

AAE374 Discussion Section
Review

1. Production Functions:

Cobb-Douglas production function $Y = F(K, L) = K^\alpha L^{1-\alpha}$ where α is capital's share of income, which is assumed to have a value between 0 and 1.

$$\text{Output per-capita: } y = \frac{Y}{L} = \left(\frac{1}{L} K\right)^\alpha \left(\frac{1}{L} L\right)^{1-\alpha} = \left(\frac{K}{L}\right)^\alpha = k^\alpha.$$

a. Returns to Scale

Constant Returns to Scale: Increase all inputs by the same factor; output will increase by the same factor (e.g., double the inputs => output is doubled).

$$F(zK, zL) = (zK)^\alpha (zL)^{1-\alpha} = zK^\alpha L^{1-\alpha} = zF(K, L) = zY \text{ (CRS).}$$

A more general form is $Y = F(K, L) = K^\alpha L^\beta$ and CRS if $\alpha + \beta = 1$, DRS if $\alpha + \beta < 1$ and IRS if $\alpha + \beta > 1$.

b. Marginal Returns

The marginal product of a particular input is the extra output produced when one more unit of the input is used in production, maintaining every other input at their initial levels. Diminishing marginal product means that as you increase only one input (leaving all other inputs at the same level) output will increase but at a decreasing rate.

2. Trade Theory:

a. Heckscher-Ohlin 2-2-2 (PS1)

(A1) Factor endowments: differences in relative factor endowments

(A2) Identical technologies across countries

(A3) Factor intensities

(A4) Homogenous consumer tastes

Differences in relative prices form a basis for trade

Heckscher-Ohlin Theorem: country with abundant capital will be able to produce relatively more of the capital intensive product

Factor Price Equalization Theorem: As trade takes place between two countries, prices adjust until relative prices are equalized

Growth Policy: Free trade (no government intervention) seems to be the best strategy for growth.

3. Economic Growth

a. Solow & Unconditional Convergence (PS2)

Consider a group of economies with a production function specified as in the Solow Model.

$$Y = K^\alpha L^{1-\alpha}$$

$$y = k^\alpha$$

We know that the growth rate of the stock of capital per capita will be:

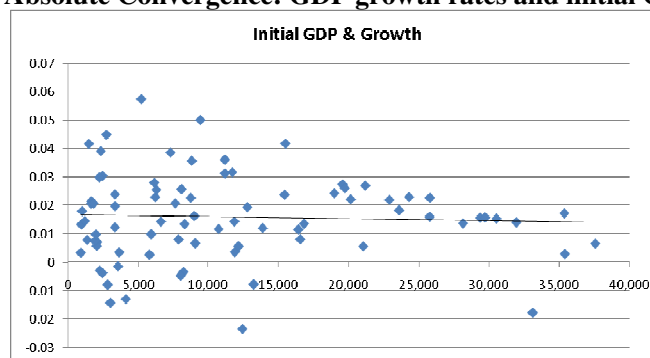
$$\Delta k = sAk^\alpha - \delta k$$

Suppose countries have access to the same technology, and have the same savings rate, depreciation rate, and population growth, then they will converge to the same steady state. This equilibrium (steady state) will be characterized by the same capital – labor ratio, output per capita, and growth rate. Recall that in steady state the growth rate of all variables (in per capita terms) is equal to zero.

Convergence occurs because the poorer countries grow at a higher rate than the rich countries. This occurs because of diminishing returns to capital, which slows growth in rich countries and speeds it up in poor countries (Solow vs. Harrod-Domar).

Growth policy: Even if the countries start with a different stock of capital per capita, in the long run they will converge to the same equilibrium: zero growth in steady state and same capital labor ratio. South will catch up with the North.

Figure 1. Absolute Convergence: GDP growth rates and initial GDP (1965)

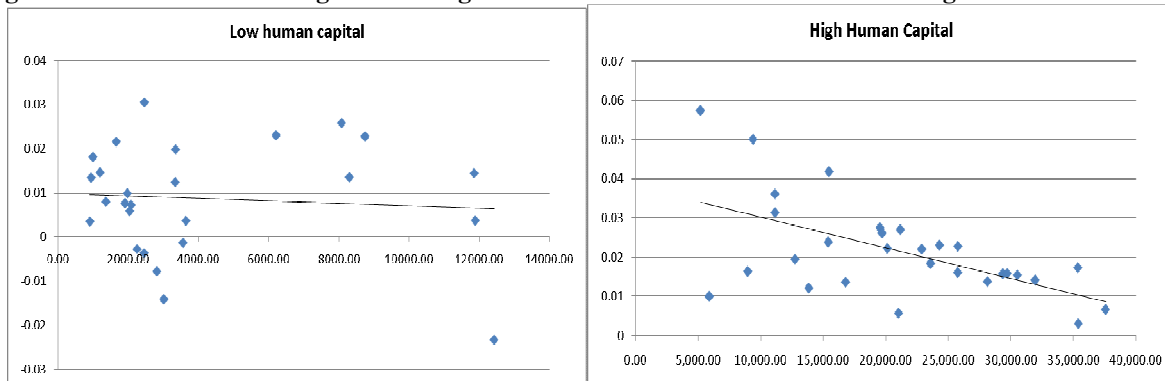


b. Savings, Human Capital & Conditional Convergence (PS3)

We next saw that countries with different savings rates (even with the same technologies, depreciation rates and population growth rates) converge to different steady state levels (e.g. k_1^* vs. k_2^*). This is the essence of conditional convergence, which can explain why countries have not converged. As we continue to change technologies, depreciation rates, and population growth rates, we can predict that countries will reach different steady states based on these criteria.

Human capital is another factor that supports the conditional convergence hypothesis. We saw from the models in class that countries with higher human capital will have different steady states of output and capital per capita. This occurs because, conditional on human capital, initially poorer countries should grow faster.

Figure 2. Conditional Convergence: GDP growth rates and initial GDP in low and high education countries



Growth policy: Improving human capital or increasing savings rates may lead to better steady states.

4. Technology, Externalities & Scale Economies

We were also able to calculate the impact of Total Factor Productivity on growth rates across countries and learned that TFP is one of the main mechanisms through which growth occurs. Consider the production of bananas and computers

$$\text{Bananas : } Y_b(t) = A_b [K_b(t)]^\alpha [L_b(t)]^{1-\alpha}$$

$$\text{Computers : } Y_c(t) = A_c(t) [K_c(t)]^\alpha [L_c(t)]^{1-\alpha}$$

Growth is a story about accumulation of technology, spillovers, externalities, and coordination.

Growth Policy: Governments can play a role by providing incentives for technological change, and solving coordination problems

5. Trade Theory + Growth + Economies of Scale (PS4)

a. No IRS:

Convergence from both convergence of capital/worker (Solow) which over time also causes convergence in H-O because as capital/worker converges the basis for comparative advantage shrinks, as seen in the % of capital in the computer sector. Convergence in capital/labor ratio, patterns of specialization, and welfare.

Growth Policy: Free market rules! (again?)

b. IRS:

Trade leads to specialization, but as the north begins to invest more and more in the computer sector, it will reach the threshold for economies of scale. When this happens, the North's comparative advantage in computers increases (H-O) the relatively slower growth in the South

keeps the South from “catching-up” in computer technology, so over time their comparative advantage goes away from computers (less % of Capital in computer sector graph).

Thus, trade worsens the “core-periphery” structure and, in terms of specialization, the South eventually “deindustrializes” its computer sector. The South becomes relatively worse off while the North becomes relatively better off. Differences in income/capita growth rate, income/capita levels and utility levels persist and get bigger.

Growth Policy: Free market does not rule!

c. Autarchy:

The South would have faster income growth, higher utility and income, and would specialize less in bananas in autarchy.

Some costs in the first periods as it can not take advantage of its comparative advantage (and that of the North in computers), but over time it will invest in the computer sector and become a computer “super star”.

Growth Policy: Does autarchy rule?

d. Coordination failure?

No single capitalist alone in the South would find it rational or profitable to invest a large amount in a computer factory, but if everyone got together and did so the country would be better off. This problem arises because of the external economies of scale (as opposed to internal economies of scale) which occur at an industry wide level.

The market maximizes individual welfare and firm profits in each time period, but this may not be socially desirable.

e. Tariffs

Over the first five years a tariff of 10% causes the country to produce a higher proportion of manufactured products (computers), a lower production of agricultural output (bananas). Hence it limits the scope for specialization.

Over the long run the tariff will make the country better off than either free trade or autarchy. This is because it protects the infant computer sector in the South and leads to a higher investment in computers than it would have with either free trade or autarchy (at least after a few years). In such case, the South will accumulate capital in the computer sector and will eventually reach the threshold where scale economies kick in, and thus would enjoy from higher productivity.

Growth Policy: Do tariffs rule?

Now we have a complex way of modeling and analyzing growth and the causes of growth. Keep using your advanced models and statistical methods to combine these theories with the empirics!