AAE 730 - Problem Set 2

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1. There are two individuals. Each individual i earns exogenous random income y_i which comes from distribution f_i. Let $\rho(y_1, y_2)$ be individual 1's consumption. Individual 2 consumes $y_1 + y_2 - \rho$. The individuals have von Neumann Morgenstern utility and are strictly risk averse. Risk is fully pooled between the two.

   (a) Show that $\rho(y_1, y_2) = \bar{\rho}(y_1 + y_2)$. In other words, show that the optimal consumption of 1 depends only on total income.

   (b) Assume CARA utility with risk aversion parameters σ_1 and σ_2. Show that $\bar{\rho}(y_1 + y_2)$ is linear. Calculate its slope as a function of σ_1 and σ_2. How does individual 1's weight affect $\bar{\rho}$

2. There is one household with no access to savings or borrowing. There are two states of nature, l and h, and consumption (which equals income for this household) is such that $c_l < c_h$. The probability of h is p. The household has von Neumann Morgenstern utility and is strictly risk averse.

   (a) In this problem you will show that the marginal value to the household of actuarially fair insurance is proportional to the coefficient of relative risk aversion and the proportional drop in consumption in state l. Note that with actuarially fair insurance, if ϵ is the payment to the household in l then the household must pay out $\frac{\epsilon(1-p)}{p}$ in h. Let v be the price of access to this (otherwise actuarially fair) insurance which is paid only in h (to simplify the algebra). Household expected utility is thus

$$pu(c_h - \frac{\epsilon(1-p)}{p} - \epsilon v) + (1-p)u(c_l + \epsilon).$$

Show that the v at which this household is indifferent to purchasing insurance is approximately proportional to $\gamma \frac{c_h - c_l}{c_h}$ where γ is the coefficient of relative risk aversion.

(b) Now suppose that the degree of risk faced by the individual is endogenous. In each state, income can be generated with the utility cost of θ_s where $\theta_h = 1 < \theta_l$. This means that in the good state it is easier to generate income, although income need not be different in the two states. Assuming CRRA utility, the household's problem in each state s is $\max_{c_s} \frac{c_s^{1-\gamma}}{1-\gamma} - \theta_s c_s$. Show that the proportional consumption drop $\frac{c_h - c_l}{c_h}$ is decreasing in γ.

(c) Gertler and Gruber (AER, 2002), using data from Indonesia, find that consumption falls by a lot when a household faces income shocks. They argue that there may be large welfare benefits from consumption insurance. Similar arguments have been made in the opposite direction (small fluctuations in consumption imply that the benefits from insurance are low). Interpret these arguments in light of the analysis you carried out in parts a and b.