HW #4

#1 $Y = 5 - 3X$ show $\sqrt{R^2} = r = -1$

$$r(X,Y) = \frac{Cov(X,Y)}{[Var(X)\cdot Var(Y)]^{.5}} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

$$Var(Y) = 9\,Var(X) \Rightarrow \sigma_Y = 3\sigma_X$$

$$Cov(X,Y) = E[X(5-3X)] - E(X)E(5-3X)$$

$$= \cancel{5E(X)} - \cancel{5E(X)} - 3E(X^2) + 3E(X)E(X)$$

$$= -3\,Var(X) = -3\sigma_X^2$$

$$r = \frac{-3\sigma_X^2}{3\sigma_X\sigma_X} = -1$$

#2 $Y_i = \hat{Y}_i + e_i$

$$\frac{\Sigma Y_i}{n} = \frac{\Sigma \hat{Y}_i}{n} + \frac{\Sigma e_i}{n}$$

$$\bar{Y} = \bar{\hat{Y}}$$

#3 Let $T_i = \frac{Y_i - \bar{Y}}{S_Y}$

$Z_i = \frac{X_i - \bar{X}}{S_X}$

$T_i = \beta Z_i + e_i$

minimizing $\Sigma e_i^2 \Rightarrow \beta = \frac{\Sigma z_i t_i}{\Sigma t_i^2}$

$$\beta = \frac{\frac{\Sigma x_i y_i}{S_x S_y}}{\frac{\Sigma x_i^2}{S_x^2}} = \frac{\Sigma x_i y_i}{\Sigma x_i} \frac{S_x}{S_y} = \frac{\Sigma x_i y_i \sqrt{\Sigma x_i^2}}{\Sigma x_i \sqrt{\Sigma y_i^2}}$$

$$= \frac{\Sigma x_i y_i}{\sqrt{\Sigma x_i^2 \Sigma y_i^2}} = r$$

#4 Not too hard

#5. Note that $\frac{X'(A+A')X}{2}$ is a scalar

$$\frac{X'AX}{2} = \frac{X'A'X}{2}$$

#6. Not graded