		–4.46%			

16. *a.*

Enter	30		±$500		$10,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		10.50%			

16. *b.*

Enter	22		±$500		$6,700
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		12.52%			

16. *c.*

Enter	8		±$6,700		$10,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		5.13%			

17.

Enter	10	11%			$150,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$52,827.67		

18.

Enter	45	10%	$2,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$145,780.97

Enter	35	10%	$2,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$56,204.87

19.

	N	I/Y	PV	PMT	FV
Enter	6	6.5%	$30,000		
Solve for					$43,774.27

20.

	N	I/Y	PV	PMT	FV
Enter		10%	±$10,000		$120,000
Solve for	26.07				

From now, you'll wait 2 + 26.07 = 28.07 years

CHAPTER 6
DISCOUNTED CASH FLOW VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. The four pieces are the present value (PV), the periodic cash flow (*C*), the discount rate (*r*), and the number of payments, or the life of the annuity, *t*.

2. Assuming positive cash flows, both the present and the future values will rise.

3. Assuming positive cash flows, the present value will fall and the future value will rise.

4. It's deceptive, but very common. The basic concept of time value of money is that a dollar today is not worth the same as a dollar tomorrow. The deception is particularly irritating given that such lotteries are usually government sponsored!

5. If the total money is fixed, you want as much as possible as soon as possible. The team (or, more accurately, the team owner) wants just the opposite.

6. The better deal is the one with equal installments.

7. Yes, they should. APRs generally don't provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.

8. A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue.

9. The subsidy is the present value (on the day the loan is made) of the interest that would have accrued up until the time it actually begins to accrue.

10. The problem is that the subsidy makes it easier to repay the loan, not obtain it. However, ability to repay the loan depends on future employment, not current need. For example, consider a student who is currently needy, but is preparing for a career in a high-paying area (such as corporate finance!). Should this student receive the subsidy? How about a student who is currently not needy, but is preparing for a relatively low-paying job (such as becoming a college professor)?

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

PV = FV / $(1 + r)^t$

PV@10% = \$1,200 / 1.10 + \$600 / 1.10^2 + \$855 / 1.10^3 + \$1,480 / 1.10^4 = \$3,240.01

PV@18% = \$1,200 / 1.18 + \$600 / 1.18^2 + \$855 / 1.18^3 + \$1,480 / 1.18^4 = \$2,731.61

PV@24% = \$1,200 / 1.24 + \$600 / 1.24^2 + \$855 / 1.24^3 + \$1,480 / 1.24^4 = \$2,432.40

2. To find the PVA, we use the equation:

PVA = $C(\{1 – [1/(1 + r)]^t \} / r)$

At a 5 percent interest rate:

X@5%: PVA = \$4,000{[1 – $(1/1.05)^9$] / .05 } = \$28,431.29

Y@5%: PVA = \$6,000{[1 – $(1/1.05)^5$] / .05 } = \$25,976.86

And at a 22 percent interest rate:

X@22%: PVA = \$4,000{[1 – $(1/1.22)^9$] / .22 } = \$15,145.14

Y@22%: PVA = \$6,000{[1 – $(1/1.22)^5$] / .22 } = \$17,181.84

Notice that the PV of Cash flow X has a greater PV at a 5 percent interest rate, but a lower PV at a 22 percent interest rate. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At the higher interest rate, these bigger cash flows early are more important since the cost of waiting (the interest rate) is so much greater.

3. To solve this problem, we must find the FV of each cash flow and add them. To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$

$FV@8\% = \$800(1.08)^3 + \$900(1.08)^2 + \$1,000(1.08) + \$1,100 = \$4,237.53$

$FV@11\% = \$800(1.11)^3 + \$900(1.11)^2 + \$1,000(1.11) + \$1,100 = \$4,412.99$

$FV@24\% = \$800(1.24)^3 + \$900(1.24)^2 + \$1,000(1.24) + \$1,100 = \$5,249.14$

Notice we are finding the value at Year 4, the cash flow at Year 4 is simply added to the FV of the other cash flows. In other words, we do not need to compound this cash flow.

4. To find the PVA, we use the equation:

$PVA = C(\{1 – [1/(1 + r)]^t \} / r)$

PVA@15 yrs: $PVA = \$3,600\{[1 – (1/1.10)^{15}] / .10\} = \$27,381.89$

PVA@40 yrs: $PVA = \$3,600\{[1 – (1/1.10)^{40}] / .10\} = \$35,204.58$

PVA@75 yrs: $PVA = \$3,600\{[1 – (1/1.10)^{75}] / .10\} = \$35,971.70$

To find the PV of a perpetuity, we use the equation:

$PV = C / r$

$PV = \$3,600 / .10 = \$36,000.00$

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75 year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $28.30.

5. Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

$PVA = C(\{1 – [1/(1 + r)]^t \} / r)$
$PVA = \$28,000 = \$C\{[1 – (1/1.0765)^{14}] / .0765\}$

We can now solve this equation for the annuity payment. Doing so, we get:

$C = \$28,000 / 8.4145 = \$3,327.58$

6. To find the PVA, we use the equation:

$PVA = C(\{1 – [1/(1 + r)]^t \} / r)$
$PVA = \$80,000\{[1 – (1/1.082)^8] / .082\} = \$456,262.25$

7. Here we need to find the FVA. The equation to find the FVA is:

FVA = $C\{[(1 + r)^t - 1] / r\}$

FVA for 20 years = \$2,000[(1.105^{20} – 1) / .105] = \$121,261.62

FVA for 40 years = \$2,000[(1.105^{40} – 1) / .105] = \$1,014,503.16

Notice that doubling the number of periods does not double the FVA.

8. Here we have the FVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the FVA equation:

FVA = $C\{[(1 + r)^t - 1] / r\}$
\$80,000 = \$$C$[($1.058^{10}$ – 1) / .058]

We can now solve this equation for the annuity payment. Doing so, we get:

C = \$80,000 / 13.05765 = \$6,126.68

9. Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

PVA = $C(\{1 - [1/(1 + r)]^t\} / r)$
\$40,000 = $C\{[1 - (1/1.09)^7] / .09\}$

We can now solve this equation for the annuity payment. Doing so, we get:

C = \$40,000 / 5.03295 = \$7,947.62

10. This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

PV = C / r
PV = \$15,000 / .08 = \$187,500.00

11. Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

PV = C / r
\$195,000 = \$15,000 / r

We can now solve for the interest rate as follows:

r = \$15,000 / \$195,000 = 7.69%

12. For discrete compounding, to find the EAR, we use the equation:

$EAR = [1 + (APR / m)]^m – 1$

$EAR = [1 + (.11 / 4)]^4 – 1 \quad = 11.46\%$

$EAR = [1 + (.07 / 12)]^{12} – 1 \quad = 7.23\%$

$EAR = [1 + (.09 / 365)]^{365} – 1 = 9.42\%$

To find the EAR with continuous compounding, we use the equation:

$EAR = e^q – 1$
$EAR = e^{.17} – 1 = 18.53\%$

13. Here we are given the EAR and need to find the APR. Using the equation for discrete compounding:

$EAR = [1 + (APR / m)]^m – 1$

We can now solve for the APR. Doing so, we get:

$APR = m[(1 + EAR)^{1/m} – 1]$

$EAR = .081 = [1 + (APR / 2)]^2 – 1 \qquad APR = 2[(1.081)^{1/2} – 1] \quad = 7.94\%$

$EAR = .076 = [1 + (APR / 12)]^{12} – 1 \qquad APR = 12[(1.076)^{1/12} – 1] \quad = 7.35\%$

$EAR = .168 = [1 + (APR / 52)]^{52} – 1 \qquad APR = 52[(1.168)^{1/52} – 1] \quad = 15.55\%$

Solving the continuous compounding EAR equation:

$EAR = e^q – 1$

We get:

$APR = \ln(1 + EAR)$
$APR = \ln(1 + .262)$
$APR = 23.27\%$

14. For discrete compounding, to find the EAR, we use the equation:

$EAR = [1 + (APR / m)]^m – 1$

So, for each bank, the EAR is:

First National: $EAR = [1 + (.122 / 12)]^{12} – 1 = 12.91\%$

First United: $EAR = [1 + (.124 / 2)]^2 – 1 = 12.78\%$

Notice that the higher APR does not necessarily mean the higher EAR. The number of compounding periods within a year will also affect the EAR.

15. The reported rate is the APR, so we need to convert the EAR to an APR as follows:

$\text{EAR} = [1 + (\text{APR} / m)]^m - 1$

$\text{APR} = m[(1 + \text{EAR})^{1/m} - 1]$
$\text{APR} = 365[(1.17)^{1/365} - 1] = 15.70\%$

This is deceptive because the borrower is actually paying annualized interest of 17% per year, not the 15.70% reported on the loan contract.

16. For this problem, we simply need to find the FV of a lump sum using the equation:

$\text{FV} = \text{PV}(1 + r)^t$

It is important to note that compounding occurs semiannually. To account for this, we will divide the interest rate by two (the number of compounding periods in a year), and multiply the number of periods by two. Doing so, we get:

$\text{FV} = \$800[1 + (.104/2)]^{40} = \$6{,}077.42$

17. For this problem, we simply need to find the FV of a lump sum using the equation:

$\text{FV} = \text{PV}(1 + r)^t$

It is important to note that compounding occurs daily. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365. Doing so, we get:

FV in 5 years $= \$6{,}000[1 + (.071/365)]^{5(365)} = \$8{,}556.79$

FV in 10 years $= \$6{,}000[1 + (.071/365)]^{10(365)} = \$12{,}203.10$

FV in 20 years $= \$6{,}000[1 + (.071/365)]^{20(365)} = \$24{,}819.30$

18. For this problem, we simply need to find the PV of a lump sum using the equation:

$\text{PV} = \text{FV} / (1 + r)^t$

It is important to note that compounding occurs daily. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365. Doing so, we get:

$\text{PV} = \$24{,}000 / [(1 + .11/365)^{6(365)}] = \$12{,}405.67$

19. The APR is simply the interest rate per period times the number of periods in a year. In this case, the interest rate is 30 percent per month, and there are 12 months in a year, so we get:

APR = 12(30%) = 360%

To find the EAR, we use the EAR formula:

$\text{EAR} = [1 + (\text{APR} / m)]^m - 1$

$\text{EAR} = (1 + .30)^{12} - 1 = 2{,}229.81\ \%$

Notice that we didn't need to divide the APR by the number of compounding periods per year. We do this division to get the interest rate per period, but in this problem we are already given the interest rate per period.

20. We first need to find the annuity payment. We have the PVA, the length of the annuity, and the interest rate. Using the PVA equation:

$\text{PVA} = C(\{1 - [1/(1 + r)]^t\ \} / r)$
$\$56{,}850 = \$C[1 - \{1 / [1 + (.082/12)]^{60}\} / (.082/12)]$

Solving for the payment, we get:

$C = \$56{,}850 / 49.086 = \$1{,}158.16$

To find the EAR, we use the EAR equation:

$\text{EAR} = [1 + (\text{APR} / m)]^m - 1$
$\text{EAR} = [1 + (.082 / 12)]^{12} - 1 = 8.52\%$

21. Here we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

$\text{PVA} = C(\{1 - [1/(1 + r)]^t\ \} / r)$
$\$16{,}500 = \$500\{\ [1 - (1/1.009)^t\ ] / .009\}$

Now we solve for t:

$1/1.009^t = 1 - [(\$16{,}500)(.009) / (\$500)]$
$1.009^t = 1/(0.703) = 1.422$
$t = \ln 1.422 / \ln 1.009 = 39.33$ months

22. Here we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

$\text{FV} = \text{PV}(1 + r)$
$\$4 = \$3(1 + r)$
$r = 4/3 - 1 = 33.33\%$ per week

The interest rate is 33.33% per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

APR = (52)33.33% = 1,733.33%

And using the equation to find the EAR:

$EAR = [1 + (APR / m)]^m – 1$
$EAR = [1 + .3333]^{52} – 1 = 313,916,515.69\%$

23. Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

$PV = C / r$
$\$58,000 = \$1,150 / r$

We can now solve for the interest rate as follows:

$r = \$1,150 / \$58,000 = 1.98\%$ per month

The interest rate is 1.98% per month. To find the APR, we multiply this rate by the number of months in a year, so:

APR = (12)1.98% = 23.79%

And using the equation to find an EAR:

$EAR = [1 + (APR / m)]^m – 1$
$EAR = [1 + .0198]^{12} – 1 = 26.57\%$

24. This problem requires us to find the FVA. The equation to find the FVA is:

$FVA = C\{[(1 + r)^t – 1] / r\}$
$FVA = \$150[\{[1 + (.11/12)\ ]^{240} – 1\} / (.11/12)] = \$129,845.71$

25. In the previous problem, the cash flows are monthly and the compounding period is monthly. This assumption still holds. Since the cash flows are annual, we need to use the EAR to calculate the future value of annual cash flows. It is important to remember that you have to make sure the compounding periods of the interest rate times with the cash flows. In this case, we have annual cash flows, so we need the EAR since it is the true annual interest rate you will earn. So, finding the EAR:

$EAR = [1 + (APR / m)]^m – 1$
$EAR = [1 + (.11/12)]^{12} – 1 = 11.57\%$

Using the FVA equation, we get:

$FVA = C\{[(1 + r)^t – 1] / r\}$
$FVA = \$1,800[(1.1157^{20} – 1) / .1157] = \$123,428.72$

26. The cash flows are simply an annuity with four payments per year for four years, or 16 payments. We can use the PVA equation:

$$PVA = C(\{1 - [1/(1 + r)]^t \} / r)$$
$$PVA = \$1,200\{[1 - (1/1.0050)^{16}] / .0050\} = \$18,407.91$$

27. The cash flows are annual and the compounding period is quarterly, so we need to calculate the EAR to make the interest rate comparable with the timing of the cash flows. Using the equation for the EAR, we get:

$$EAR = [1 + (APR / m)]^m - 1$$
$$EAR = [1 + (.13/4)]^4 - 1 = 13.65\%$$

And now we use the EAR to find the PV of each cash flow as a lump sum and add them together:

$$PV = \$900 / 1.1365 + \$750 / 1.1365^2 + \$1,140 / 1.1365^4 = \$2,055.99$$

28. Here the cash flows are annual and the given interest rate is annual, so we can use the interest rate given. We simply find the PV of each cash flow and add them together.

$$PV = \$2,800 / 1.0975 + \$8,100 / 1.0975^3 + \$1,940 / 1.0975^4 = \$10,015.75$$

Intermediate

29. The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest by First Simple Bank paid over 10 years will be:

$$.08(10) = .8$$

First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of $1, or:

$$(1 + r)^{10}$$

Setting the two equal, we get:

$(.08)(10) = (1 + r)^{10} – 1$

$r = 1.8^{1/10} – 1 = 6.05\%$

30. Here we need to convert an EAR into interest rates for different compounding periods. Using the equation for the EAR, we get:

$\text{EAR} = [1 + (\text{APR} / m)]^m – 1$

$\text{EAR} = .16 = (1 + r)^2 – 1; \quad r = (1.16)^{1/2} – 1 \quad = 7.70\%$ per six months

$\text{EAR} = .16 = (1 + r)^4 – 1; \quad r = (1.16)^{1/4} – 1 \quad = 3.78\%$ per quarter

$\text{EAR} = .16 = (1 + r)^{12} – 1; \quad r = (1.16)^{1/12} – 1 \quad = 1.24\%$ per month

Notice that the effective six month rate is not twice the effective quarterly rate because of the effect of compounding.

31. Here we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts. After the first six months, the balance will be:

$\text{FV} = \$4,000\ [1 + (.019/12)]^6 = \$4,038.15$

This is the balance in six months. The FV in another six months will be:

$\text{FV} = \$4,038.15\ [1 + (.16/12)]^6 = \$4,372.16$

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

Interest = $4,372.16 – 4,000.00 = $372.16

32. We need to find the annuity payment in retirement. Our retirement savings ends and the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

Stock account: $\text{FVA} = \$700[\{[1 + (.11/12)\ ]^{360} – 1\} / (.11/12)] = \$1,963,163.82$

Bond account: $\text{FVA} = \$300[\{[1 + (.07/12)\ ]^{360} – 1\} / (.07/12)] = \$365,991.30$

So, the total amount saved at retirement is:

$1,963,163.82 + 365,991.30 = $2,329,155.11

Solving for the withdrawal amount in retirement using the PVA equation gives us:

$\text{PVA} = \$2,329,155.11 = \$C[1 – \{1 / [1 + (.09/12)]^{300}\} / (.09/12)]$
$C = \$2,329,155.11 / 119.1616 = \$19,546.19$ withdrawal per month

33. We need to find the FV of a lump sum in one year and two years. It is important that we use the number of months in compounding since interest is compounded monthly in this case. So:

FV in one year $= \$1(1.0116)^{12} = \$1.15$

FV in two years $= \$1(1.0116)^{24} = \$1.32$

There is also another common alternative solution. We could find the EAR, and use the number of years as our compounding periods. So we will find the EAR first:

$\text{EAR} = (1 + .0116)^{12} - 1 = 14.84\%$

Using the EAR and the number of years to find the FV, we get:

FV in one year $= \$1(1.1484)^{1} = \$1.15$

FV in two years $= \$1(1.1484)^{2} = \$1.32$

Either method is correct and acceptable. We have simply made sure that the interest compounding period is the same as the number of periods we use to calculate the FV.

34. Here we are finding the annuity payment necessary to achieve the same FV. The interest rate given is a 10 percent APR, with monthly deposits. We must make sure to use the number of months in the equation. So, using the FVA equation:

FVA in 40 years $= C[\{[1 + (.10/12)\ ]^{480} - 1\} / (.10/12)]$
$C = \$1{,}000{,}000 / 6{,}324.0796 = \$158.13$

FVA in 30 years $= C[\{[1 + (.10/12)\ ]^{360} - 1\} / (.10/12)]$
$C = \$1{,}000{,}000 / 2{,}260.4879 = \$442.38$

FVA in 20 years $= C[\{[1 + (.10/12)\ ]^{240} - 1\} / (.10/12)]$
$C = \$1{,}000{,}000 / 759.3688 = \$1{,}316.88$

Notice that a deposit for half the length of time, i.e. 20 years versus 40 years, does not mean that the annuity payment is doubled. In this example, by reducing the savings period by one-half, the deposit necessary to achieve the same terminal value is about nine times as large.

35. Since we are looking to triple our money, the PV and FV are irrelevant as long as the FV is three times as large as the PV. The number of periods is four, the number of quarters per year. So:

$\text{FV} = \$3 = \$1(1 + r)^{(12/3)}$
$r = 31.61\%$

36. Since we have an APR compounded monthly and an annual payment, we must first convert the interest rate to an EAR so that the compounding period is the same as the cash flows.

EAR = $[1 + (.10 / 12)]^{12} – 1$ = 10.4713%

PVA_1 = \$80,000 $\{[1 – (1 / 1.104713)^2] / .104713\}$ = \$137,969.76

PVA_2 = \$35,000 + \$60,000$\{[1 – (1/1.104713)^2] / .104713\}$ = \$138,477.32

You would choose the second option since it has a higher PV.

37. The interest rate given is monthly and the cash flows are annual. We need to find the EAR so that the compounding period is the same as the cash flows. The EAR is:

EAR = $(1 + .007)^{12} – 1$ = 8.73%

And the PV of the annuity using the EAR is:

PVA = \$10,000 $[1 – (1/1.0873)^{20} / .0873]$ = \$93,062.18

38. Here we need to find the interest rate for two possible investments. Each investment is a lump sum, so:

G: PV = \$50,000 = \$85,000 / $(1 + r)^5$
$(1 + r)^5$ = \$85,000 / \$50,000
$r = (1.700)^{1/5} – 1$ = 11.20%

H: PV = \$50,000 = \$175,000 / $(1 + r)^{11}$
$(1 + r)^{11}$ = \$175,000 / \$50,000
$r = (3.500)^{1/11} – 1$ = 12.06%

39. The relationship between the PVA and the interest rate is:

PVA falls as *r* increases, and PVA rises as *r* decreases
FVA rises as *r* increases, and FVA falls as *r* decreases

The present values of \$5,000 per year for 10 years at the various interest rates given are:

PVA@10% = \$5,000$\{[1 – (1/1.10)^{10}] / .10\}$ = \$30,722.84

PVA@5% = \$5,000$\{[1 – (1/1.05)^{10}] / .05\}$ = \$38,608.67

PVA@15% = \$5,000$\{[1 – (1/1.15)^{10}] / .15\}$ = \$25,093.84

40. Here we are given the FVA, the interest rate, and the amount of the annuity. We need to solve for the number of payments. Using the FVA equation:

FVA = $20,000 = $125[{[1 + (.10/12)]^t – 1 } / (.10/12)]

Solving for *t*, we get:

1.00833^t = 1 + [($20,000)(.10/12) / 125]
t = ln 2.33333 / ln 1.00833 = 102.10 payments

41. Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

PVA = $45,000 = $950[{1 – [1 / (1 + *r*)]^{60}}/ *r*]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and increasing the interest rate decreases the PVA. Using a spreadsheet, we find:

r = 0.810%

The APR is the periodic interest rate times the number of periods in the year, so:

APR = 12(0.810) = 9.72%

42. The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the $1,000 monthly payments is:

PVA = $1,000[(1 – {1 / [1 + (.068/12)]}^{360}) / (.068/12)] = $153,391.83

The monthly payments of $1,000 will amount to a principal payment of $153,391.83. The amount of principal you will still owe is:

$200,000 – 153,391.83 = $46,608.17

This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal will be:

Balloon payment = $46,608.17 [1 + (.068/12)]^{360} = $356,387.10

43. We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV pf the cash flows we know are:

PV of Year 1 CF: $1,000 / 1.10 = $909.09

PV of Year 3 CF: $2,000 / 1.10^3 = $1,502.63

PV of Year 4 CF: $2,000 / 1.10^4 = $1,366.03

So, the PV of the missing CF is:

$5,979 – 909.09 – 1,502.63 – 1,366.03 = $2,201.25

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

$2,201.25$(1.10)^2$ = $2,663.52

44. To solve this problem, we simply need to find the PV of each lump sum and add them together. It is important to note that the first cash flow of $1 million occurs today, so we do not need to discount that cash flow. The PV of the lottery winnings is:

$1,000,000 + $1,400,000/1.10 + $1,800,000/$1.10^2$ + $2,200,000/$1.10^3$ + $2,600,000/$1.10^4$ + $3,000,000/$1.10^5$ + $3,400,000/$1.10^6$ + $3,800,000/$1.10^7$ + $4,200,000/$1.10^8$ + $4,600,000/$1.10^9$ + $5,000,000/$1.10^{10}$ = $18,758,930.79

45. Here we are finding interest rate for an annuity cash flow. We are given the PVA, number of periods, and the amount of the annuity. We need to solve for the number of payments. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

Amount borrowed = 0.80($1,600,000) = $1,280,000

Using the PVA equation:

PVA = $1,280,000 = $10,000[{1 – [1 / (1 + r)]^{360}}/ r]

Unfortunately this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and increasing the interest rate decreases the PVA. Using a spreadsheet, we find:

r = 0.7228%

The APR is the monthly interest rate times the number of months in the year, so:

APR = 12(0.7228) = 8.67%

And the EAR is:

EAR = $(1 + .007228)^{12} - 1$ = 9.03%

46. The profit the firm earns is just the PV of the sales price minus the cost to produce the asset. We find the PV of the sales price as the PV of a lump sum:

PV = $115,000 / 1.13^3 = $79,700.77

And the firm's profit is:

Profit = $79,700.77 – 72,000.00 = $7,700.77

To find the interest rate at which the firm will break even, we need to find the interest rate using the PV (or FV) of a lump sum. Using the PV equation for a lump sum, we get:

$72,000 = $115,000 / $(1 + r)^3$
r = $(\$115,000 / \$72,000)^{1/3} - 1$ = 16.89%

47. We want to find the value of the cash flows today, so we will find the PV of the annuity, and then bring the lump sum PV back to today. The annuity has 17 payments, so the PV of the annuity is:

PVA = $2,000{[1 – $(1/1.12)^{17}$] / .12} = $14,239.26

Since this is an ordinary annuity equation, this is the PV one period before the first payment, so it is the PV at t = 8. To find the value today, we find the PV of this lump sum. The value today is:

PV = $14,239.26 / 1.12^8 = $5,751.00

48. This question is asking for the present value of an annuity, but the interest rate changes during the life of the annuity. We need to find the present value of the cash flows for the last eight years first. The PV of these cash flows is:

PVA_2 = $1,500 [{1 – 1 / $[1 + (.12/12)]^{96}$} / (.12/12)] = $92,291.55

Note that this is the PV of this annuity exactly seven years from today. Now we can discount this lump sum to today. The value of this cash flow today is:

PV = $92,291.55 / $[1 + (.15/12)]^{84}$ = $32,507.18

Now we need to find the PV of the annuity for the first seven years. The value of these cash flows today is:

PVA_1 = $1,500 [{1 – 1 / $[1 + (.15/12)]^{84}$} / (.15/12)] = $77,733.28

The value of the cash flows today is the sum of these two cash flows, so:

PV = $77,733.28 + 32,507.18 = $110,240.46

49. Here we are trying to find the dollar amount invested today that will equal the FVA with a known interest rate, and payments. First we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

$$FVA = \$1,000\ [\{[\ 1 + (.105/12)]^{180} - 1\}\ /\ (.105/12)] = \$434,029.81$$

Now we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

$$FV = \$434,029.81 = PVe^{.09(15)}$$
$$PV = \$434,029.81\ e^{-1.35} = \$112,518.00$$

50. To find the value of the perpetuity at $t = 7$, we first need to use the PV of a perpetuity equation. Using this equation we find:

$$PV = \$3,000\ /\ .065 = \$46,153.85$$

Remember that the PV of a perpetuity (and annuity) equations give the PV one period before the first payment, so, this is the value of the perpetuity at $t = 14$. To find the value at $t = 7$, we find the PV of this lump sum as:

$$PV = \$46,153.85\ /\ 1.065^7 = \$29,700.29$$

51. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The interest rate for the cash flows of the loan is:

$$PVA = \$20,000 = \$1,900\{(1 - [1\ /\ (1 + r)]^{12}\ )\ /\ r\ \}$$

Again, we cannot solve this equation for r, so we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. Using a spreadsheet, we find:

r = 2.076% per month

So the APR is:

APR = 12(2.076%) = 24.91%

And the EAR is:

$$EAR = (1.0276)^{12} - 1 = 27.96\%$$

52. The cash flows in this problem are semiannual, so we need the effective semiannual rate. The interest rate given is the APR, so the monthly interest rate is:

Monthly rate = .12 / 12 = .01

To get the semiannual interest rate, we can use the EAR equation, but instead of using 12 months as the exponent, we will use 6 months. The effective semiannual rate is:

Semiannual rate = $(1.01)^6 - 1 = 6.15\%$

We can now use this rate to find the PV of the annuity. The PV of the annuity is:

PVA @ $t = 9$: $\$6,000\{[1 - (1 / 1.0615)^{10}] / .0615\} = \$43,844.21$

Note, this is the value one period (six months) before the first payment, so it is the value at $t = 9$. So, the value at the various times the questions asked for uses this value 9 years from now.

PV @ $t = 5$: $\$43,844.21 / 1.0615^8 = \$27,194.83$

Note, you can also calculate this present value (as well as the remaining present values) using the number of years. To do this, you need the EAR. The EAR is:

EAR = $(1 + .01)^{12} - 1 = 12.68\%$

So, we can find the PV at $t = 5$ using the following method as well:

PV @ $t = 5$: $\$43,844.21 / 1.1268^4 = \$27,194.83$

The value of the annuity at the other times in the problem is:

PV @ $t = 3$: $\$43,844.21 / 1.0615^{12} = \$21,417.72$
PV @ $t = 3$: $\$43,844.21 / 1.1268^6 = \$21,417.72$

PV @ $t = 0$: $\$43,844.21 / 1.0615^{18} = \$14,969.38$
PV @ $t = 0$: $\$43,844.21 / 1.1268^9 = \$14,969.38$

53. *a.* Calculating the PV of an ordinary annuity, we get:

PVA = $\$525\{[1 - (1/1.095)^6] / .095\} = \$2,320.41$

b. To calculate the PVA due, we calculate the PV of an ordinary annuity for $t - 1$ payments, and add the payment that occurs today. So, the PV of the annuity due is:

PVA = $\$525 + \$525\{[1 - (1/1.095)^5] / .095\} = \$2,540.85$

54. We need to use the PVA due equation, that is:

$PVA_{due} = (1 + r)\ PVA$

Using this equation:

$PVA_{due} = \$56{,}000 = [1 + (.0815/12)] \times C[\{1 - 1 / [1 + (.0815/12)]^{48}\} / (.0815/12)$

$\$55{,}622.23 = \$C\{1 - [1 / (1 + .0815/12)^{48}]\} / (.0815/12)$

$C = \$1{,}361.82$

Notice, when we find the payment for the PVA due, we simply discount the PV of the annuity due back one period. We then use this value as the PV of an ordinary annuity.

55. The payment for a loan repaid with equal payments is the annuity payment with the loan value as the PV of the annuity. So, the loan payment will be:

$PVA = \$30{,}000 = C\ \{[1 - 1 / (1 + .10)^5] / .10\}$

$C = \$7{,}913.92$

The interest payment is the beginning balance times the interest rate for the period, and the principal payment is the total payment minus the interest payment. The ending balance is the beginning balance minus the principal payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal payment is:

Year	Beginning Balance	Total Payment	Interest Payment	Principal Payment	Ending Balance
1	$30,000.00	$7,913.92	$3,000.00	$4,913.92	$25,086.08
2	25,086.08	7,913.92	2,508.61	5,405.32	19,680.76
3	19,680.76	7,913.92	1,968.08	5,945.85	13,734.91
4	13,734.91	7,913.92	1,373.49	6,540.43	7,194.48
5	7,194.48	7,913.92	719.45	7,194.48	0.00

In the third year, $1,968.08 of interest is paid.

Total interest over life of the loan = $3,000 + 2,508.61 + 1,968.08 + 1,373.49 + 719.45 = $9,569.62

56. This amortization table calls for equal principal payments of $6,000 per year. The interest payment is the beginning balance times the interest rate for the period, and the total payment is the principal payment plus the interest payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal principal reduction is:

Year	Beginning Balance	Total Payment	Interest Payment	Principal Payment	Ending Balance
1	$30,000.00	$9,000.00	$3,000.00	$6,000.00	$24,000.00
2	24,000.00	8,400.00	2,400.00	6,000.00	18,000.00
3	18,000.00	7,800.00	1,800.00	6,000.00	12,000.00
4	12,000.00	7,200.00	1,200.00	6,000.00	6,000.00
5	6,000.00	6,600.00	600.00	6,000.00	0.00

In the third year, $1,800 of interest is paid.

Total interest over life of the loan = $3,000 + 2,400 + 1,800 + 1,200 + 600 = $9,000.00

Notice that the total payments for the equal principal reduction loan are lower. This is because more principal is repaid early in the loan, which reduces the total interest expense over the life of the loan.

Challenge

57. The cash flows for this problem occur monthly, and the interest rate given is the EAR. Since the cash flows occur monthly, we must get the effective monthly rate. One way to do this is to find the APR based on monthly compounding, and then divide by 12. So, the pre-retirement APR is:

$\text{EAR} = .1011 = [1 + (\text{APR} / 12)]^{12} – 1; \qquad \text{APR} = 12[(1.11)^{1/12} – 1] = 10.48\%$

And the post-retirement APR is:

$\text{EAR} = .08 = [1 + (\text{APR} / 12)]^{12} – 1; \qquad \text{APR} = 12[(1.08)^{1/12} – 1]= 7.72\%$

First, we will calculate how much he needs at retirement. The amount needed at retirement is the PV of the monthly spending plus the PV of the inheritance. The PV of these two cash flows is:

$\text{PVA} = \$25,000\{1 – [1 / (1 + .0772/12)^{12(20)}]\} / (.0772/12) = \$3,051,943.26$

$\text{PV} = \$750,000 / [1 + (.0772/12)]^{240} = \$160,911.16$

So, at retirement, he needs:

$3,051,943.26 + 160,911.16 = $3,212,854.42

He will be saving $2,100 per month for the next 10 years until he purchases the cabin. The value of his savings after 10 years will be:

$\text{FVA} = \$2,100[\{[1 + (.1048/12)]^{12(10)} – 1\} / (.1048/12)] = \$442,239.69$

After he purchases the cabin, the amount he will have left is:

$442,239.69 – 350,000 = $92,239.69

He still has 20 years until retirement. When he is ready to retire, this amount will have grown to:

$FV = \$92,239.69[1 + (.1048/12)]^{12(20)} = \$743,665.12$

So, when he is ready to retire, based on his current savings, he will be short:

$3,212,854.41 – 743,665.12 = $2,469,189.29

This amount is the FV of the monthly savings he must make between years 10 and 30. So, finding the annuity payment using the FVA equation, we find his monthly savings will need to be:

$FVA = \$2,469,189.29 = C[\{[1 + (.1048/12)]^{12(20)} – 1\} / (.1048/12)]$
$C = \$3,053.87$

58. To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing is simply the PV of the lease payments, plus the $1. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:

$PV = \$1 + \$450\{1 – [1 / (1 + .08/12)^{12(3)}]\} / (.08/12) = \$14,361.31$

The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:

$PV = \$23,000 / [1 + (.08/12)]^{12(3)} = \$18,106.86$

The PV of the decision to purchase is:

$35,000 – $18,106.86 = $16,893.14

In this case, it is cheaper to lease the car than buy it since the PV of the leasing cash flows is lower. To find the breakeven resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

$35,000 – PV of resale price = $14,361.31
PV of resale price = $20,638.69

The resale price that would make the PV of the lease versus buy decision is the FV of this value, so:

Breakeven resale price $= \$20,638.69[1 + (.08/12)]^{12(3)} = \$26,216.03$

59. To find the quarterly salary for the player, we first need to find the PV of the current contract. The cash flows for the contract are annual, and we are given a daily interest rate. We need to find the EAR so the interest compounding is the same as the timing of the cash flows. The EAR is:

$$\text{EAR} = [1 + (.045/365)]^{365} - 1 = 4.60\%$$

The PV of the current contract offer is the sum of the PV of the cash flows. So, the PV is:

$$\text{PV} = \$8{,}000{,}000 + \$4{,}000{,}000/1.046 + \$4{,}800{,}000/1.046^2 + \$5{,}700{,}000/1.046^3 + \$6{,}400{,}000/1.046^4 + \$7{,}000{,}000/1.046^5 + \$7{,}500{,}000/1.046^6$$
$$\text{PV} = \$37{,}852{,}037.91$$

The player wants the contract increased in value by $750,000, so the PV of the new contract will be:

$$\text{PV} = \$37{,}852{,}037.91 + 750{,}000 = \$38{,}602{,}037.91$$

The player has also requested a signing bonus payable today in the amount of $9 million. We can simply subtract this amount from the PV of the new contract. The remaining amount will be the PV of the future quarterly paychecks.

$$\$38{,}602{,}037.91 - 9{,}000{,}000 = \$29{,}602{,}037.91$$

To find the quarterly payments, first realize that the interest rate we need is the effective quarterly rate. Using the daily interest rate, we can find the quarterly interest rate using the EAR equation, with the number of days being 91.25, the number of days in a quarter (365 / 4). The effective quarterly rate is:

$$\text{Effective quarterly rate} = [1 + (.045/365)]^{91.25} - 1 = 1.131\%$$

Now we have the interest rate, the length of the annuity, and the PV. Using the PVA equation and solving for the payment, we get:

$$\text{PVA} = \$29{,}602{,}037.91 = C\{[1 - (1/1.01131)^{24}] / .01131\}$$
$$C = \$1{,}415{,}348.37$$

60. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the $20,000 you must repay in one year, and the $17,600 you borrow today. The interest rate of the loan is:

$$\$20{,}000 = \$17{,}600(1 + r)$$
$$r = (\$20{,}000 - 17{,}600) - 1 = 13.64\%$$

Because of the discount, you only get the use of $17,600, and the interest you pay on that amount is 13.64%, not 12%.

61. Here we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:

$$APR = 12[(1.09)^{1/12} - 1] = 8.65\%$$

To find the value today of the back pay from two years ago, we will find the FV of the annuity, and then find the FV of the lump sum. Doing so gives us:

$$FVA = (\$40{,}000/12)\ [\{[\ 1 + (.0865/12)]^{12} - 1\}\ /\ (.0865/12)] = \$41{,}624.33$$
$$FV = \$41{,}624.33(1.09) = \$45{,}370.52$$

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year's back pay:

$$FVA = (\$43{,}000/12)\ [\{[\ 1 + (.0865/12)]^{12} - 1\}\ /\ (.0865/12)] = \$44{,}746.15$$

Next, we find the value today of the five year's future salary:

$$PVA = (\$45{,}000/12)\{[\{1 - \{1\ /\ [1 + (.0865/12)]^{12(5)}\}]\ /\ (.0865/12)\} = \$182{,}142.14$$

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

Award = \$45,370.52 + 44,746.15 + 182,142.14 + 100,000 + 20,000 = \$392,258.81

As the plaintiff, you would prefer a lower interest rate. In this problem, we are calculating both the PV and FV of annuities. A lower interest rate will decrease the FVA, but increase the PVA. So, by a lower interest rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time, this is the more important cash flow to the plaintiff.

62. Again, to find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

Loan repayment amount = \$10,000(1.10) = \$11,000

The amount you will receive today is the principal amount of the loan times one minus the points.

Amount received = \$10,000(1 – .03) = \$9,700

Now, we simply find the interest rate for this PV and FV.

$\$11,000 = \$9,700(1 + r)$
$r = (\$11,000 / \$9,700) – 1 = 13.40\%$

63. This is the same question as before, with different values. So:

Loan repayment amount = \$10,000(1.13) = \$11,300

Amount received = \$10,000(1 – .02) = \$9,800

$\$11,300 = \$9,800(1 + r)$
$r = (\$11,300 / \$9,800) – 1 = 15.31\%$

The effective rate is not affected by the loan amount, since it drops out when solving for *r*.

64. First we will find the APR and EAR for the loan with the refundable fee. Remember, we need to use the actual cash flows of the loan to find the interest rate. With the \$1,500 application fee, you will need to borrow \$201,500 to have \$200,000 after deducting the fee. Solving for the payment under these circumstances, we get:

$\text{PVA} = \$201,500 = C\,\{[1 – 1/(1.00625)^{360}]/.00625\}$ where .00625 = .075/12
$C = \$1,408.92$

We can now use this amount in the PVA equation with the original amount we wished to borrow, \$200,000. Solving for *r*, we find:

$\text{PVA} = \$200,000 = \$1,408.92[\{1 – [1 / (1 + r)]^{360}\}/ r]$

Solving for *r* with a spreadsheet, on a financial calculator, or by trial and error, gives:

$r = 0.6314\%$ per month

$\text{APR} = 12(0.6314\%) = 7.58\%$

$\text{EAR} = (1 + .006314)^{12} – 1 = 7.85\%$

With the nonrefundable fee, the APR of the loan is simply the quoted APR since the fee is not considered part of the loan. So:

APR = 7.50%

EAR = $[1 + (.075/12)]^{12} – 1$ = 7.76%

65. Be careful of interest rate quotations. The actual interest rate of a loan is determined by the cash flows. Here, we are told that the PV of the loan is $1,000, and the payments are $42.25 per month for three years, so the interest rate on the loan is:

PVA = \$1,000 = \$42.25$[\{1 – [1 / (1 + r)]^{36} \} / r]$

Solving for *r* with a spreadsheet, on a financial calculator, or by trial and error, gives:

r = 2.47% per month

APR = 12(2.47%) = 29.63%

EAR = $(1 + .0247)^{12} – 1$ = 34.00%

It's called add-on interest because the interest amount of the loan is added to the principal amount of the loan before the loan payments are calculated.

66. Here we are solving a two-step time value of money problem. Each question asks for a different possible cash flow to fund the same retirement plan. Each savings possibility has the same FV, that is, the PV of the retirement spending when your friend is ready to retire. The amount needed when your friend is ready to retire is:

PVA = \$90,000$\{[1 – (1/1.08)^{15}] / .08\}$ = \$770,353.08

This amount is the same for all three parts of this question.

a. If your friend makes equal annual deposits into the account, this is an annuity with the FVA equal to the amount needed in retirement. The required savings each year will be:

FVA = \$770,353.08 = $C[(1.08^{30} – 1) / .08]$
C = \$6,800.24

b. Here we need to find a lump sum savings amount. Using the FV for a lump sum equation, we get:

FV = \$770,353.08 = PV$(1.08)^{30}$
PV = \$76,555.63

c. In this problem, we have a lump sum savings in addition to an annual deposit. Since we already know the value needed at retirement, we can subtract the value of the lump sum savings at retirement to find out how much your friend is short. Doing so gives us:

FV of trust fund deposit = $\$25,000(1.08)^{10} = \$53,973.12$

So, the amount your friend still needs at retirement is:

FV = $770,353.08 – 53,973.12 = $716,379.96

Using the FVA equation, and solving for the payment, we get:

$\$716,379.96 = C[(1.08^{30} – 1) / .08]$

$C = \$6,323.80$

This is the total annual contribution, but your friend’s employer will contribute $1,500 per year, so your friend must contribute:

Friend's contribution = $6,323.80 – 1,500 = $4,823.80

67. We will calculate the number of periods necessary to repay the balance with no fee first. We simply need to use the PVA equation and solve for the number of payments.

Without fee and annual rate = 19.20%:

PVA = $\$10,000 = \$200\{[1 – (1/1.016)^t] / .016 \}$ where .016 = .192/12

Solving for t, we get:

$t = \ln\{1 / [1 – (\$10,000/\$200)(.016)]\} / \ln(1.016)$
$t = \ln 5 / \ln 1.016$
$t = 101.39$ months

Without fee and annual rate = 9.20%:

PVA = $\$10,000 = \$200\{[1 – (1/1.0076667)^t] / .0076667 \}$ where .0076667 = .092/12

Solving for t, we get:

$t = \ln\{1 / [1 – (\$10,000/\$200)(.0076667)]\} / \ln(1.0076667)$
$t = \ln 1.6216 / \ln 1.0076667$
$t = 63.30$ months

Note that we do not need to calculate the time necessary to repay your current credit card with a fee since no fee will be incurred. The time to repay the new card with a transfer fee is:

With fee and annual rate = 9.20%:

PVA = $10,200 = $200{ [1 – (1/1.0076667)^t] / .0076667 } where .0076667 = .092/12

Solving for t, we get:

t = ln{1 / [1 – ($10,200/$200)(.0076667)]} / ln(1.0076667)
t = ln 1.6420 / ln 1.0076667
t = 64.94 months

68. We need to find the FV of the premiums to compare with the cash payment promised at age 65. We have to find the value of the premiums at year 6 first since the interest rate changes at that time. So:

FV_1 = $750(1.11)^5 = $1,263.79

FV_2 = $750(1.11)^4 = $1,138.55

FV_3 = $850(1.11)^3 = $1,162.49

FV_4 = $850(1.11)^2 = $1,047.29

FV_5 = $950(1.11)^1 = $1,054.50

Value at year six = $1,263.79 + 1,138.55 + 1,162.49 + 1,047.29 + 1,054.50 + 950.00 = $6,616.62

Finding the FV of this lump sum at the child's 65th birthday:

FV = $6,616.62(1.07)^{59} = $358,326.50

The policy is not worth buying; the future value of the policy is $358,326.50, but the policy contract will pay off $250,000. The premiums are worth $108,326.50 more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:

PV = $750/1.11 + $750/1.11^2 + $850/1.11^3 + $850/1.11^4 + $950/1.11^5 + $950/1.11^6 = $3,537.51

And the value today of the $250,000 at age 65 is:

PV = $250,000/1.07^{59} = $4,616.33

PV = $4,616.33/1.11^6 = $2,468.08

The premiums still have the higher cash flow. At time zero, the difference is $2,148.25. Whenever you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest $2,148.25, the difference in the cash flows at time zero, for six years at an 11 percent interest rate, and then for 59 years at a seven percent interest rate. How much will it be worth? Without doing calculations, you know it will be worth $108,326.50, the difference in the cash flows at time 65!

69. The monthly payments with a balloon payment loan are calculated assuming a longer amortization schedule, in this case, 30 years. The payments based on a 30-year repayment schedule would be:

$PVA = \$250{,}000 = C(\{1 - [1 / (1.085/12)]^{360}\} / (.085/12))$
$C = \$1{,}922.28$

Now, at time = 8, we need to find the PV of the payments which have not been made. The balloon payment will be:

$PVA = \$1{,}922.28\ (\{1 - [1 / (1.085/12)]^{12(22)}\} / (.085/12))$
$PVA = \$229{,}278.34$

70. Here we need to find the interest rate that makes the PVA, the college costs, equal to the FVA, the savings. The PV of the college costs are:

$PVA = \$20{,}000[\{1 - [1 / (1 + r)]^4\} / r]$

And the FV of the savings is:

$FVA = \$8{,}000\{[(1 + r)^6 - 1] / r\}$

Setting these two equations equal to each other, we get:

$\$20{,}000[\{1 - [1 / (1 + r)]^4\} / r] = \$8{,}000\{[(1 + r)^6 - 1] / r\}$

Reducing the equation gives us:

$(1 + r)^{10} - 4.00(1 + r)^4 + 40.00 = 0$

Now we need to find the roots of this equation. We can solve using trial and error, a root-solving calculator routine, or a spreadsheet. Using a spreadsheet, we find:

$r = 10.57\%$

71. Here we need to find the interest rate that makes us indifferent between an annuity and a perpetuity. To solve this problem, we need to find the PV of the two options and set them equal to each other. The PV of the perpetuity is:

$PV = \$10{,}000 / r$

And the PV of the annuity is:

$PVA = \$22{,}000[\{1 - [1 / (1 + r)]^{10}\} / r]$

Setting them equal and solving for r, we get:

$\$10{,}000 / r = \$22{,}000[\{1 - [1 / (1 + r)]^{10}\} / r]$
$\$10{,}000 / \$22{,}000 = 1 - [1 / (1 + r)]^{10}$
$.5455^{1/10} = 1 / (1 + r)$
$r = 6.25\%$

72. The cash flows in this problem occur every two years, so we need to find the effective two year rate. One way to find the effective two year rate is to use an equation similar to the EAR, except use the number of days in two years as the exponent. (We use the number of days in two years since it is daily compounding; if monthly compounding was assumed, we would use the number of months in two years.) So, the effective two-year interest rate is:

Effective 2-year rate = $[1 + (.13/365)]^{365(2)} - 1 = 29.69\%$

We can use this interest rate to find the PV of the perpetuity. Doing so, we find:

PV = \$6,700 /.2969 = \$22,568.80

This is an important point: Remember that the PV equation for a perpetuity (and an ordinary annuity) tells you the PV one period before the first cash flow. In this problem, since the cash flows are two years apart, we have found the value of the perpetuity one period (two years) before the first payment, which is one year ago. We need to compound this value for one year to find the value today. The value of the cash flows today is:

PV = $\$22,568.80(1 + .13/365)^{365} = \$25,701.39$

The second part of the question assumes the perpetuity cash flows begin in four years. In this case, when we use the PV of a perpetuity equation, we find the value of the perpetuity two years from today. So, the value of these cash flows today is:

PV = $\$22,568.80 / (1 + .13/365)^{2(365)} = \$17,402.51$

73. To solve for the PVA due:

$$\text{PVA} = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + ... + \frac{C}{(1+r)^t}$$

$$\text{PVA}_{\text{due}} = C + \frac{C}{(1+r)} + ... + \frac{C}{(1+r)^{t-1}}$$

$$\text{PVA}_{\text{due}} = (1+r)\left(\frac{C}{(1+r)} + \frac{C}{(1+r)^2} + ... + \frac{C}{(1+r)^t}\right)$$

$$\text{PVA}_{\text{due}} = (1 + r)\ \text{PVA}$$

And the FVA due is:

$$\text{FVA} = C + C(1 + r) + C(1 + r)^2 + \ldots + C(1 + r)^{t-1}$$
$$\text{FVA}_{\text{due}} = C(1 + r) + C(1 + r)^2 + \ldots + C(1 + r)^t$$
$$\text{FVA}_{\text{due}} = (1 + r)[C + C(1 + r) + \ldots + C(1 + r)^{t-1}]$$
$$\text{FVA}_{\text{due}} = (1 + r)\text{FVA}$$

74. We are given the value today of an annuity with the first payment occurring seven years from today. We need to find the value of the lump sum six years from now (one period before the first payment). This will be the PVA. So, the value of the lump sum in six years is:

FV = \$75,000(1.10)^6 = \$132,867.08

Now we can use the PVA equation and solve for the annuity payment:

PVA = \$132,867.08 = C\{[1 – (1/1.10)^{10}] / .10\}

C = \$21,623.50

75. *a.* The APR is the interest rate per week times 52 weeks in a year, so:

APR = 52(10%) = 520%

EAR = (1 + .10)^{52} – 1 = 14,104.29%

b. In a discount loan, the amount you receive is lowered by the discount, and you repay the full principal. With a 10 percent discount, you would receive \$9 for every \$10 in principal, so the weekly interest rate would be:

\$10 = \$9(1 + r)
r = (\$10 / \$9) – 1 = 11.11%

Note the dollar amount we use is irrelevant. In other words, we could use \$0.90 and \$1, \$90 and \$100, or any other combination and we would get the same interest rate. Now we can find the APR and the EAR:

APR = 52(11.11%) = 577.78%

EAR = (1 + .1111)^{52} – 1 = 23,854.63%

c. Using the cash flows from the loan, we have the PVA and the annuity payments and need to find the interest rate, so:

PVA = \$58.84 = \$25[\{1 – [1 / (1 + r)]^4\}/ r]

Using a spreadsheet, trial and error, or a financial calculator, we find:

r = 25.18% per week

APR = 52(25.18%) = 1,309.92%

EAR = 1.2518^{52} – 1 = 11,851,501.94%

Calculator Solutions

1.

CFo	$0	**CFo**	$0	**CFo**	$0
C01	$1,200	**C01**	$1,200	**C01**	$1,200
F01	1	**F01**	1	**F01**	1
C02	$600	**C02**	$600	**C02**	$600
F02	1	**F02**	1	**F02**	1
C03	$855	**C03**	$855	**C03**	$855
F03	1	**F03**	1	**F03**	1
C04	$1,480	**C04**	$1,480	**C04**	$1,480
F04	1	**F04**	1	**F04**	1
I = 10		I = 18		I = 24	
NPV CPT		NPV CPT		NPV CPT	
$3,240.01		$2,731.61		$2,432.40	

2.

Enter	9	5%		$4,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$28,431.29		

Enter	5	5%		$6,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$25,976.86		

Enter	9	22%		$4,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$15,145.14		

Enter	5	22%		$6,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$17,181.84		

3.

Enter	3	8%	$800		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,007.77

Enter	2	8%	$900		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,049.76

Enter	1	8%	$1,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,080.00

FV = $1,007.77 + 1,049.76 + 1,080.00 + 1,100.00 = $4,237.53

Enter	3	11%	$800		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,094.10

Enter	2	11%	$900		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,108.89

Enter	1	11%	$1,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,110.00

FV = $1,094.10 + 1,108.89+ 1,110 + 1,100.00 = $4,412.99

Enter	3	24%	$800		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,525.30

Enter	2	24%	$900		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,383.84

Enter	1	24%	$1,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,240.00

FV = $1,525.30 + 1,383.84 + 1,240.00 + 1,100.00 = $5,249.14

4.

Enter	15	10%		$3,600	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$27,381.89		

Enter	40	10%		$3,600	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$35,204.58		

Enter	75	10%		$3,600	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$35,971.70		

5.

Enter	14	7.65%	$28,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$3,327.58	

6.

Enter	8	8.2%		$80,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$456,262.25		

7.

Enter	20	10.5%		$2,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$121,261.62

Enter	40	10.5%		$2,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,014,503.16

8.

Enter	10	5.8%			$80,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$6,126.68	

9.

Enter	7	9%	$40,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$7,947.62	

12.

Enter	11%		4
	NOM	**EFF**	**C/Y**
Solve for		11.46%	

Enter	7%		12
	NOM	**EFF**	**C/Y**
Solve for		7.23%	

Enter	9%		365
	NOM	**EFF**	**C/Y**
Solve for		9.42%	

13.

Enter		8.1%	2
	NOM	**EFF**	**C/Y**
Solve for	7.94%		

Enter		7.6%	12
	NOM	**EFF**	**C/Y**
Solve for	7.35%		

Enter		16.8%	52
	NOM	**EFF**	**C/Y**
Solve for	15.55%		

14.

Enter	12.2%		12
	NOM	**EFF**	**C/Y**
Solve for		12.91%	

Enter	12.4%		2
	NOM	**EFF**	**C/Y**
Solve for		12.78%	

15.

Enter		17%	365
	NOM	**EFF**	**C/Y**
Solve for	15.70%		

16.

Enter	40	5.2%	$800		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$6,077.42

17.

Enter	5 × 365	7.1% / 365	$6,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$8,556.79

Enter	10 × 365	7.1% / 365	$6,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$12,203.10

Enter	20 × 365	7.1% / 365	$6,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$24,819.30

18.

Enter	6 × 365	11% / 365			$24,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$12,405.67		

19.

Enter	360%		12
	NOM	**EFF**	**C/Y**
Solve for		2,229.81%	

20.

Enter	60	8.2% / 12	$56,850		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$1,158.16	

Enter	8.2%		12
	NOM	**EFF**	**C/Y**
Solve for		8.52%	

21.

Enter		0.9%	$16,500	±$500	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	39.33				

22.

Enter	1,733.33%		52
	NOM	**EFF**	**C/Y**
Solve for		313,916,515.69%	

23.

Enter	23.79%		12
	NOM	**EFF**	**C/Y**
Solve for		26.57%	

24.

Enter	20 × 12	11% / 12		$150	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$129,845.71

25.

Enter	11.00%		12		
	NOM	**EFF**	**C/Y**		
Solve for		11.57%			

Enter	20	11.57%		$1,800	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$123,428.72

26.

Enter	4 × 4	0.50%		$1,200	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$18,407.91		

27.

Enter	13.00%		4
	NOM	**EFF**	**C/Y**
Solve for		13.65%	

CFo	$0
C01	$900
F01	1
C02	$750
F02	1
C03	$0
F03	1
C04	$1,140
F04	1

I = 13.65%
NPV CPT
$2,055.99

28.

CFo	$0
C01	$2,800
F01	1
C02	$0
F02	1
C03	$8,100
F03	1
C04	$1,940
F04	1

I = 9.75%
NPV CPT
$10,015.75

30.

Enter		16%	2
	NOM	**EFF**	**C/Y**
Solve for	15.41%		

15.41% / 2 = 7.70%

Enter		16%	4
	NOM	**EFF**	**C/Y**
Solve for	15.12%		

15.12% / 4 = 3.78%

Enter		16%	12
	NOM	**EFF**	**C/Y**
Solve for	14.93%		

14.93% / 12 = 1.24%

31.

Enter	6	1.90% / 12	$4,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$4,038.15

Enter	6	16% / 12	$4,038.15		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$4,372.16

$4,372.16 – 4,000 = $372.16

32. Stock account:

Enter	360	11% / 12		$700	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,963,163.82

Bond account:

Enter	360	7% / 12		$300	
	N	I/Y	PV	PMT	FV
Solve for					$365,991.30

Savings at retirement = $1,963,163.82 + 365,991.30 = $2,329,155.11

Enter	300	9% / 12	$2,329,155.11		
	N	I/Y	PV	PMT	FV
Solve for				$19,546.19	

33.

Enter	12	1.16%	$1		
	N	I/Y	PV	PMT	FV
Solve for					$1.15

Enter	24	1.16%	$1		
	N	I/Y	PV	PMT	FV
Solve for					$1.32

34.

Enter	480	10% / 12			$1,000,000
	N	I/Y	PV	PMT	FV
Solve for				$158.13	

Enter	360	10% / 12			$1,000,000
	N	I/Y	PV	PMT	FV
Solve for				$442.38	

Enter	240	10% / 12			$1,000,000
	N	I/Y	PV	PMT	FV
Solve for				$1,316.88	

35.

Enter	12 / 3		±$1		$3
	N	I/Y	PV	PMT	FV
Solve for		31.61%			

36.

Enter	10.00%		12
	NOM	EFF	C/Y
Solve for		10.47%	

	N	I/Y	PV	PMT	FV
Enter	2	10.47%		$80,000	
Solve for			$137,969.76		

CFo	$35,000
C01	$60,000
F01	2

I = 10.47%
NPV CPT
$138,477.32

37.

	NOM	EFF	C/Y
Enter	.007% × 12		12
Solve for		8.73%	

	N	I/Y	PV	PMT	FV
Enter	20	8.73%		$10,000	
Solve for			$93,062.18		

38.

	N	I/Y	PV	PMT	FV
Enter	5		±$50,000		$85,000
Solve for		11.20%			

	N	I/Y	PV	PMT	FV
Enter	11		±50,000		$175,000
Solve for		12.06%			

39.

	N	I/Y	PV	PMT	FV
Enter	10	10%		$5,000	
Solve for			$30,722.84		

	N	I/Y	PV	PMT	FV
Enter	10	5%		$5,000	
Solve for			$38,608.67		

	N	I/Y	PV	PMT	FV
Enter	10	15%		$5,000	
Solve for			$25,093.84		

40.

	N	I/Y	PV	PMT	FV
Enter		10% / 12		±$125	$20,000
Solve for	102.10				

41.

	N	I/Y	PV	PMT	FV
Enter	60		$45,000	±$950	
Solve for		0.810%			

0.810% × 12 = 9.72%

42.

	N	I/Y	PV	PMT	FV
Enter	360	6.8% / 12		$1,000	
Solve for			$153,391.83		

$200,000 – 153,391.83 = $46,608.17

	N	I/Y	PV	PMT	FV
Enter	360	6.8% / 12	$46,608.17		
Solve for					$356,387.10

43.

CFo	$0
C01	$1,000
F01	1
C02	$0
F02	1
C03	$2,000
F03	1
C04	$2,000
F04	1

I = 10%
NPV CPT
$3,777.75

PV of missing CF = $5,979 – 3,777.75 = $2,201.25
Value of missing CF:

	N	I/Y	PV	PMT	FV
Enter	2	10%	$2,201.25		
Solve for					$2,663.52

44.

CFo	$1,000,000
C01	$1,400,000
F01	1
C02	$1,800,000
F02	1
C03	$2,200,000
F03	1
C04	$2,600,000
F04	1
C05	$3,000,000
F05	1
C06	$3,400,000
F06	1
C07	$3,800,000
F07	1
C08	$4,200,000
F08	1
C09	$4,600,000
F09	1
C010	$5,000,000

I = 10%
NPV CPT
$18,758,930.79

45.

Enter	360		.80($1,600,000)	±$10,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		0.7228%			

APR = 0.7228% × 12 = 8.67%

Enter	8.67%		12
	NOM	**EFF**	**C/Y**
Solve for		9.03%	

46.

Enter	3	13%			$115,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$79,700.77		

Profit = $79,700.77 – 72,000 = $7,700.77

Enter	3		±$72,000		$115,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		16.89%			

47.

	N	I/Y	PV	PMT	FV
Enter	17	12%		$2,000	
Solve for			$14,239.26		

	N	I/Y	PV	PMT	FV
Enter	8	12%			$14,239.26
Solve for			$5,751.00		

48.

	N	I/Y	PV	PMT	FV
Enter	84	15% / 12		$1,500	
Solve for			$77,733.28		

	N	I/Y	PV	PMT	FV
Enter	96	12% / 12		$1,500	
Solve for			$92,291.55		

	N	I/Y	PV	PMT	FV
Enter	84	15% / 12			$92,291.55
Solve for			$32,507.18		

$77,733.28 + 32,507.18 = $110,240.46

49.

	N	I/Y	PV	PMT	FV
Enter	15 × 12	10.5%/12		$1,000	
Solve for					$434,029.81

FV = $434,029.81 = PV $e^{.09(15)}$; PV = $434,029.81 $e^{-1.35}$ = $112,518.00

50. PV@ t = 14: $3,000 / 0.065 = $46,153.85

	N	I/Y	PV	PMT	FV
Enter	7	6.5%			$46,153.85
Solve for			$29,700.29		

51.

	N	I/Y	PV	PMT	FV
Enter	12		$20,000	±$1,900	
Solve for		2.076%			

APR = 2.076% × 12 = 24.91%

	NOM	EFF	C/Y
Enter	24.91%		12
Solve for		27.96%	

52. Monthly rate = .12 / 12 = .01; semiannual rate = $(1.01)^6 - 1 = 6.15\%$

Enter	10	6.15%		$6,000	
	N	I/Y	PV	PMT	FV
Solve for			$43,844.21		

Enter	8	6.15%			$43,844.21
	N	I/Y	PV	PMT	FV
Solve for			$27,194.83		

Enter	12	6.15%			$43,844.21
	N	I/Y	PV	PMT	FV
Solve for			$21,417.72		

Enter	18	6.15%			$43,844.21
	N	I/Y	PV	PMT	FV
Solve for			$14,969.38		

53.

a.

Enter	6	9.5%		$525	
	N	I/Y	PV	PMT	FV
Solve for			$2,320.41		

b. 2nd BGN 2nd SET

.

Enter	6	9.5%		$525	
	N	I/Y	PV	PMT	FV
Solve for			$2,540.85		

54. 2nd BGN 2nd SET

Enter	48	8.15% / 12	$56,000		
	N	I/Y	PV	PMT	FV
Solve for				$1,361.82	

57. Pre-retirement APR:

Enter		11%	12
	NOM	EFF	C/Y
Solve for	10.48%		

Post-retirement APR:

Enter		8%	12
	NOM	EFF	C/Y
Solve for	7.72%		

At retirement, he needs:

Enter	240	7.72% / 12		$25,000	$750,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$3,3212,854.41		

In 10 years, his savings will be worth:

Enter	120	10.48% / 12		$2,100	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$442,239.69

After purchasing the cabin, he will have: $442,239.69 – 350,000 = $92,239.69

Each month between years 10 and 30, he needs to save:

Enter	240	10.48% / 12	$92,239.69		$3,212,854.42
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$3,053.87	

58. PV of purchase:

Enter	36	8% / 12			$23,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$18,106.86		

$35,000 – 18,106.86 = $16,893.14

PV of lease:

Enter	36	8% / 12		$450	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$14,360.31		

$14,360.31 + 1 = $14,361.31

Lease the car.

You would be indifferent when the PV of the two cash flows are equal. The present value of the purchase decision must be $14,361.31. Since the difference in the two cash flows is $35,000 – 14,361.31 = $20,638.69, this must be the present value of the future resale price of the car. The break-even resale price of the car is:

Enter	36	8% / 12	$20,638.69		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$26,216.03

59.

Enter	4.50%		365
	NOM	**EFF**	**C/Y**
Solve for		4.60%	

CFo	$8,000,000
C01	$4,000,000
F01	1
C02	$4,800,000
F02	1
C03	$5,700,000
F03	1
C04	$6,400,000
F04	1
C05	$7,000,000
F05	1
C06	$7,500,000
F06	1

I = 4.60%
NPV CPT
$37,852,037.91

New contract value = $37,852,037.91 + 750,000 = $38,602,037.91

PV of payments = $38,602,037.91 – 9,000,000 = $29,602,037.91
Effective quarterly rate = $[1 + (.045/365)]^{91.25} - 1 = 1.131\%$

Enter	24	1.131%	$29,602,037.91		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$1,415,348.37	

60.

Enter	1		$17,600		±$20,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		13.64%			

61.

Enter		9%	12
	NOM	**EFF**	**C/Y**
Solve for	8.65%		

Enter	12	8.65% / 12		$40,000 / 12	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$41,624.33

Enter	1	9%	$41,624.33		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$45,370.52

Enter	12	8.65% / 12		$43,000 / 12	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$44,746.15

Enter	60	8.65% / 12		$45,000 / 12	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$182,142.14		

Award = $45,370.52 + 44,746.15 + 182,142.14 + 100,000 + 20,000 = $392,258.81

62.

Enter	1		$9,700		±$11,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		13.40%			

63.

Enter	1		$9,800		±$11,300
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		15.31%			

64. Refundable fee: With the $1,500 application fee, you will need to borrow $201,500 to have $200,000 after deducting the fee. Solve for the payment under these circumstances.

Enter	30 × 12	7.50% / 12	$201,500		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$1,408.92	

Enter	30 × 12		$200,000	±$1,408.92	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		0.6314%			

APR = 0.6314% × 12 = 7.58%

Enter	7.58%		12
	NOM	**EFF**	**C/Y**
Solve for		7.85%	

Without refundable fee: APR = 7.50%

Enter	7.50%		12
	NOM	**EFF**	**C/Y**
Solve for		7.76%	

65.

Enter	36		$1,000	±$42.25	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		2.47%			

APR = 2.47% × 12 = 29.63%

Enter	29.63%		12
	NOM	**EFF**	**C/Y**
Solve for		34.00%	

66. What she needs at age 65:

Enter	15	8%		$90,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$770,353.08		

a.

Enter	30	8%			$770,353.08
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$6,800.24	

b.

Enter	30	8%			$770,353.08
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$76,555.63		

c.

Enter	10	8%	$25,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$53,973.12

At 65, she is short: $770,353.08 – 53,973.12 = $716,379.96

Enter	30	8%			±$716,379.96
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$6,323.80	

Her employer will contribute $1,500 per year, so she must contribute:

$6,323.80 – 1,500 = $4,823.80 per year

67. Without fee:

Enter		19.2% / 12	$10,000	±$200	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	101.39				

Enter		9.2% / 12	$10,000	±$200	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	63.30				

With fee:

Enter		9.2% / 12	$10,200	±$200	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	64.94				

68. Value at Year 6:

Enter	5	11%	$750		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,263.79

Enter	4	11%	$750		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,138.55

Enter	3	11%	$850		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,162.49

Enter	2	11%	$850		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,047.29

Enter	1	11%	$950		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,054.50

So, at Year 5, the value is: $1,263.79 + 1,138.55 + 1,162.49 + 1,047.29 + 1,054.50 + 950 = $6,612.62

At Year 65, the value is:

Enter	59	7%	$6,612.62		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$358,326.50

The policy is not worth buying; the future value of the policy is $358K, but the policy contract will pay off $250K.

69.

Enter	30 × 12	8.5% / 12	$250,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$1,922.28	

Enter	22 × 12	8.5% / 12		$1,922.28	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$229,278.34		

70.

CFo	±$8,000
C01	±$8,000
F01	5
C02	$20,000
F02	4

IRR CPT
10.57%

74. The value at $t = 7$:

Enter	6	10%	$75,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$132,867.08

Enter	10	10%	$132,867.08		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$21,623.50	

75.

a. APR = 10% × 52 = 520%

Enter	520%		52
	NOM	**EFF**	**C/Y**
Solve for		14,104.29%	

b.

Enter	1		$9.00		±$10.00
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		11.11%			

APR = 11.11% × 52 = 577.78%

Enter	577.78%		52
	NOM	**EFF**	**C/Y**
Solve for		23,854.63%	

c.

Enter	4		$58.84	±$25	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		25.18%			

APR = 25.18% × 52 = 1,309.92%

Enter	1,309.92 %		52
	NOM	**EFF**	**C/Y**
Solve for		11,851,501.94%	

CHAPTER 7
INTEREST RATES AND BOND VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.

2. All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.

3. No. If the bid were higher than the ask, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?

4. Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.

5. There are two benefits. First, the company can take advantage of interest rate declines by calling in an issue and replacing it with a lower coupon issue. Second, a company might wish to eliminate a covenant for some reason. Calling the issue does this. The cost to the company is a higher coupon. A put provision is desirable from an investor's standpoint, so it helps the company by reducing the coupon rate on the bond. The cost to the company is that it may have to buy back the bond at an unattractive price.

6. Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also simply ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and simply determines what the bond's coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly at par.

7. Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provide the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.

8. Companies pay to have their bonds rated simply because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.

9. Treasury bonds have no credit risk since it is backed by the U.S. government, so a rating is not necessary. Junk bonds often are not rated because there would be no point in an issuer paying a rating agency to assign its bonds a low rating (it's like paying someone to kick you!).

10. The term structure is based on pure discount bonds. The yield curve is based on coupon-bearing issues.

11. Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.

12. As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other's securities is referred to as "reciprocal immunity."

13. Lack of transparency means that a buyer or seller can't see recent transactions, so it is much harder to determine what the best bid and ask prices are at any point in time.

14. One measure of liquidity is the bid-ask spread. Liquid instruments have relatively small spreads. Looking at Figure 7.4, the bellwether bond has a spread of one tick; it is one of the most liquid of all investments. Generally, liquidity declines after a bond is issued. Some older bonds, including some of the callable issues, have spreads as wide as six ticks.

15. Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.

16. A 100-year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the "equity in disguise" has a significant tax advantage.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

<u>*Basic*</u>

1. The yield to maturity is the required rate of return on a bond expressed as a nominal annual interest rate. For noncallable bonds, the yield to maturity and required rate of return are interchangeable terms. Unlike YTM and required return, the coupon rate is not a return used as the interest rate in bond cash flow valuation, but is a fixed percentage of par over the life of the bond used to set the coupon payment amount. For the example given, the coupon rate on the bond is still 10 percent, and the YTM is 8 percent.

2. Price and yield move in opposite directions; if interest rates rise, the price of the bond will fall. This is because the fixed coupon payments determined by the fixed coupon rate are not as valuable when interest rates rise—hence, the price of the bond decreases.

NOTE: Most problems do not explicitly list a par value for bonds. Even though a bond can have any par value, in general, corporate bonds in the United States will have a par value of $1,000. We will use this par value in all problems unless a different par value is explicitly stated.

3. The price of any bond is the PV of the interest payment, plus the PV of the par value. Notice this problem assumes an annual coupon. The price of the bond will be:

$P = \$80(\{1 – [1/(1 + .06)]^{10} \} / .06) + \$1,000[1 / (1 + .06)^{10}] = \$1,147.20$

We would like to introduce shorthand notation here. Rather than write (or type, as the case may be) the entire equation for the PV of a lump sum, or the PVA equation, it is common to abbreviate the equations as:

$\text{PVIF}_{R,t} = 1 / (1 + r)^t$

which stands for Present Value Interest Factor

$\text{PVIFA}_{R,t} = (\{1 – [1/(1 + r)]^t \} / r)$

which stands for Present Value Interest Factor of an Annuity

These abbreviations are short hand notation for the equations in which the interest rate and the number of periods are substituted into the equation and solved. We will use this shorthand notation in remainder of the solutions key.

4. Here we need to find the YTM of a bond. The equation for the bond price is:

$P = \$884.50 = \$90(\text{PVIFA}_{R\%,9}) + \$1,000(\text{PVIF}_{R\%,9})$

Notice the equation cannot be solved directly for *R*. Using a spreadsheet, a financial calculator, or trial and error, we find:

$R = \text{YTM} = 11.09\%$

If you are using trial and error to find the YTM of the bond, you might be wondering how to pick an interest rate to start the process. First, we know the YTM has to be higher than the coupon rate since the bond is a discount bond. That still leaves a lot of interest rates to check. One way to get a starting point is to use the following equation, which will give you an approximation of the YTM:

Approximate YTM = [Annual interest payment + (Price difference from par / Years to maturity)] / [(Price + Par value) / 2]

Solving for this problem, we get:

Approximate YTM = [$90 + ($115.50 / 9] / [($884.50 + 1,000) / 2] = 10.91%

This is not the exact YTM, but it is close, and it will give you a place to start.

5. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

$P = \$870 = C(PVIFA_{6.8\%,16}) + \$1,000(PVIF_{6.8\%,16})$

Solving for the coupon payment, we get:

$C = \$54.42$

The coupon payment is the coupon rate times par value. Using this relationship, we get:

Coupon rate = \$54.42 / \$1,000 = 5.44%

6. To find the price of this bond, we need to realize that the maturity of the bond is 10 years. The bond was issued one year ago, with 11 years to maturity, so there are 10 years left on the bond. Also, the coupons are semiannual, so we need to use the semiannual interest rate and the number of semiannual periods. The price of the bond is:

$P = \$41.00(PVIFA_{3.7\%,20}) + \$1,000(PVIF_{3.7\%,20}) = \$1,055.83$

7. Here we are finding the YTM of a semiannual coupon bond. The bond price equation is:

$P = \$970 = \$43(PVIFA_{R\%,20}) + \$1,000(PVIF_{R\%,20})$

Since we cannot solve the equation directly for R, using a spreadsheet, a financial calculator, or trial and error, we find:

$R = 4.531\%$

Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:

$YTM = 2 \times 4.531\% = 9.06\%$

8. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

$P = \$1,145 = C(PVIFA_{3.75\%,29}) + \$1,000(PVIF_{3.75\%,29})$

Solving for the coupon payment, we get:

$C = \$45.79$

Since this is the semiannual payment, the annual coupon payment is:

$2 \times \$45.79 = \$91.58$

And the coupon rate is the coupon rate divided by par value, so:

Coupon rate = \$91.58 / \$1,000 = 9.16%

9. The approximate relationship between nominal interest rates (R), real interest rates (r), and inflation (h) is:

$R = r + h$

Approximate $r = .06 - .045 = .015$ or 1.50%

The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$(1 + .06) = (1 + r)(1 + .045)$

Exact $r = [(1 + .06) / (1 + .045)] - 1 = .0144$ or 1.44%

10. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$R = (1 + .04)(1 + .025) - 1 = .0660$ or 6.60%

11. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$h = [(1 + .15) / (1 + .09)] - 1 = .0550$ or 5.50%

12. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$r = [(1 + .134) / (1.045)] - 1 = .0852$ or 8.52%

13. This is a bond since the maturity is greater than 10 years. The coupon rate, located in the first column of the quote is 6.125%. The bid price is:

Bid price = 110:27 = 110 27/32 = 110.84375% × $1,000 = $1,108.4375

The previous day's ask price is found by:

Previous day's asked price = Today's asked price – Change = 110 27/32 – (–10/32) = 111 05/32

The previous day's price in dollars was:

Previous day's dollar price = 111.15625% × $1,000 = $1,111.5625

14. This is a premium bond because it sells for more than 100% of face value. The current yield is:

Current yield = Annual coupon payment / Price = $75/$1,282.1875 = 5.85%

The YTM is located under the "ASK YLD" column, so the YTM is 5.24%.

The bid-ask spread is the difference between the bid price and the ask price, so:

Bid-Ask spread = 128:07 – 128:06 = 1/32

Intermediate

15. Here we are finding the YTM of semiannual coupon bonds for various maturity lengths. The bond price equation is:

$P = C(\text{PVIFA}_{R\%,t}) + \$1{,}000(\text{PVIF}_{R\%,t})$

X: $P_0 = \$80(\text{PVIFA}_{6\%,13}) + \$1{,}000(\text{PVIF}_{6\%,13}) = \$1{,}177.05$
$P_1 = \$80(\text{PVIFA}_{6\%,12}) + \$1{,}000(\text{PVIF}_{6\%,12}) = \$1{,}167.68$
$P_3 = \$80(\text{PVIFA}_{6\%,10}) + \$1{,}000(\text{PVIF}_{6\%,10}) = \$1{,}147.20$
$P_8 = \$80(\text{PVIFA}_{6\%,5}) + \$1{,}000(\text{PVIF}_{6\%,5}) = \$1{,}084.25$
$P_{12} = \$80(\text{PVIFA}_{6\%,1}) + \$1{,}000(\text{PVIF}_{6\%,1}) = \$1{,}018.87$
$P_{13} = \$1{,}000$

Y: $P_0 = \$60(\text{PVIFA}_{8\%,13}) + \$1{,}000(\text{PVIF}_{8\%,13}) = \$841.92$
$P_1 = \$60(\text{PVIFA}_{8\%,12}) + \$1{,}000(\text{PVIF}_{8\%,12}) = \$849.28$
$P_3 = \$60(\text{PVIFA}_{8\%,10}) + \$1{,}000(\text{PVIF}_{8\%,10}) = \$865.80$
$P_8 = \$60(\text{PVIFA}_{8\%,5}) + \$1{,}000(\text{PVIF}_{8\%,5}) = \$920.15$
$P_{12} = \$60(\text{PVIFA}_{8\%,1}) + \$1{,}000(\text{PVIF}_{8\%,1}) = \$981.48$
$P_{13} = \$1{,}000$

All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called "pull to par." In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

16. Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 10 percent. If the YTM suddenly rises to 12 percent:

P_{Sam} = \$50($PVIFA_{6\%,4}$) + \$1,000($PVIF_{6\%,4}$) = \$965.35

P_{Dave} = \$50($PVIFA_{6\%,30}$) + \$1,000($PVIF_{6\%,30}$) = \$862.35

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$\Delta P_{Sam}\%$ = (\$965.35 – 1,000) / \$1,000 = – 3.47%

$\Delta P_{Dave}\%$ = (\$862.35 – 1,000) / \$1,000 = – 13.76%

If the YTM suddenly falls to 8 percent:

P_{Sam} = \$50($PVIFA_{4\%,4}$) + \$1,000($PVIF_{4\%,4}$) = \$1,036.30

P_{Dave} = \$50($PVIFA_{4\%,30}$) + \$1,000($PVIF_{4\%,30}$) = \$1,172.92

$\Delta P_{Sam}\%$ = (\$1,036.30 – 1,000) / \$1,000 = + 3.63%

$\Delta P_{Dave}\%$ = (\$1,172.92 – 1,000) / \$1,000 = + 17.29%

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

17. Initially, at a YTM of 7 percent, the prices of the two bonds are:

P_J = \$25($PVIFA_{3.5\%,16}$) + \$1,000($PVIF_{3.5\%,16}$) = \$879.06

P_K = \$55($PVIFA_{3.5\%,16}$) + \$1,000($PVIF_{3.5\%,16}$) = \$1,241.88

If the YTM rises from 7 percent to 9 percent:

P_J = \$25($PVIFA_{4.5\%,16}$) + \$1,000($PVIF_{4.5\%,16}$) = \$775.32

P_K = \$55($PVIFA_{4.5\%,16}$) + \$1,000($PVIF_{4.5\%,16}$) = \$1,112.34

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$\Delta P_J\%$ = (\$775.32 – 879.06) / \$879.06 = – 11.80%
$\Delta P_K\%$ = (\$1,112.34 – 1,241.88) / \$1,241.88 = – 10.43%

If the YTM declines from 7 percent to 5 percent:

$P_J \quad = \$25(PVIFA_{2.5\%,16}) + \$1,000(PVIF_{2.5\%,16}) = \$1,000.000$

$P_K \quad = \$55(PVIFA_{2.5\%,16}) + \$1,000(PVIF_{2.5\%,16}) = \$1,391.65$

$\Delta P_J\% \quad = (\$1,000.00 – 879.06) / \$879.06 \quad = + 13.76\%$

$\Delta P_K\% \quad = (\$1,391.65 – 1,241.88) / \$1,241.88 \quad = + 12.06\%$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

18. The bond price equation for this bond is:

$P_0 = \$1,040 = \$42(PVIFA_{R\%,18}) + \$1,000(PVIF_{R\%,18})$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 3.887\%$

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 3.887% = 7.77%

The current yield is:

Current yield = Annual coupon payment / Price = $84 / $1,040 = 8.08%

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

Effective annual yield = $(1 + 0.03887)^2 – 1 = 7.92\%$

19. The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

$P = \$1,095 = \$40(PVIFA_{R\%,40}) + \$1,000(PVIF_{R\%,40})$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 3.55\%$

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 3.55% = 7.10%

20. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are five months until the next coupon payment, so one month has passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = $72/2 × 1/6 = $6

And we calculate the clean price as:

Clean price = Dirty price – Accrued interest = $1,140 – 6 = $1,134

21. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are three months until the next coupon payment, so three months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = $65/2 × 3/6 = $16.25

And we calculate the dirty price as:

Dirty price = Clean price + Accrued interest = $865 + 16.25 = $881.25

22. To find the number of years to maturity for the bond, we need to find the price of the bond. Since we already have the coupon rate, we can use the bond price equation, and solve for the number of years to maturity. We are given the current yield of the bond, so we can calculate the price as:

Current yield = .0906 = $110/$P_0$
P_0 = $110/.0906 = $1,214.13

Now that we have the price of the bond, the bond price equation is:

$P = \$1,214.13 = \$110[(1 – (1/1.085)^t) / .085] + \$1,000/1.085^t$

We can solve this equation for t as follows:

$\$1,214.13 (1.085)^t = \$1,294.12 (1.085)^t – 1,294.12 + 1,000$

$294.12 = 79.99(1.085)^t$

$3.6769 = 1.085^t$

$t = \log 3.6769 / \log 1.085 = 15.96 \approx 16$ years

The bond has 16 years to maturity.

23. The bond has 10 years to maturity, so the bond price equation is:

P = \$769.355 = \$36.875($PVIFA_{R\%,20}$) + \$1,000($PVIF_{R\%,20}$)

Using a spreadsheet, financial calculator, or trial and error we find:

R = 5.64%

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 5.64% = 11.28%

The current yield is the annual coupon payment divided by the bond price, so:

Current yield = \$73.75 / \$769.355 = 9.59%

The "EST Spread" column shows the difference between the YTM of the bond quoted and the YTM of the U.S. Treasury bond with a similar maturity. The column lists the spread in basis points. One basis point is one-hundredth of one percent, so 100 basis points equals one percent. The spread for this bond is 468 basis points, or 4.68%. This makes the equivalent Treasury yield:

Equivalent Treasury yield = 11.28% – 4.68% = 6.60%

24. *a.* The bond price is the present value of the cash flows from a bond. The YTM is the interest rate used in valuing the cash flows from a bond.

b. If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.

c. Current yield is defined as the annual coupon payment divided by the current bond price. For premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.

25. The price of a zero coupon bond is the PV of the par, so:

a. P_0 = \$1,000/1.08^{20} = \$214.55

b. In one year, the bond will have 19 years to maturity, so the price will be:

P_1 = \$1,000/1.08^{19} = \$231.71

The interest deduction is the price of the bond at the end of the year, minus the price at the beginning of the year, so:

Year 1 interest deduction = $231.71 – 214.55 = $17.16

The price of the bond when it has one year left to maturity will be:

P_{19} = $1,000/1.08 = $925.93

Year 19 interest deduction = $1,000 – 925.93 = $74.07

c. Previous IRS regulations required a straight-line calculation of interest. The total interest received by the bondholder is:

Total interest = $1,000 – 214.55 = $785.45

The annual interest deduction is simply the total interest divided by the maturity of the bond, so the straight-line deduction is:

Annual interest deduction = $785.45 / 20 = $39.27

d. The company will prefer straight-line methods when allowed because the valuable interest deductions occur earlier in the life of the bond.

26. *a.* The coupon bonds have a 7% coupon which matches the 7% required return, so they will sell at par. The number of bonds that must be sold is the amount needed divided by the bond price, so:

Number of coupon bonds to sell = $15,000,000 / $1,000 = 15,000

The number of zero coupon bonds to sell would be:

Price of zero coupon bonds = $1,000/1.07^{30} = $131.37

Number of zero coupon bonds to sell = $15,000,000 / $131.37 = 114,181

Note: In this case, the price of the bond was rounded to the number of cents when calculating the number of bonds to sell.

b. The repayment of the coupon bond will be the par value plus the last coupon payment times the number of bonds issued. So:

Coupon bonds repayment = 15,000($1,070) = $16,050,000

The repayment of the zero coupon bond will be the par value times the number of bonds issued, so:

Zeroes: repayment = 114,181($1,000) = $114,181,000 *c*. The total coupon payment for the coupon bonds will be the number bonds times the coupon payment. For the cash flow of the coupon bonds, we need to account for the tax deductibility of the interest payments. To do this, we will multiply the total coupon payment times one minus the tax rate. So:

Coupon bonds: (15,000)($70)(1–.35) = $682,500 cash outflow

Note that this is cash outflow since the company is making the interest payment.

For the zero coupon bonds, the first year interest payment is the difference in the price of the zero at the end of the year and the beginning of the year. The price of the zeroes in one year will be:

$P_1 = \$1,000/1.07^{29} = \$140.56$

The year 1 interest deduction per bond will be this price minus the price at the beginning of the year, which we found in part *b*, so:

Year 1 interest deduction per bond = $140.56 – 131.37 = $9.19

The total cash flow for the zeroes will be the interest deduction for the year times the number of zeroes sold, times the tax rate. The cash flow for the zeroes in year 1 will be:

Cash flows for zeroes in Year 1 = (114,181)($9.19)(.35) = $367,490.91

Notice the cash flow for the zeroes is a cash inflow. This is because of the tax deductibility of the imputed interest expense. That is, the company gets to write off the interest expense for the year even though the company did not have a cash flow for the interest expense. This reduces the company's tax liability, which is a cash inflow.

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt. We should note an important point here: If you find the PV of the cash flows from the coupon bond and the zero coupon bond, they will be the same. This is because of the much larger repayment amount for the zeroes.

27. We found the maturity of a bond in Problem 22. However, in this case, the maturity is indeterminate. A bond selling at par can have any length of maturity. In other words, when we solve the bond pricing equation as we did in Problem 22, the number of periods can be any positive number.

Challenge

28. To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year is:

P: $P_0 = \$100(PVIFA_{8\%,5}) + \$1,000(PVIF_{8\%,5}) = \$1,079.85$

$P_1 = \$100(PVIFA_{8\%,4}) + \$1,000(PVIF_{8\%,4}) = \$1,066.24$

Current yield = $100 / $1,079.85 = 9.26%

The capital gains yield is:

Capital gains yield = (New price – Original price) / Original price

Capital gains yield = ($1,066.24 – 1,079.85) / $1,079.85 = –1.26%

The current price of Bond D and the price of Bond D in one year is:

D: $P_0 = \$60(PVIFA_{8\%,5}) + \$1,000(PVIF_{8\%,5}) = \$920.15$

$P_1 = \$60(PVIFA_{8\%,4}) + \$1,000(PVIF_{8\%,4}) = \$933.76$

Current yield = $60 / $920.15 = 6.52%

Capital gains yield = ($933.76 – 920.15) / $920.15 = +1.48%

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 8%, but this return is distributed differently between current income and capital gains.

29. *a*. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:

$P_0 = \$1,150 = \$80(PVIFA_{R\%,10}) + \$1,000(PVIF_{R\%,10})$

Using a spreadsheet, financial calculator, or trial and error we find:

R = YTM = 5.97%

b. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:

$P_2 = \$80(PVIFA_{4.97\%,8}) + \$1,000(PVIF_{4.97\%,8}) = \$1,196.41$

To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were $80 each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:

$P_0 = \$1,150 = \$80(PVIFA_{R\%,2}) + \$1,196.41(PVIF_{R\%,2})$

Solving for R, we get:

R = HPY = 8.89%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

30. The price of any bond (or financial instrument) is the PV of the future cash flows. Even though Bond M makes different coupons payments, to find the price of the bond, we just find the PV of the cash flows. The PV of the cash flows for Bond M is:

P_M = $1,200($PVIFA_{5\%,16}$)($PVIF_{5\%,12}$) + $1,500($PVIFA_{5\%,12}$)($PVIF_{5\%,28}$) + $20,000($PVIF_{5\%,40}$)
P_M = $13,474.20

Notice that for the coupon payments of $1,500, we found the PVA for the coupon payments, and then discounted the lump sum back to today.

Bond N is a zero coupon bond with a $20,000 par value, therefore, the price of the bond is the PV of the par, or:

P_N = $20,000($PVIF_{5\%,40}$) = $2,840.91

31. To calculate this, we need to set up an equation with the callable bond equal to a weighted average of the noncallable bonds. We will invest X percent of our money in the first noncallable bond, which means our investment in Bond 3 (the other noncallable bond) will be (1 – X). The equation is:

$C_2 = C_1 X + C_3(1 - X)$
$8.25 = 6.50 X + 12(1 - X)$
$8.25 = 6.50 X + 12 - 12 X$
$X = 0.68181$

So, we invest about 68 percent of our money in Bond 1, and about 32 percent in Bond 3. This combination of bonds should have the same value as the callable bond, excluding the value of the call. So:

$P_2 = 0.68181P_1 + 0.31819P_3$
$P_2 = 0.68181(106.375) + 0.31819(134.96875)$
$P_2 = 115.4730$

The call value is the difference between this implied bond value and the actual bond price. So, the call value is:

Call value = 115.4730 – 103.50 = 11.9730

Assuming $1,000 par value, the call value is $119.73.

32. In general, this is not likely to happen, although it can (and did). The reason this bond has a negative YTM is that it is a callable U.S. Treasury bond. Market participants know this. Given the high coupon rate of the bond, it is extremely likely to be called, which means the bondholder will not receive all the cash flows promised. A better measure of the return on a callable bond is the yield to call (YTC). The YTC calculation is the basically the same as the YTM calculation, but the number of periods is the number of periods until the call date. If the YTC were calculated on this bond, it would be positive.

Calculator Solutions

3.

Enter	10	6%		$80	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,147.20		

4.

Enter	9		±$884.50	$90	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		11.09%			

5.

Enter	16	6.8%	±$870		$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$54.42	

Coupon rate = $54.42 / $1,000 = 5.44%

6.

Enter	20	3.70%		$41	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,055.83		

7.

Enter	20		±$970	$43	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		4.531%			

4.531% × 2 = 9.06%

8.

Enter	29	3.75%	±$1,145		$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$45.79	

$45.79 × 2 = $91.58

$91.58 / $1,000 = 9.16%

15. Bond X

P_0

Enter	13	6%		$80	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,177.05		

P_1

Enter	12	6%		$80	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,167.68		

P_3

	N	I/Y	PV	PMT	FV
Enter	10	6%		$80	$1,000
Solve for			$1,147.20		

P_8

	N	I/Y	PV	PMT	FV
Enter	5	6%		$80	$1,000
Solve for			$1,084.25		

P_{12}

	N	I/Y	PV	PMT	FV
Enter	1	6%		$80	$1,000
Solve for			$1,018.87		

Bond Y

P_0

	N	I/Y	PV	PMT	FV
Enter	13	8%		$60	$1,000
Solve for			$841.92		

P_1

	N	I/Y	PV	PMT	FV
Enter	12	8%		$60	$1,000
Solve for			$849.28		

P_3

	N	I/Y	PV	PMT	FV
Enter	10	8%		$60	$1,000
Solve for			$865.80		

P_8

	N	I/Y	PV	PMT	FV
Enter	5	8%		$60	$1,000
Solve for			$920.15		

P_{12}

	N	I/Y	PV	PMT	FV
Enter	1	8%		$60	$1,000
Solve for			$981.48		

16. If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 10 percent. If the YTM suddenly rises to 12 percent:

P_{Sam}

	N	I/Y	PV	PMT	FV
Enter	4	6%		$50	$1,000
Solve for			$965.35		

$\Delta P_{Sam}\%$ = ($965.35 – 1,000) / $1,000 = – 3.47%

P_{Dave}

Enter	30	6%		$50	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$862.35		

$\Delta P_{Dave}\%$ = ($862.35 – 1,000) / $1,000 = – 13.76%

If the YTM suddenly falls to 8 percent:

P_{Sam}

Enter	4	4%		$50	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,036.30		

$\Delta P_{Sam}\%$ = ($1,036.30 – 1,000) / $1,000 = + 3.63%

P_{Dave}

Enter	30	4%		$50	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,172.92		

$\Delta P_{Dave}\%$ = ($1,172.92 – 1,000) / $1,000 = + 17.29%

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

17. Initially, at a YTM of 7 percent, the prices of the two bonds are:

P_J

Enter	16	3.5%		$25	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$879.06		

P_K

Enter	16	3.5%		$55	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,241.88		

If the YTM rises from 7 percent to 9 percent:

P_J

Enter	16	4.5%		$25	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$775.32		

$\Delta P_J\%$ = ($775.32 – 897.06) / $897.06 = – 11.80%

P_K

Enter	16	4.5%		$55	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,112.34		

$\Delta P_K\%$ = ($1,112.34 – 1,241.88) / $1,241.88 = – 10.43%

If the YTM declines from 7 percent to 5 percent:

P_J

Enter	16	2.5%		$25	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,000.00		

$\Delta P_J\%$ = ($1,000.00 – 879.06) / $879.06 = + 13.76%

P_K

Enter	16	2.5%		$55	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,391.65		

$\Delta P_K\%$ = ($1,391.65 – 1,241.88) / $1,241.88 = + 12.06%

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

18.

Enter	18		±$1,040	$42	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		3.887%			

3.887% × 2 = 7.77%

Enter	7.77 %		2
	NOM	**EFF**	**C/Y**
Solve for		7.92%	

19. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

Enter	40		±$1,095	$40	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		3.55%			

3.55% × 2 = 7.10%

22. Current yield = .0906 = $110/$P_0$; P_0 = $110/.0906 = $1,214.13

Enter		8.5%	±$1,214.13	$110	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	15.96				

15.96 or ≈ 16 years

23.

Enter	20		±$769.355	$36.875	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		5.64%			

5.64% × 2 = 11.28%

25.

a. P_o

Enter	20	8%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$214.55		

b. P_1

Enter	19	8%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$231.71		

year 1 interest deduction = $231.71 – 214.55 = $17.16

P_{19}

Enter	1	8%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$925.93		

year 19 interest deduction = $1,000 – 925.93 = $74.07

c. Total interest = $1,000 – $214.55 = $785.45
Annual interest deduction = $785.45 / 20 = $39.27

d. The company will prefer straight-line methods when allowed because the valuable interest deductions occur earlier in the life of the bond.

26. *a.* The coupon bonds have a 7% coupon rate, which matches the 7% required return, so they will sell at par; # of bonds = $15M/$1,000 = 15,000.

For the zeroes:

Enter	30	7%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$131.37		

$15M/$131.37 = 114,181 will be issued.

b. Coupon bonds: repayment = 15,000($1,070) = $16.05M
Zeroes: repayment = 114,181($1,000) = $114,181,000

c. Coupon bonds: (15,000)($70)(1–.35) = $682,500 cash outflow
Zeroes:

Enter	29	7%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$140.56		

year 1 interest deduction = $140.56 – 131.37 = $9.19
(114,181)($9.19)(.35) = $367,497.34 cash inflow
During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt.

28.

Bond P

P_0

	N	I/Y	PV	PMT	FV
Enter	5	8%		$100	$1,000
Solve for			$1,079.85		

P_1

	N	I/Y	PV	PMT	FV
Enter	4	8%		$100	$1,000
Solve for			$1,066.24		

Current yield = $100 / $1,079.85 = 9.26%

Capital gains yield = ($1,066.24 – 1,079.85) / $1,079.85 = –1.26%

Bond D

P_0

	N	I/Y	PV	PMT	FV
Enter	5	8%		$60	$1,000
Solve for			$920.15		

P_1

	N	I/Y	PV	PMT	FV
Enter	4	8%		$60	$1,000
Solve for			$933.76		

Current yield = $60 / $920.15 = 6.52%

Capital gains yield = ($933.76 – 920.15) / $920.15 = +1.48%

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 8%, but this return is distributed differently between current income and capital gains.

29.

a.

	N	I/Y	PV	PMT	FV
Enter	10		±$1,150	$80	$1,000
Solve for		5.97%			

This is the rate of return you expect to earn on your investment when you purchase the bond.

b.

	N	I/Y	PV	PMT	FV
Enter	8	4.97%		$90	$1,000
Solve for			$1,196.41		

The HPY is:

	N	I/Y	PV	PMT	FV
Enter	2		±$1,150	$90	$1,196.41
Solve for		8.89%			

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

30.

P_M

CFo	$0
C01	$0
F01	12
C02	$1,200
F02	16
C03	$1,500
F03	11
C04	$21,500
F04	1

I = 5%
NPV CPT
$13,474.20

P_N

Enter	40	5%			$20,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$2,840.91		

CHAPTER 8
STOCK VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. The value of any investment depends on its cash flows; i.e., what investors will actually receive. The cash flows from a share of stock are the dividends.

2. Investors believe the company will eventually start paying dividends (or be sold to another company).

3. In general, companies that need the cash will often forgo dividends since dividends are a cash expense. Young, growing companies with profitable investment opportunities are one example; another example is a company in financial distress. This question is examined in depth in a later chapter.

4. The general method for valuing a share of stock is to find the present value of all expected future dividends. The dividend growth model presented in the text is only valid (i) if dividends are expected to occur forever, that is, the stock provides dividends in perpetuity, and (ii) if a constant growth rate of dividends occurs forever. A violation of the first assumption might be a company that is expected to cease operations and dissolve itself some finite number of years from now. The stock of such a company would be valued by applying the general method of valuation explained in this chapter. A violation of the second assumption might be a start-up firm that isn't currently paying any dividends, but is expected to eventually start making dividend payments some number of years from now. This stock would also be valued by the general dividend valuation method explained in this chapter.

5. The common stock probably has a higher price because the dividend can grow, whereas it is fixed on the preferred. However, the preferred is less risky because of the dividend and liquidation preference, so it is possible the preferred could be worth more, depending on the circumstances.

6. The two components are the dividend yield and the capital gains yield. For most companies, the capital gains yield is larger. This is easy to see for companies that pay no dividends. For companies that do pay dividends, the dividend yields are rarely over five percent and are often much less.

7. Yes. If the dividend grows at a steady rate, so does the stock price. In other words, the dividend growth rate and the capital gains yield are the same.

8. In a corporate election, you can buy votes (by buying shares), so money can be used to influence or even determine the outcome. Many would argue the same is true in political elections, but, in principle at least, no one has more than one vote.

9. It wouldn't seem to be. Investors who don't like the voting features of a particular class of stock are under no obligation to buy it.

10. Investors buy such stock because they want it, recognizing that the shares have no voting power. Presumably, investors pay a little less for such shares than they would otherwise.

11. Presumably, the current stock value reflects the risk, timing and magnitude of all future cash flows, both short-term and long-term. If this is correct, then the statement is false.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The constant dividend growth model is:

$P_t = D_t \times (1 + g) / (R - g)$

So the price of the stock today is:

$P_0 = D_0 (1 + g) / (R - g) = \$1.40 (1.06) / (.12 - .06) = \$24.73$

The dividend at year 4 is the dividend today times the FVIF for the growth rate in dividends and four years, so:

$P_3 = D_3 (1 + g) / (R - g) = D_0 (1 + g)^4 / (R - g) = \$1.40 (1.06)^4 / (.12 - .06) = \$29.46$

We can do the same thing to find the dividend in Year 16, which gives us the price in Year 15, so:

$P_{15} = D_{15} (1 + g) / (R - g) = D_0 (1 + g)^{16} / (R - g) = \$1.40 (1.06)^{16} / (.12 - .06) = \$59.27$

There is another feature of the constant dividend growth model: The stock price grows at the dividend growth rate. So, if we know the stock price today, we can find the future value for any time in the future we want to calculate the stock price. In this problem, we want to know the stock price in three years, and we have already calculated the stock price today. The stock price in three years will be:

$P_3 = P_0(1 + g)^3 = \$24.73(1 + .06)^3 = \$29.46$

And the stock price in 15 years will be:

$P_{15} = P_0(1 + g)^{15} = \$24.73(1 + .06)^{15} = \$59.27$

2. We need to find the required return of the stock. Using the constant growth model, we can solve the equation for R. Doing so, we find:

$R = (D_1 / P_0) + g = (\$3.10 / \$48.00) + .05 = 11.46\%$

3. The dividend yield is the dividend next year divided by the current price, so the dividend yield is:

Dividend yield = $D_1 / P_0 = \$3.10 / \$48.00 = 6.46\%$

The capital gains yield, or percentage increase in the stock price, is the same as the dividend growth rate, so:

Capital gains yield = 5%

4. Using the constant growth model, we find the price of the stock today is:

$P_0 = D_1 / (R - g) = \$3.60 / (.13 - .045) = \$42.35$

5. The required return of a stock is made up of two parts: The dividend yield and the capital gains yield. So, the required return of this stock is:

R = Dividend yield + Capital gains yield = .039 + .09 = 9.90%

6. We know the stock has a required return of 12 percent, and the dividend and capital gains yield are equal, so:

Dividend yield = 1/2(.12) = .06 = Capital gains yield

Now we know both the dividend yield and capital gains yield. The dividend is simply the stock price times the dividend yield, so:

$D_1 = .06(\$70) = \$4.20$

This is the dividend next year. The question asks for the dividend this year. Using the relationship between the dividend this year and the dividend next year:

$D_1 = D_0(1 + g)$

We can solve for the dividend that was just paid:

$D_1 = \$4.20(1 + .06)$

$D_0 = \$4.20 / 1.06 = \$3.96$

7. The price of any financial instrument is the PV of the future cash flows. The future dividends of this stock are an annuity for eight years, so the price of the stock is the PVA, which will be:

$P_0 = \$12.00(\text{PVIFA}_{10\%,8}) = \$64.02$

8. The price a share of preferred stock is the dividend divided by the required return. This is the same equation as the constant growth model, with a dividend growth rate of zero percent. Remember, most preferred stock pays a fixed dividend, so the growth rate is zero. Using this equation, we find the price per share of the preferred stock is:

$R = D/P_0 = \$8.25/\$113 = 7.30\%$

<u>*Intermediate*</u>

9. This stock has a constant growth rate of dividends, but the required return changes twice. To find the value of the stock today, we will begin by finding the price of the stock at Year 6, when both the dividend growth rate and the required return are stable forever. The price of the stock in Year 6 will be the dividend in Year 7, divided by the required return minus the growth rate in dividends. So:

$P_6 = D_6 (1 + g) / (R – g) = D_0 (1 + g)^7 / (R – g) = \$3.00 (1.05)^7 / (.11 – .05) = \$70.36$

Now we can find the price of the stock in Year 3. We need to find the price here since the required return changes at that time. The price of the stock in Year 3 is the PV of the dividends in Years 4, 5, and 6, plus the PV of the stock price in Year 6. The price of the stock in Year 3 is:

$P_3 = \$3.00(1.050)^4 / 1.14 + \$3.00(1.050)^5 / 1.14^2 + \$3.00(1.05)^6 / 1.14^3 + \$70.36 / 1.14^3$
$P_3 = \$56.35$

Finally, we can find the price of the stock today. The price today will be the PV of the dividends in Years 1, 2, and 3, plus the PV of the stock in Year 3. The price of the stock today is:

$P_0 = \$3.00(1.050) / 1.16 + \$3.00(1.050)^2 / (1.16)^2 + \$3.00(1.050)^3 / (1.16)^3 + \$56.35 / (1.16)^3$
$= \$43.50$

10. Here we have a stock that pays no dividends for 10 years. Once the stock begins paying dividends, it will have a constant growth rate of dividends. We can use the constant growth model at that point. It is important to remember that general constant dividend growth formula is:

$P_t = [D_t \times (1 + g)] / (R – g)$

This means that since we will use the dividend in Year 10, we will be finding the stock price in Year 9. The dividend growth model is similar to the PVA and the PV of a perpetuity: The equation gives you the PV one period before the first payment. So, the price of the stock in Year 9 will be:

$P_9 = D_{10} / (R – g) = \$8.00 / (.13 – .06) = \$114.29$

The price of the stock today is simply the PV of the stock price in the future. We simply discount the future stock price at the required return. The price of the stock today will be:

$P_0 = \$114.29 / 1.13^9 = \$38.04$

11. The price of a stock is the PV of the future dividends. This stock is paying four dividends, so the price of the stock is the PV of these dividends using the required return. The price of the stock is:

$P_0 = \$12 / 1.11 + \$15 / 1.11^2 + \$18 / 1.11^3 + \$21 / 1.11^4 = \$49.98$

12. With supernormal dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the supernormal growth period. The stock begins constant growth in Year 4, so we can find the price of the stock in Year 3, one year before the constant dividend growth begins, as:

$P_4 = D_4 (1 + g) / (R – g) = \$2.00(1.05) / (.13 – .05) = \$26.25$

The price of the stock today is the PV of the first three dividends, plus the PV of the Year 3 stock price. So, the price of the stock today will be:

P_0 = \$8.00 / 1.13 + \$6.00 / 1.13^2 + \$3.00 / 1.13^3 + \$26.25 / 1.13^3 = \$31.18

13. With supernormal dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the futuresstock price, plus the PV of all dividends during the supernormal growth period. The stock begins constant growth in Year 4, so we can find the price of the stock in Year 3, one year before the constant dividend growth begins as:

$P_3 = D_3 (1 + g) / (R – g) = D_0 (1 + g_1)^3 (1 + g_2) / (R – g)$ = \$2.80$(1.25)^3$(1.07) / (.13 – .07) = \$97.53

The price of the stock today is the PV of the first three dividends, plus the PV of the Year 3 stock price. The price of the stock today will be:

P_0 = 2.80(1.25) / 1.13 + \$2.80$(1.25)^2$ / 1.13^2 + \$2.80$(1.25)^3$ / 1.13^3 + \$97.53 / 1.13^3
P_0 = \$77.90

14. Here we need to find the dividend next year for a stock experiencing supernormal growth. We know the stock price, the dividend growth rates, and the required return, but not the dividend. First, we need to realize that the dividend in Year 3 is the current dividend times the FVIF. The dividend in Year 3 will be:

$D_3 = D_0 (1.30)^3$

And the dividend in Year 4 will be the dividend in Year 3 times one plus the growth rate, or:

$D_4 = D_0 (1.30)^3 (1.18)$

The stock begins constant growth in Year 4, so we can find the price of the stock in Year 4 as the dividend in Year 5, divided by the required return minus the growth rate. The equation for the price of the stock in Year 4 is:

$P_4 = D_4 (1 + g) / (R – g)$

Now we can substitute the previous dividend in Year 4 into this equation as follows:

$P_4 = D_0 (1 + g_1)^3 (1 + g_2) (1 + g_3) / (R – g)$

$P_4 = D_0 (1.30)^3 (1.18) (1.08) / (.14 – .08) = 46.66D_0$

When we solve this equation, we find that the stock price in Year 4 is 46.66 times as large as the dividend today. Now we need to find the equation for the stock price today. The stock price today is the PV of the dividends in Years 1, 2, 3, and 4, plus the PV of the Year 4 price. So:

$P_0 = D_0(1.30)/1.14 + D_0(1.30)^2/1.14^2 + D_0(1.30)^3/1.14^3 + D_0(1.30)^3(1.18)/1.14^4 + 46.66D_0/1.14^4$

We can factor out D_0 in the equation, and combine the last two terms. Doing so, we get:

P_0 = \$70.00 = $D_0\{1.30/1.14 + 1.30^2/1.14^2 + 1.30^3/1.14^3 + [(1.30)^3(1.18) + 46.66] / 1.14^4\}$

Reducing the equation even further by solving all of the terms in the braces, we get:

$\$70 = \$33.04D_0$

$D_0 = \$70.00 / \$33.04 = \$2.12$

This is the dividend today, so the projected dividend for the next year will be:

$D_1 = \$2.12(1.30) = \$2.75$

15. The constant growth model can be applied even if the dividends are declining by a constant percentage, just make sure to recognize the negative growth. So, the price of the stock today will be:

$P_0 = D_0\,(1 + g) / (R - g) = \$10.00(1 - .08) / [(.11 - (-.08)] = \$48.42$

16. We are given the stock price, the dividend growth rate, and the required return, and are asked to find the dividend. Using the constant dividend growth model, we get:

$P_0 = \$50 = D_0\,(1 + g) / (R - g)$

Solving this equation for the dividend gives us:

$D_0 = \$50(.14 - .08) / (1.08) = \$2.78$

17. The price of a share of preferred stock is the dividend payment divided by the required return. We know the dividend payment in Year 6, so we can find the price of the stock in Year 5, one year before the first dividend payment. Doing so, we get:

$P_5 = \$9.00 / .07 = \$128.57$

The price of the stock today is the PV of the stock price in the future, so the price today will be:

$P_0 = \$128.57 / (1.07)^5 = \$91.67$

18. The annual dividend paid to stockholders is \$0.15, and the dividend yield is 0.2 percent. Using the equation for the dividend yield:

Dividend yield = Dividend / Stock price

We can plug the numbers in and solve for the stock price:

$.002 = \$0.15 / P_0$

$P_0 = \$0.15/.002 = \$75.00$

The "Net Chg" of the stock shows the stock increased by \$2.20 on this day, so the closing stock price yesterday was:

Yesterday's closing price = \$75.00 – 2.20 = \$72.80

To find the net income, we need to find the EPS. The stock quote tells us the P/E ratio for the stock is 14. Since we know the stock price as well, we can use the P/E ratio to solve for EPS as follows:

P/E = 14 = Stock price / EPS = \$75.00 / EPS

EPS = \$75.00 / 14 = \$5.357

We know that EPS is just the total net income divided by the number of shares outstanding, so:

EPS = NI / Shares = \$5.357 = NI / 25,000,000

NI = \$5.357(25,000,000) = \$133,928,571

Challenge

19. We are asked to find the dividend yield and capital gains yield for each of the stocks. All of the stocks have a 15 percent required return, which is the sum of the dividend yield and the capital gains yield. To find the components of the total return, we need to find the stock price for each stock. Using this stock price and the dividend, we can calculate the dividend yield. The capital gains yield for the stock will be the total return (required return) minus the dividend yield.

W: $P_0 = D_0(1 + g) / (R - g) = \$4.50(1.10)/(.15 - .10) = \$99.00$

$\text{Dividend yield} = D_1/P_0 = 4.50(1.10)/99.00 = 5\%$

Capital gains yield = .15 – .05 = 10%

X: $P_0 = D_0(1 + g) / (R - g) = \$4.50/(.15 - 0) = \$30.00$

$\text{Dividend yield} = D_1/P_0 = 4.50/30.00 = 15\%$

Capital gains yield = .15 – .15 = 0%

Y: $P_0 = D_0(1 + g) / (R - g) = \$4.50(1 - .05)/(.15 + .05) = \$21.38$

$\text{Dividend yield} = D_1/P_0 = 4.50(0.95)/21.38 = 20\%$

Capital gains yield = .15 – .20 = – 5%

Z: $P_2 = D_2(1 + g) / (R - g) = D_0(1 + g_1)^2(1 + g_2)/(R - g) = \$4.50(1.20)^2(1.12)/(.15 - .12) = \$241.92$

$P_0 = \$4.50\,(1.20) / (1.15) + \$4.50\,(1.20)^2 / (1.15)^2 + \$241.92 / (1.15)^2 = \$192.52$

$\text{Dividend yield} = D_1/P_0 = \$4.50(1.20)/\$192.52 = 2.8\%$

Capital gains yield = .15 – .028 = 12.2%

In all cases, the required return is 15%, but the return is distributed differently between current income and capital gains. High growth stocks have an appreciable capital gains component but a relatively small current income yield; conversely, mature, negative-growth stocks provide a high current income but also price depreciation over time.

20. *a*. Using the constant growth model, the price of the stock paying annual dividends will be:

$P_0 = D_0(1 + g) / (R - g) = \$3.00(1.06)/(.14 - .06) = \$39.75$

b. If the company pays quarterly dividends instead of annual dividends, the quarterly dividend will be one-fourth of annual dividend, or:

Quarterly dividend: $3.00(1.06)/4 = $0.795

To find the equivalent annual dividend, we must assume that the quarterly dividends are reinvested at the required return. We can then use this interest rate to find the equivalent annual dividend. In other words, when we receive the quarterly dividend, we reinvest it at the required return on the stock. So, the effective quarterly rate is:

Effective quarterly rate: $1.14^{.25} - 1 = .0333$

The effective annual dividend will be the FVA of the quarterly dividend payments at the effective quarterly required return. In this case, the effective annual dividend will be:

Effective $D_1 = \$0.795(\text{FVIFA}_{3.33\%,4}) = \$3.34$

Now, we can use the constant growth model to find the current stock price as:

$P_0 = \$3.34/(.14 - .06) = \$41.78$

Note that we can not simply find the quarterly effective required return and growth rate to find the value of the stock. This would assume the dividends increased each quarter, not each year.

21. Here we have a stock with supernormal growth, but the dividend growth changes every year for the first four years. We can find the price of the stock in Year 3 since the dividend growth rate is constant after the third dividend. The price of the stock in Year 3 will be the dividend in Year 4, divided by the required return minus the constant dividend growth rate. So, the price in Year 3 will be:

$P_3 = \$3.50(1.20)(1.15)(1.10)(1.05) / (.13 - .05) = \$69.73$

The price of the stock today will be the PV of the first three dividends, plus the PV of the stock price in Year 3, so:

$P_0 = \$3.50(1.20)/(1.13) + \$3.50(1.20)(1.15)/1.13^2 + \$3.50(1.20)(1.15)(1.10)/1.13^3 + \$69.73/1.13^3$
$P_0 = \$59.51$

22. Here we want to find the required return that makes the PV of the dividends equal to the current stock price. The equation for the stock price is:

$$P = \$3.50(1.20)/(1 + R) + \$3.50(1.20)(1.15)/(1 + R)^2 + \$3.50(1.20)(1.15)(1.10)/(1 + R)^3$$
$$+ [\$3.50(1.20)(1.15)(1.10)(1.05)/(R - .05)]/(1 + R)^3 = \$98.65$$

We need to find the roots of this equation. Using spreadsheet, trial and error, or a calculator with a root solving function, we find that:

$R = 9.85\%$

CHAPTER 9
NET PRESENT VALUE AND OTHER INVESTMENT CRITERIA

Answers to Concepts Review and Critical Thinking Questions

1. A payback period less than the project's life means that the NPV is positive for a zero discount rate, but nothing more definitive can be said. For discount rates greater than zero, the payback period will still be less than the project's life, but the NPV may be positive, zero, or negative, depending on whether the discount rate is less than, equal to, or greater than the IRR. The discounted payback includes the effect of the relevant discount rate. If a project's discounted payback period is less than the project's life, it must be the case that NPV is positive.

2. If a project has a positive NPV for a certain discount rate, then it will also have a positive NPV for a zero discount rate; thus, the payback period must be less than the project life. Since discounted payback is calculated at the same discount rate as is NPV, if NPV is positive, the discounted payback period must be less than the project's life. If NPV is positive, then the present value of future cash inflows is greater than the initial investment cost; thus PI must be greater than 1. If NPV is positive for a certain discount rate R, then it will be zero for some larger discount rate R*; thus the IRR must be greater than the required return.

3. *a.* Payback period is simply the accounting break-even point of a series of cash flows. To actually compute the payback period, it is assumed that any cash flow occurring during a given period is realized continuously throughout the period, and not at a single point in time. The payback is then the point in time for the series of cash flows when the initial cash outlays are fully recovered. Given some predetermined cutoff for the payback period, the decision rule is to accept projects that payback before this cutoff, and reject projects that take longer to payback.

b. The worst problem associated with payback period is that it ignores the time value of money. In addition, the selection of a hurdle point for payback period is an arbitrary exercise that lacks any steadfast rule or method. The payback period is biased towards short-term projects; it fully ignores any cash flows that occur after the cutoff point.

c. Despite its shortcomings, payback is often used because (1) the analysis is straightforward and simple and (2) accounting numbers and estimates are readily available. Materiality considerations often warrant a payback analysis as sufficient; maintenance projects are another example where the detailed analysis of other methods is often not needed. Since payback is biased towards liquidity, it may be a useful and appropriate analysis method for short-term projects where cash management is most important.

4. *a.* The discounted payback is calculated the same as is regular payback, with the exception that each cash flow in the series is first converted to its present value. Thus discounted payback provides a measure of financial/economic break-even because of this discounting, just as regular payback provides a measure of accounting break-even because it does not discount the cash flows. Given some predetermined cutoff for the discounted payback period, the decision rule is to accept projects whose discounted cash flows payback before this cutoff period, and to reject all other projects.

b. The primary disadvantage to using the discounted payback method is that it ignores all cash flows that occur after the cutoff date, thus biasing this criterion towards short-term projects. As a result, the method may reject projects that in fact have positive NPVs, or it may accept projects with large future cash outlays resulting in negative NPVs. In addition, the selection of a cutoff point is again an arbitrary exercise.

c. Discounted payback is an improvement on regular payback because it takes into account the time value of money. For conventional cash flows and strictly positive discount rates, the discounted payback will always be greater than the regular payback period.

5. *a.* The average accounting return is interpreted as an average measure of the accounting performance of a project over time, computed as some average profit measure attributable to the project divided by some average balance sheet value for the project. This text computes AAR as average net income with respect to average (total) book value. Given some predetermined cutoff for AAR, the decision rule is to accept projects with an AAR in excess of the target measure, and reject all other projects.

b. AAR is not a measure of cash flows and market value, but a measure of financial statement accounts that often bear little resemblance to the relevant value of a project. In addition, the selection of a cutoff is arbitrary, and the time value of money is ignored. For a financial manager, both the reliance on accounting numbers rather than relevant market data and the exclusion of time value of money considerations are troubling. Despite these problems, AAR continues to be used in practice because (1) the accounting information is usually available, (2) analysts often use accounting ratios to analyze firm performance, and (3) managerial compensation is often tied to the attainment of certain target accounting ratio goals.

6. *a.* NPV is simply the present value of a project's cash flows. NPV specifically measures, after considering the time value of money, the net increase or decrease in firm wealth due to the project. The decision rule is to accept projects that have a positive NPV, and reject projects with a negative NPV.

b. NPV is superior to the other methods of analysis presented in the text because it has no serious flaws. The method unambiguously ranks mutually exclusive projects, and can differentiate between projects of different scale and time horizon. The only drawback to NPV is that it relies on cash flow and discount rate values that are often estimates and not certain, but this is a problem shared by the other performance criteria as well. A project with NPV = $2,500 implies that the total shareholder wealth of the firm will increase by $2,500 if the project is accepted.

7. *a.* The IRR is the discount rate that causes the NPV of a series of cash flows to be identically zero. IRR can thus be interpreted as a financial break-even rate of return; at the IRR discount rate, the net value of the project is zero. The IRR decision rule is to accept projects with IRRs greater than the discount rate, and to reject projects with IRRs less than the discount rate.

b. IRR is the interest rate that causes NPV for a series of cash flows to be zero. NPV is preferred in all situations to IRR; IRR can lead to ambiguous results if there are non-conventional cash flows, and it also ambiguously ranks some mutually exclusive projects. However, for stand-alone projects with conventional cash flows, IRR and NPV are interchangeable techniques.

c. IRR is frequently used because it is easier for many financial managers and analysts to rate performance in relative terms, such as "12%", than in absolute terms, such as "$46,000." IRR may be a preferred method to NPV in situations where an appropriate discount rate is unknown are uncertain; in this situation, IRR would provide more information about the project than would NPV.

8. *a.* The profitability index is the present value of cash inflows relative to the project cost. As such, it is a benefit/cost ratio, providing a measure of the relative profitability of a project. The profitability index decision rule is to accept projects with a PI greater than one, and to reject projects with a PI less than one.

b. PI = (NPV + cost)/cost = 1 + (NPV/cost). If a firm has a basket of positive NPV projects and is subject to capital rationing, PI may provide a good ranking measure of the projects, indicating the "bang for the buck" of each particular project.

9. For a project with future cash flows that are an annuity:

Payback = I / C

And the IRR is:

0 = – I + C / IRR

Solving the IRR equation for IRR, we get:

IRR = C / I

Notice this is just the reciprocal of the payback. So:

IRR = 1 / PB

For long-lived projects with relatively constant cash flows, the sooner the project pays back, the greater is the IRR.

10. There are a number of reasons. Two of the most important have to do with transportation costs and exchange rates. Manufacturing in the U.S. places the finished product much closer to the point of sale, resulting in significant savings in transportation costs. It also reduces inventories because goods spend less time in transit. Higher labor costs tend to offset these savings to some degree, at least compared to other possible manufacturing locations. Of great importance is the fact that manufacturing in the U.S. means that a much higher proportion of the costs are paid in dollars. Since sales are in dollars, the net effect is to immunize profits to a large extent against fluctuations in exchange rates. This issue is discussed in greater detail in the chapter on international finance.

11. The single biggest difficulty, by far, is coming up with reliable cash flow estimates. Determining an appropriate discount rate is also not a simple task. These issues are discussed in greater depth in the next several chapters. The payback approach is probably the simplest, followed by the AAR, but even these require revenue and cost projections. The discounted cash flow measures (discounted payback, NPV, IRR, and profitability index) are really only slightly more difficult in practice.

12. Yes, they are. Such entities generally need to allocate available capital efficiently, just as for-profits do. However, it is frequently the case that the "revenues" from not-for-profit ventures are not tangible. For example, charitable giving has real opportunity costs, but the benefits are generally hard to measure. To the extent that benefits are measurable, the question of an appropriate required return remains. Payback rules are commonly used in such cases. Finally, realistic cost/benefit analysis along the lines indicated should definitely be used by the U.S. government and would go a long way toward balancing the budget!

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

<u>Basic</u>

1. To calculate the payback period, we need to find the time that the project has recovered its initial investment. After two years, the project has created:

$1,200 + 2,500 = $3,700

in cash flows. The project still needs to create another:

$4,800 – 3,700 = $1,100

in cash flows. During the third year, the cash flows from the project will be $3,400. So, the payback period will be 2 years, plus what we still need to make divided by what we will make during the third year. The payback period is:

Payback = 2 + ($1,100 / $3,400) = 2.32 years

2. To calculate the payback period, we need to find the time that the project has recovered its initial investment. The cash flows in this problem are an annuity, so the calculation is simpler. If the initial cost is $3,000, the payback period is:

Payback = 3 + ($480 / $840) = 3.57 years

There is a shortcut to calculate the future cash flows are an annuity. Just divide the initial cost by the annual cash flow. For the $3,000 cost, the payback period is:

Payback = $3,000 / $840 = 3.57 years

For an initial cost of $5,000, the payback period is:

Payback = 5 + ($800 / $840) = 5.95 years

The payback period for an initial cost of $7,000 is a little trickier. Notice that the total cash inflows after eight years will be:

Total cash inflows = 8($840) = $6,720

If the initial cost is $7,000, the project never pays back. Notice that if you use the shortcut for annuity cash flows, you get:

Payback = $7,000 / $840 = 8.33 years.

This answer does not make sense since the cash flows stop after eight years, so again, we must conclude the payback period is never.

3. Project A has cash flows of:

Cash flows = \$30,000 + 18,000 =\$48,000

during this first two years. The cash flows are still short by \$2,000 of recapturing the initial investment, so the payback for Project A is:

Payback = 2 + (\$2,000 / \$10,000) = 2.20 years

Project B has cash flows of:

Cash flows = \$9,000 + 25,000 + 35,000 =\$69,000

during this first two years. The cash flows are still short by \$1,000 of recapturing the initial investment, so the payback for Project B is:

B: Payback = 3 + (\$1,000 / \$425,000) = 3.002 years

Using the payback criterion and a cutoff of 3 years, accept project A and reject project B.

4. When we use discounted payback, we need to find the value of all cash flows today. The value today of the project cash flows for the first four years is:

Value today of Year 1 cash flow = \$7,000/1.14 = \$6,140.35
Value today of Year 2 cash flow = \$7,500/1.14^2 = \$5,771.01
Value today of Year 3 cash flow = \$8,000/1.14^3 = \$5,399.77
Value today of Year 4 cash flow = \$8,500/1.14^4 = \$5,032.68

To find the discounted payback, we use these values to find the payback period. The discounted first year cash flow is \$6,140.35, so the discounted payback for an \$8,000 initial cost is:

Discounted payback = 1 + (\$8,000 – 6,140.35)/\$5,771.01 = 1.32 years

For an initial cost of \$13,000, the discounted payback is:

Discounted payback = 2 + (\$13,000 – 6,140.35 – 5,771.01)/\$5,399.77 = 2.20 years

Notice the calculation of discounted payback. We know the payback period is between two and three years, so we subtract the discounted values of the Year 1 and Year 2 cash flows from the initial cost. This is the numerator, which is the discounted amount we still need to make to recover our initial investment. We divide this amount by the discounted amount we will earn in Year 3 to get the fractional portion of the discounted payback.

If the initial cost is \$18,000, the discounted payback is:

Discounted payback = 3 + (\$18,000 – 6,140.35 – 5,771.01 – 5,399.77) / \$5,032.68 = 3.14 years

5. R = 0%: 4 + ($1,600 / $2,100) = 4.76 years
discounted payback = regular payback = 4.76 years

R = 5%: $\$2,100/1.05 + \$2,100/1.05^2 + \$2,100/1.05^3 + \$2,100/1.05^4 + \$2,100/1.05^5 = \$9,091.90$
$\$2,100/1.05^6 = \$1,567.05$
discounted payback = 5 + ($10,000 – 9,091.90) / $1,567.05 = 5.58 years

R = 15%: $\$2,100/1.15 + \$2,100/1.15^2 + \$2,100/1.15^3 + \$2,100/1.15^4 + \$2,100/1.15^5 + \$2,100/1.15^6$ = $7,947.41; The project never pays back.

6. Our definition of AAR is the average net income divided by the average book value. The average net income for this project is:

Average net income = ($1,416,000 + 1,868,000 + 1,562,000 + 985,000) / 4 = $1,457,750

And the average book value is:

Average book value = ($15M + 0) / 2 = $7.5M

So, the AAR for this project is:

AAR = Average net income / Average book value = $1,457,750 / $7,500,000 = 19.44%

7. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

$0 = – \$30,000 + \$20,000/(1+IRR) + \$14,000/(1+IRR)^2 + \$11,000/(1+IRR)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 26.48%

Since the IRR is greater than the required return we would accept the project.

8. The NPV of a project is the PV of the outflows minus by the PV of the inflows. The equation for the NPV of this project at an 11 percent required return is:

$NPV = – \$30,000 + \$20,000/1.11 + \$14,000/1.11^2 + \$11,000/1.11^3 = \$7,423.84$

At an 11 percent required return, the NPV is positive, so we would accept the project.

The equation for the NPV of the project at a 30 percent required return is:

$NPV = – \$30,000 + \$20,000/1.30 + \$14,000/1.30^2 + \$11,000/1.30^3 = – \$1,324.53$

At a 30 percent required return, the NPV is negative, so we would reject the project.

9. The NPV of a project is the PV of the outflows minus by the PV of the inflows. Since the cash inflows are an annuity, the equation for the NPV of this project at an 8 percent required return is:

NPV = – \$70,000 + \$14,000($PVIFA_{8\%, 9}$) = \$17,456.43

At an 8 percent required return, the NPV is positive, so we would accept the project.

The equation for the NPV of the project at a 16 percent required return is:

NPV = – \$70,000 + \$14,000($PVIFA_{16\%, 9}$) = –\$5,508.39

At a 16 percent required return, the NPV is negative, so we would reject the project.

We would be indifferent to the project if the required return was equal to the IRR of the project, since at that required return the NPV is zero. The IRR of the project is:

0 = – \$40,000 + \$14,000($PVIFA_{IRR, 9}$)

IRR = 13.70%

10. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

$0 = -\$8,000 + \$3,200/(1+IRR) + \$4,000/(1+IRR)^2 + \$6,100/(1+IRR)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 26.83%

11. The NPV of a project is the PV of the outflows minus by the PV of the inflows. At a zero discount rate (and only at a zero discount rate), the cash flows can be added together across time. So, the NPV of the project at a zero percent required return is:

NPV = – \$8,000 + 3,200 + 4,000 + 6,100 = \$5,300

The NPV at a 10 percent required return is:

$NPV = -\$8,000 + \$3,200/1.1 + \$4,000/1.1^2 + \$6,100/1.1^3 = \$2,797.90$

The NPV at a 20 percent required return is:

$NPV = -\$8,000 + \$3,200/1.2 + \$4,000/1.2^2 + \$6,100/1.2^3 = \$974.54$

And the NPV at a 30 percent required return is:

$NPV = -\$8,000 + \$3,200/1.3 + \$4,000/1.3^2 + \$6,100/1.3^3 = -\$395.08$

Notice that as the required return increases, the NPV of the project decreases. This will always be true for projects with conventional cash flows. Conventional cash flows are negative at the beginning of the project and positive throughout the rest of the project.

12. *a.* The IRR is the interest rate that makes the NPV of the project equal to zero. The equation for the IRR of Project A is:

$0 = –\$34,000 + \$16,500/(1+IRR) + \$14,000/(1+IRR)^2 + \$10,000/(1+IRR)^3 + \$6,000/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 16.60%

The equation for the IRR of Project B is:

$0 = –\$34,000 + \$5,000/(1+IRR) + \$10,000/(1+IRR)^2 + \$18,000/(1+IRR)^3 + \$19,000/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 15.72%

Examining the IRRs of the projects, we see that the IRR_A is greater than the IRR_B, so IRR decision rule implies accepting project A. This may not be a correct decision; however, because the IRR criterion has a ranking problem for mutually exclusive projects. To see if the IRR decision rule is correct or not, we need to evaluate the project NPVs.

b. The NPV of Project A is:

$NPV_A = –\$34,000 + \$16,500/1.11 + \$14,000/1.11^2 + \$10,000/1.11^3 + \$6,000/1.11^4$
$NPV_A = \$3,491.88$

And the NPV of Project B is:

$NPV_B = –\$34,000 + \$5,000/1.11 + \$10,000/1.11^2 + \$18,000/1.11^3 + \$19,000/1.11^4$
$NPV_B = \$4,298.06$

The NPV_B is greater than the NPV_A, so we should accept Project B.

c. To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project. Here, we will subtract the cash flows for Project B from the cash flows of Project A. Once we find these differential cash flows, we find the IRR. The equation for the crossover rate is:

Crossover rate: $0 = \$11,500/(1+R) + \$4,000/(1+R)^2 – \$8,000/(1+R)^3 – \$13,000/(1+R)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

R = 13.75%

At discount rates above 13.75% choose project A; for discount rates below 13.75% choose project B; indifferent between A and B at a discount rate of 13.75%.

13. The IRR is the interest rate that makes the NPV of the project equal to zero. The equation to calculate the IRR of Project X is:

$0 = -\$5{,}000 + \$2{,}700/(1+IRR) + \$1{,}700/(1+IRR)^2 + \$2{,}300/(1+IRR)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 16.82%

For Project Y, the equation to find the IRR is:

$0 = -\$5{,}000 + \$2{,}300/(1+IRR) + \$1{,}800/(1+IRR)^2 + \$2{,}700/(1+IRR)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 16.60%

To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project Y from the cash flows from Project X. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

Crossover rate: $0 = \$400/(1+R) - \$100/(1+R)^2 - \$400/(1+R)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

R = 13.28%

The table below shows the NPV of each project for different required returns. Notice that Project Y always has a higher NPV for discount rates below 13.28 percent, and always has a lower NPV for discount rates above 13.28 percent.

R	$\$NPV_X$	$\$NPV_Y$
0%	1,700.00	1,800.00
5%	1,100.21	1,155.49
10%	587.53	607.06
15%	145.56	136.35
20%	(238.43)	(270.83)
25%	(574.40)	(625.60)

14. *a.* The equation for the NPV of the project is:

$$NPV = – \$28M + \$53M/1.1 – \$8M/1.1^2 = \$13,570,247.93$$

The NPV is greater than 0, so we would accept the project.

b. The equation for the IRR of the project is:

$$0 = –\$28M + \$53M/(1+IRR) – \$8M/(1+IRR)^2$$

From Descartes rule of signs, we know there are two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:

$$IRR = 72.75\%, –83.46\%$$

When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct, that is, both interest rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.

15. The profitability index is defined as the PV of the cash inflows divided by the PV of the cash outflows. The equation for the profitability index at a required return of 10 percent is:

$$PI = [\$3,200/1.1 + \$3,900/1.1^2 + \$2,600/1.1^3] / \$7,000 = 1.155$$

The equation for the profitability index at a required return of 15 percent is:

$$PI = [\$3,200/1.15 + \$3,900/1.15^2 + \$2,600/1.15^3] / \$7,000 = 1.063$$

The equation for the profitability index at a required return of 22 percent is:

$$PI = [\$3,200/1.22 + \$3,900/1.22^2 + \$2,600/1.22^3] / \$7,000 = 0.954$$

We would accept the project if the required return were 10 percent or 15 percent since the PI is greater than one. We would reject the project if the required return were 22 percent since the PI is less than one.

16. *a.* The profitability index is the PV of the future cash flows divided by the initial investment. The cash flows for both projects are an annuity, so:

$$PI_I = \$15,000(PVIFA_{10\%,3}) / \$30,000 = 1.243$$

$$PI_{II} = \$2,800(PVIFA_{10\%,3}) / \$5,000 = 1.393$$

The profitability index decision rule implies that we accept project II, since PI_{II} is greater than the PI_I.

b. The NPV of each project is:

$$NPV_I = – \$30,000 + \$15,000(PVIFA_{10\%,3}) = \$7,302.78$$

$$NPV_{II} = – \$5,000 + \$2,800(PVIFA_{10\%,3}) = \$1,963.19$$

The NPV decision rule implies accepting Project I, since the NPV_I is greater than the NPV_{II}.

c. Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitude of the cash flows for the two projects are of different scale. In this problem, project I is roughly 3 times as large as project II and produces a larger NPV, yet the profitability index criterion implies that project II is more acceptable.

17. *a.* The payback period for each project is:

A: 3 + ($135K/$370K) = 3.36 years

B: 2 + ($1K/$11K) = 2.09 years

The payback criterion implies accepting project B, because it pays back sooner than project A.

b. The discounted payback for each project is:

A: $\$15K/1.15 + \$30K/1.15^2 + \$30K/1.15^3 = \$55,453.28$
$\$370K/1.15^4 = \$211,548.70$

Discounted payback = 3 + ($210,000 – 55,453.28)/$211,548.70 = 3.73 years

B: $\$11K/1.15 + \$9K/1.15^2 = \$16,370.51$
$\$11K/1.15^3 = \$7,232.68$

Discounted payback = 2 + ($21,000 – 16,370.51)/$7,232.68 = 2.64 years

The discounted payback criterion implies accepting project B because it pays back sooner than A

c. The NPV for each project is:

A: $NPV = -\$210K + \$15K/1.15 + \$30K/1.15^2 + \$30K/1.15^3 + \$370K/1.15^4 = \$57,001.98$

B: $NPV = -\$21K + \$11K/1.15 + \$9K/1.15^2 + \$11K/1.15^3 + \$9K/1.15^4 = \$7,748.97$

NPV criterion implies we accept project A because project A has a higher NPV than project B.

d. The IRR for each project is:

A: $\$210K = \$15K/(1+IRR) + \$30K/(1+IRR)^2 + \$30K/(1+IRR)^3 + \$370K/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 22.97%

B: $\$21K = \$11K/(1+IRR) + \$9K/(1+IRR)^2 + \$11K/(1+IRR)^3 + \$9K/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 32.73%

IRR decision rule implies we accept project B because IRR for B is greater than IRR for A.

e. The profitability index for each project is:

A: $PI = (\$15K/1.15 + \$30K/1.15^2 + \$30K/1.15^3 + \$370K/1.15^4) / \$210K = 1.271$

B: $PI = (\$11K/1.15 + \$9K/1.15^2 + \$11K/1.15^3 + \$9K/1.15^4) / \$21K = 1.369$

Profitability index criterion implies accept project A because its PI is greater than project B's.

f. In this instance, the NPV and PI criterion imply that you should accept project A, while payback period, discounted payback and IRR imply that you should accept project B. The final decision should be based on the NPV since it does not have the ranking problem associated with the other capital budgeting techniques. Therefore, you should accept project A.

18. At a zero discount rate (and only at a zero discount rate), the cash flows can be added together across time. So, the NPV of the project at a zero percent required return is:

NPV = – $568,240 + 289,348 + 196,374 + 114,865 + 93,169 = $125,516

If the required return is infinite, future cash flows have no value. Even if the cash flow in one year is $1 trillion, at an infinite rate of interest, the value of this cash flow today is zero. So, if the future cash flows have no value today, the NPV of the project is simply the cash flow today, so at an infinite interest rate:

NPV = – $568,240

The interest rate that makes the NPV of a project equal to zero is the IRR. The equation for the IRR of this project is:

$0 = -\$568,240 + \$289,348/(1+IRR) + \$196,374/(1+IRR)^2 + \$114,865/(1+IRR)^3 + \$93,169/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 10.71%

<u>*Intermediate*</u>

19. Since the NPV index has the cost subtracted in the numerator, NPV index = PI – 1.

20. *a.* To have a payback equal to the project's life, given C is a constant cash flow for N years:

$C = \text{I/N}$

b. To have a positive NPV, $\text{I} < C\,(\text{PVIFA}_{R\%,\,N})$. Thus, $C > \text{I} / (\text{PVIFA}_{R\%,\,N})$.

c. Benefits = $C\,(\text{PVIFA}_{R\%,\,N}) = 2 \times \text{costs} = 2\text{I}$
$C = 2\text{I} / (\text{PVIFA}_{R\%,\,N})$

<u>*Challenge*</u>

21. Given the seven year payback, the worst case is that the payback occurs at the end of the seventh year. Thus, the worst-case:

$\text{NPV} = -\$483{,}000 + \$483{,}000/1.12^7 = -\$264{,}515.33$

The best case has infinite cash flows beyond the payback point. Thus, the best-case NPV is infinite.

22. The equation for the IRR of the project is:

$0 = -\$504 + \$2{,}862/(1 + \text{IRR}) - \$6{,}070/(1 + \text{IRR})^2 + \$5{,}700/(1 + \text{IRR})^3 - \$2{,}000/(1 + \text{IRR})^4$

Using Descartes rule of signs, from looking at the cash flows we know there are four IRRs for this project. Even with most computer spreadsheets, we have to do some trial and error. From trial and error, IRRs of 25%, 33.33%, 42.86%, and 66.67% are found.

We would accept the project when the NPV is greater than zero. See for yourself if that NPV is greater than zero for required returns between 25% and 33.33% or between 42.86% and 66.67%.

23. *a.* Here the cash inflows of the project go on forever, which is a perpetuity. Unlike ordinary perpetuity cash flows, the cash flows here grow at a constant rate forever, which is a growing perpetuity. If you remember back to the chapter on stock valuation, we presented a formula for valuing a stock with constant growth in dividends. This formula is actually the formula for a growing perpetuity, so we can use it here. The PV of the future cash flows from the project is:

PV of cash inflows = $C_1/(R - g)$
PV of cash inflows = $\$50{,}000/(.13 - .06) = \$714{,}285.71$

NPV is the PV of the outflows minus by the PV of the inflows, so the NPV is:

NPV of the project = $-\$780{,}000 + 714{,}285.71 = -\$65{,}714.29$

The NPV is negative, so we would reject the project.

b. Here we want to know the minimum growth rate in cash flows necessary to accept the project. The minimum growth rate is the growth rate at which we would have a zero NPV. The equation for a zero NPV, using the equation for the PV of a growing perpetuity is:

$0 = – \$780,000 + \$50,000/(.13 – g)$

Solving for g, we get:

$g = 6.59\%$

Calculator Solutions

7.

CFo	–$30,000
C01	$20,000
F01	1
C02	$14,000
F02	1
C03	$11,000
F03	1

IRR CPT
26.48%

8.

CFo	–$30,000
C01	$20,000
F01	1
C02	$14,000
F02	1
C03	$11,000
F03	1

I = 11%
NPV CPT
$7,423.84

CFo	–$30,000
C01	$20,000
F01	1
C02	$14,000
F02	1
C03	$11,000
F03	1

I = 30%
NPV CPT
–$1,324.53

9.

CFo	–$70,000
C01	$14,000
F01	9

I = 8%
NPV CPT
$17,456.43

CFo	–$70,000
C01	$14,000
F01	9

I = 16%
NPV CPT
–$5,508.39

CFo	–$70,000
C01	$14,000
F01	9

IRR CPT
13.70%

10.

CFo	–$8,000
C01	$3,200
F01	1
C02	$4,000
F02	1
C03	$6,100
F03	1

IRR CPT
26.83%

11.

CFo	–$8,000
C01	$3,200
F01	1
C02	$4,000
F02	1
C03	$6,100
F03	1

I = 0%
NPV CPT
$5,300

CFo	–$8,000
C01	$3,200
F01	1
C02	$4,000
F02	1
C03	$6,100
F03	1

I = 10%
NPV CPT
$2,797.90

CFo	–$8,000
C01	$3,200
F01	1
C02	$4,000
F02	1
C03	$6,100
F03	1

I = 20%
NPV CPT
$974.54

CFo	–$8,000
C01	$3,200
F01	1
C02	$4,000
F02	1
C03	$6,100
F03	1

I = 30%
NPV CPT
–$395.08

12. *Project A*

CFo	–$34,000
C01	$16,500
F01	1
C02	$14,000
F02	1
C03	$10,000
F03	1
C04	$6,000
F04	1

IRR CPT
16.60%

CFo	–$34,000
C01	$16,500
F01	1
C02	$14,000
F02	1
C03	$10,000
F03	1
C04	$6,000
F04	1

I = 11%
NPV CPT
$3,491.88

Project B

CFo	–$34,000	**CFo**	–$34,000
C01	$5,000	**C01**	$5,000
F01	1	**F01**	1
C02	$10,000	**C02**	$10,000
F02	1	**F02**	1
C03	$18,000	**C03**	$18,000
F03	1	**F03**	1
C04	$19,000	**C04**	$19,000
F04	1	**F04**	1
IRR CPT		I = 11%	
15.72%		NPV CPT	
		$4,298.06	

Crossover rate

CFo	$0
C01	$11,500
F01	1
C02	$4,000
F02	1
C03	–$8,000
F03	1
C04	–$13,000
F04	1
IRR CPT	
13.75%	

13. *Project X*

CFo	–$5,000	**CFo**	–$5,000	**CFo**	–$5,000
C01	$2,700	**C01**	$2,700	**C01**	$2,700
F01	1	**F01**	1	**F01**	1
C02	$1,700	**C02**	$1,700	**C02**	$1,700
F02	1	**F02**	1	**F02**	1
C03	$2,300	**C03**	$2,300	**C03**	$2,300
F03	1	**F03**	1	**F03**	1
I = 0%		I = 15%		I = 25%	
NPV CPT		NPV CPT		NPV CPT	
$1,700.00		$145.56		–$574.40	

Project Y

CFo	–$5,000	**CFo**	–$5,000	**CFo**	–$5,000
C01	$2,300	**C01**	$2,300	**C01**	$2,300
F01	1	**F01**	1	**F01**	1
C02	$1,800	**C02**	$1,800	**C02**	$1,800
F02	1	**F02**	1	**F02**	1
C03	$2,700	**C03**	$2,700	**C03**	$2,700
F03	1	**F03**	1	**F03**	1
I = 0%		I = 15%		I = 25%	
NPV CPT		NPV CPT		NPV CPT	
$1,800.00		$136.35		–$625.60	

Crossover rate

CFo	$0
C01	$400
F01	1
C02	–$100
F02	1
C03	–$400
F03	1

IRR CPT
13.28%

14.

CFo	–$28,000,000	**CFo**	–$28,000,000
C01	$53,000,000	**C01**	$53,000,000
F01	1	**F01**	1
C02	–$8,000,000	**C02**	–$8,000,000
F02	1	**F02**	1
I = 10%		IRR CPT	
NPV CPT		72.75%	
$13,570,247.93			

Financial calculators will only give you one IRR, even if there are multiple IRRs. Using trial and error, or a root solving calculator, the other IRR is –83.46%.

15.

CFo	$0	**CFo**	$0	**CFo**	$0
C01	$3,200	**C01**	$3,200	**C01**	$3,200
F01	1	**F01**	1	**F01**	1
C02	$3,900	**C02**	$3,900	**C02**	$3,900
F02	1	**F02**	1	**F02**	1
C03	$2,600	**C03**	$2,600	**C03**	$2,600
F03	1	**F03**	1	**F03**	1
I = 10%		I = 15%		I = 22%	
NPV CPT		NPV CPT		NPV CPT	
$8,085.65		$7,441.11		$6,675.05	

@10%: PI = $8,085.65 / $7,000 = 1.155
@15%: PI = $7,441.11 / $7,000 = 1.063
@22%: PI = $6,675.05 / $7,000 = 0.954

16. *Project I*

CFo	$0	**CFo**	–$30,000
C01	$15,000	**C01**	$15,000
F01	3	**F01**	3
I = 10%		I = 10%	
NPV CPT		NPV CPT	
$37,302.78		$7,302.78	

PI = $37,302.78 / $30,000 = 1.243

Project II

CFo	$0	**CFo**	–$5,000
C01	$2,800	**C01**	$2,800
F01	3	**F01**	3
I = 10%		I = 10%	
NPV CPT		NPV CPT	
$6,963.19		$1,963.19	

PI = $6,963.19 / $5,000 = 1.393

17.

CF(A)

c.		*d.*		*e.*	
Cfo	–$210,000	**CFo**	–$210,000	**CFo**	$0
C01	$15,000	**C01**	$15,000	**C01**	$15,000
F01	1	**F01**	1	**F01**	1
C02	$30,000	**C02**	$30,000	**C02**	$30,000
F02	2	**F02**	2	**F02**	2
C03	$370,000	**C03**	$370,000	**C03**	$370,000
F03	1	**F03**	1	**F03**	1
I = 15%		IRR CPT		I = 15%	
NPV CPT		22.97%		NPV CPT	
$57,001.98				$267,001.98	

PI = $267,001.98 / $210,000 = 1.271

CF(B)

c.		*d.*		*e.*	
CFo	–$21,000	**CFo**	–$21,000	**CFo**	$0
C01	$11,000	**C01**	$11,000	**C01**	$11,000
F01	1	**F01**	1	**F01**	1
C02	$9,000	**C02**	$9,000	**C02**	$9,000
F02	1	**F02**	1	**F02**	1
C03	$11,000	**C03**	$11,000	**C03**	$11,000
F03	1	**F03**	1	**F03**	1
C04	$9,000	**C04**	$9,000	**C04**	$9,000
F04	1	**F04**	1	**F04**	1
I = 15%		IRR CPT		I = 15%	
NPV CPT		32.73%		NPV CPT	
$7,748.97				$28,748.97	

PI = $28,748. 97 / $21,000 = 1.369

f. In this instance, the NPV and PI criterion imply that you should accept project A, while payback period, discounted payback and IRR imply that you should accept project B. The final decision should be based on the NPV since it does not have the ranking problem associated with the other capital budgeting techniques. Therefore, you should accept project A.

18.

CFo	–$568,240
C01	$289,348
F01	1
C02	$196,374
F02	1
C03	$114,865
F03	1
C04	$93,169
F04	1

I = 0%
NPV CPT
$125,516.00

CFo	–$568,240
C01	$289,348
F01	1
C02	$196,374
F02	1
C03	$114,865
F03	1
C04	$93,169
F04	1

IRR CPT
10.71%