Exam 1, AAE636, 2010

1. (5) Describe (in words mainly) what the standard F-statistic for a regression is trying to evaluate.

Answer: The regression F evaluates the restriction that all of the coefficients of the model except the intercept coefficient are jointly equal to zero.

2. (5) Suppose you have a random variable X with mean μ and variance σ^2. What is the expected value and variance of $Z2 = X*5+7$.

Answer: E[Z2] = 5*E[X] + 7 = 5 μ + 7.

Var[Z2] = $25\sigma^2$

3. Suppose you have the following information about a regression result:

$$(X'X)^{-1} = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 2 & 3 & 3 \\ 2 & 3 & 3 & 4 \end{bmatrix} \text{ and } X'Y = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} \text{ and } n=64$$

a) (10) What are the OLS parameter estimates from this model? Present this result in appropriate matrix or vector form.

Answer:

$$\widehat{\beta} = (X'X)^{-1}X'Y = \begin{bmatrix} 3+4+2+2 \\ 6+4+2+3 \\ 6+4+3+3 \\ 6+6+3+4 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 16 \\ 19 \end{bmatrix}$$

b) (15) Suppose the t-value for the second parameter in the parameter vector is equal to 2.5. What are the other t-ratios (Just show the numerical formulae)?

Answer:

$$\frac{\widehat{\beta}_2 - \beta_2}{s_{\widehat{\beta}_2}} = \frac{15-0}{\sqrt{s^2 a_{22}}} = \frac{15}{\sqrt{s^2 2}} = 2.5 \Rightarrow s^2 = 18$$

Which implies that

$$\frac{\widehat{\beta}_1}{s_{\widehat{\beta}_1}} = \frac{11}{\sqrt{18*1}} = \frac{11}{\sqrt{18}}; \quad \frac{\widehat{\beta}_3}{s_{\widehat{\beta}_3}} = \frac{16}{\sqrt{18*3}} = \frac{16}{\sqrt{54}}; \quad \frac{\widehat{\beta}_4}{s_{\widehat{\beta}_4}} = \frac{19}{\sqrt{18*4}} = \frac{19}{\sqrt{72}}$$

c) (5) Suppose the t-ratio for the second parameter in the parameter vector is equal to 2.5. What is the covariance between the second and third parameter estimates?

Answer is Easy: $\text{cov}(\widehat{\beta}_2, \widehat{\beta}_3) = s^2 a_{ij} = 18*2 = 36$

d) (10) The critical t-value at the 95% level is 2.0. What is the 95% confidence interval for the second parameter estimate?

Answer:

$$\widehat{\beta}_2 \pm t_{.025,60} * \sqrt{s^2 a_{22}} = 15 \pm 2*6 = (3, 27)$$

e) (10) Calculate the t-value (or F-value) for the hypothesis test that second parameter estimate is equal to the third parameter estimate (reduce and report the final numerical formula: do not calculate the final result)?

Noting that $V = s^2 (X'X)^{-1}$ and that s^2=18

$$\frac{\widehat{\beta}_2 - \widehat{\beta}_3 - (\beta_2 - \beta_3)}{s_{\widehat{\beta}_2 - \widehat{\beta}_3}} = \frac{15-16-0}{\sqrt{A'VA}} = \frac{15-16}{\sqrt{18 * A'(X'X)^{-1} A}} \text{ where } A' = [0 \quad 1 \quad -1 \quad 0]$$

Dropping the rows and columns in the (X'X)-1 matrix that are associated with the zeros in the A' matrix, we can write:

$$= \frac{15-16}{\left\{18*[1 \quad -1]\begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}^{0.5}} = \frac{-1}{\sqrt{18}}$$

f) (10) What is the sum of the squared errors of the model?

$$s^2 = \frac{e'e}{n-k} = \frac{e'e}{64-4} = 18 \Rightarrow e'e = 18*60 = 1080$$

4. Suppose you are estimating the following model:

$$Z = a_1 + a_2 M2 + a_3 M3 + a_4 \ln(M2 * M3) + 2a_5 (M2 / M3)^{0.5} + e$$

The variance-covariance matrix for the parameters is V, the a's are parameter estimates, M2 and M3 are independent variables, e is the error term, and Z is the dependent variable.

a) (10) The model's use of a log transformation of M2*M3 implies a restriction. What is that restriction and what would you do the free up the model to avoid such a restriction?

Answer:

$a_4 \ln(M2 * M3) = a_4 \ln(M2) + a_4 \ln(M3)$ Thus, the implied restriction is that the ln(M2) is required by construction to have the same estimated impact on Z that ln(M3) has.

An unrestricted version of the original model needs an extra and unique parameter on either ln(M2) or ln(M3):

$$Z = a_1 + a_2 M2 + a_3 M3 + a_4 \ln(M2) + a_5 \ln(M3) + 2a_6 (M2 / M3)^{0.5}$$

b) (10) Show how the marginal effect of M2 on Z would be calculated at the mean of each data point (i.e. $\partial Z / \partial M2$ =T) .

Answer:

$$\partial Z / \partial M2 = \mathrm{T} = a_2 + a_4 \frac{1}{\overline{M2}} + a_5 \frac{1}{(\overline{M2} * \overline{M3})^{0.5}}$$

c) (15) Set up and show how the t-ratio is calculated for the elasticity measure of M2 on Z (i.e., show how to evaluate statistically $\frac{\partial Z}{\partial M2}\frac{\overline{M2}}{\overline{Z}}$).

Answer:

The elasticity measured at the means of the data is:

$$\frac{\partial Z}{\partial M2}\frac{\overline{\mathrm{M2}}}{\overline{Z}} = \left[a_2 + a_4 \frac{1}{\overline{M2}} + a_5 \frac{1}{(\overline{M2} * \overline{M3})^{0.5}} \right]\left[\frac{\overline{\mathrm{M2}}}{\overline{Z}}\right] = \left\{ a_2 \left[\frac{\overline{\mathrm{M2}}}{\overline{Z}}\right] + a_4 \frac{1}{\overline{Z}} + a_5 \frac{1}{\overline{Z}}\left[\frac{\overline{M2}}{\overline{M3}}\right]^{0.5} \right\}$$

V is a 5x5 symmetric matrix of variances and covariances of the regression parameters.

We only need to define the A (the gradient vector) for $\frac{\partial Z}{\partial M2}\frac{\overline{M2}}{\overline{Z}}$ to get the standard error on $\frac{\partial Z}{\partial M2}\frac{\overline{M2}}{\overline{Z}}$. Using the elasticity measure in braces, the gradient vector is:

$$A' = \left[0 \quad \left[\frac{\overline{\mathrm{M2}}}{\overline{Z}}\right] \quad 0 \quad \frac{1}{\overline{Z}} \quad \frac{1}{\overline{Z}}\left[\frac{\overline{M2}}{\overline{M3}}\right]^{0.5} \right]$$

The t-ratio for evaluating the marginal effects of T is:

$$t = \frac{\left\{ a_2 \left[\frac{\overline{\mathrm{M2}}}{\overline{Z}}\right] + a_4 \frac{1}{\overline{Z}} + a_5 \frac{1}{\overline{Z}}\left[\frac{\overline{M2}}{\overline{M3}}\right]^{0.5} \right\}}{\left[A'VA\right]^{0.5}}$$