

IS/AAE374 Discussion Section
PS3: Total Factor Productivity and Factor Decomposition

Factor decomposition (A: Total Factor Productivity)

Thus far in the course, we have mainly analyzed growth through changes in inputs, including capital and labor. We will also consider technological improvements as an important factor in growth. Countries often discover and adopt new technologies that have higher output for any given level in inputs (represented by the A term in the Cobb-Douglas functions below) and the adoption of advanced technologies is an important factor in growth and the potential for poor countries to converge. Growth rates are comprised of a combination of factors, including changes in capital, human capital, and technological change. Factor decomposition enables us to analyze the impact of these different changes on overall output. We will now consider the two and three-factor examples:

(Two factors)

$$Y = AK^\alpha L^{1-\alpha}$$

$$Y/L = A(K/L)^\alpha \quad (y = Ak^\alpha)$$

Take a log, $\log y = \log A + \alpha \log k$

Take a derivative, $\frac{\Delta y}{y} = \frac{\Delta A}{A} + \alpha \frac{\Delta k}{k}$

$$\hat{y} = \hat{A} + \alpha \hat{k}$$

where the hat terms are growth rates of the respective terms. A similar process provides us with the formula for the three factor case:

(Three factors)

$$\hat{y} = \hat{A} + \alpha \hat{k} + \beta \hat{s}$$

This formula can be modified into two useful formulas:

$$\hat{A} = \hat{y} - \alpha \hat{k} - \beta \hat{s}$$

$$1 = \hat{A} / \hat{y} + \alpha \hat{k} / \hat{y} + \beta \hat{s} / \hat{y}$$

The first formula shows how we can calculate the change in total factor productivity (which is not easily observed) as a function of growth rates that we can easily calculate from the data. The second formula divides the three factor formula above by y-hat in order to determine the percentage of the total change in output comprised of each of the three terms (here growth in TFP, capital, and human capital).

Please see the associated excel file (pset3) from the website for an example of these calculations.