11.1 Introduction

Consider an economy facing the allocation of n goods, involving M firms and N households. The i-th household has preferences represented by a utility function $U_i(\mathbf{x}_i)$, where $\mathbf{x}_i = (x_{i1}, \ldots, x_{in}) \in \mathbf{R}^n$ is a vector of the goods consumed by the i-th household, $\mathbf{x}_i \geq 0$, $i = 1, \ldots, N$. We assume throughout that $U_i(\mathbf{x}_i)$ is a quasi-concave function of $\mathbf{x}_i$. Denote by $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$ the vector of all consumption decisions.

We assume that the consumer goods are private goods. In other words, $\mathbf{x}_i$ is a vector of goods consumed only by the i-th household. This means of the benefits generated by the goods x_i are captured entirely by the i-th household. This rules out the existence of externalities and/or public goods for the consumption goods $\mathbf{x}$. The case of external effects or public goods in consumption activities will be discussed later.

The j-th firm chooses netputs $\mathbf{y}_j = (y_{j1}, \ldots, y_{jn}) \in \mathbf{R}^n$, where outputs are positive and inputs negative, $j = 1, \ldots, M$. Denote by $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_M)$ the vector of all production decisions. The technological feasibility of producing $\mathbf{y}$ is denoted by the feasible set $\mathbf{Y}$, with $\mathbf{y} \in \mathbf{Y} \subset \mathbf{R}^{Mn}$. This allows the production goods (denoted by the netputs $\mathbf{y} \in \mathbf{Y}$) to include private goods, public goods, as well as externalities. Indeed, depending on the nature of the feasible set $\mathbf{Y}$, the netput $\mathbf{y}_j$ associated with the j-th firm can affect the production possibility of any or all of the M firms.

Definition: A feasible allocation is an allocation $\mathbf{z} = (\mathbf{x}, \mathbf{y})$ satisfying

$$\mathbf{x}_i \geq 0,\ i = 1, \ldots, N, \tag{1a}$$

$$\mathbf{y} \in \mathbf{Y}, \tag{1b}$$

and

$$\textstyle\sum_i \mathbf{x}_i \leq \sum_j \mathbf{y}_j. \tag{1c}$$

Expression (1c) is simply a quantity balance: it states that aggregate consumption cannot exceed aggregate production.

Throughout, we will make the following two assumptions.

Assumption A1: A feasible allocation exists.

Assumption A2: There exists a feasible allocation such that $\sum_j \mathbf{y}_j > 0$.

A1 says there must exist some allocations to choose from and A2 states that it must be feasible to produce a positive aggregate quantity of all commodities. Note that, from (1a), the lower bound for aggregate consumption is 0. In this context, A2 implies that there exists a feasible allocation such that aggregate production of each commodity is higher than its aggregate consumption. Among all the feasible allocations, we would like to identify the ones that seem more desirable from a social viewpoint.

11.2 Pareto Efficiency

Definition: An allocation is **Pareto efficient** if it is feasible and there is no other feasible allocation that can make one household better off without making any other worse off.

Let $\mathbf{z}^* = (\mathbf{x}_1^*, \ldots, \mathbf{x}_N^*; \mathbf{y}_1^*, \ldots, \mathbf{y}_M^*) = (\mathbf{x}^*, \mathbf{y}^*)$ be a Pareto efficient allocation. It follows that there does not exist any other feasible allocation $\mathbf{z} = (\mathbf{x}, \mathbf{y})$ such that $U_i(\mathbf{x}_i) \geq U_i(\mathbf{x}_i^*)$ for all households $i = 1, \ldots, N$, and $U_{i'}(\mathbf{x}_{i'}) > U_{i'}(\mathbf{x}_{i'}^*)$ for some household i'.

Alternatively, Pareto inefficiency means that it is possible to make at least one household better off without making any other worth off. Intuitively, Pareto inefficiency seems quite undesirable from a social viewpoint. It states that some resources are being wasted because they could be used better so as to improve the welfare of households in general.

Using the Pareto criterion, welfare levels are expressed entirely in terms of household welfare. This does not mean that firm welfare is irrelevant. Rather, it means that that firm welfare is relevant, but only to the extent that production activities contribute to increasing consumer welfare. In this context, production activities are not an end; rather they are a means of generating goods that will eventually be consumed by households.

It would be very useful to develop insights into the identification of Pareto efficient/inefficient allocations. This would help address two important issues:

- Identifying Pareto inefficient allocations can help discover which existing decision rules are inefficient.
- By identifying Pareto efficient allocations, we can gain some insights into improved decision rules that can help enhance household and social welfare.

Note: The efficiency of decision rules apply at all levels: the micro level (e.g., firm or household) as well as the aggregate level (e.g., government policy, trade policy).

11.2.1 Identifying efficient allocations

To identify Pareto efficient allocations, we need to rely on some household welfare measurement. It will be convenient for us to rely on the benefit function. As we have seen earlier, the benefit function for the i-th household is

$$B_i(\mathbf{x}_i, U_i) = \underset{\beta}{Max}\ \{\beta : U_i(\mathbf{x}_i - \beta\mathbf{g}) \geq U_i, (\mathbf{x}_i - \beta\mathbf{g}) \geq \mathbf{0}\}, \tag{2}$$

where $\mathbf{g} \in \mathbf{R}^n$ is reference bundle satisfying $\mathbf{g} \geq 0$, $\mathbf{g} \neq 0$. $B_i(\mathbf{x}_i, U_i)$ measures the number of units of bundle $\mathbf{g}$ the i-th household is willing to trade starting from utility U_i to obtain $\mathbf{x}_i$. As such, it is a measurement of the i-th household's benefit associated with consuming the private goods $\mathbf{x}_i$. If the value of one unit of the bundle $\mathbf{g}$ is worth \$1, then $B_i(\mathbf{x}_i, U_i)$ is the i-th household willingness to pay starting from utility U_i to obtain $\mathbf{x}_i$. We have shown earlier that, under the assumption that $U_i(\mathbf{x}_i)$ is a quasi-concave function, the benefit function $B_i(\mathbf{x}_i, U_i)$ is concave in $\mathbf{x}_i$, and non-increasing in U_i.

For a given bundle $\mathbf{g}$, we will focus our attention on the aggregate benefit function defined as the sum of the individual benefit functions across all households:

$$B(\mathbf{x}, \mathbf{U}) = \sum_i B_i(\mathbf{x}_i, U_i), \tag{3}$$

where $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$ and $\mathbf{U} = (U_1, \ldots, U_N)$. From the properties of $B_i(\mathbf{x}_i, U_i)$, it follows that the aggregate benefit function $B(\mathbf{x}, \mathbf{U})$ is concave in $\mathbf{x}$, and non-increasing in U.

Intuitively, we expect that efficient allocations will maximize aggregate benefit. This suggest considering the following allocations.

Definition: An allocation **z** is maximal if it maximizes the aggregate benefit function $B(\mathbf{x}, \mathbf{U})$ in (3) subject to the feasibility conditions (1a)-(1c). Thus a maximal allocation is a solution **z***(**U**) to following optimization problem

$$W(\mathbf{U}) = \underset{\mathbf{z}}{Max} \ \{B(\mathbf{x}, \mathbf{U}): \text{ equations (1a) - (1c)}\}, \quad (4)$$

where W(**U**) is the indirect objective function.

The indirect objective function $W(\mathbf{U}) = B(\mathbf{x}^*(\mathbf{U}), \mathbf{U})$ is the largest feasible aggregate benefit that can be obtained to reach utility levels $\mathbf{U} = (U_1, \ldots, U_N)$. There are three possibilities.

- If the largest feasible aggregate benefit W(**U**) is negative, this means that reaching utility **U** requires a negative aggregate quantity of the bundle **g**. This implies that reaching utility level $\mathbf{U} = (U_1, \ldots, U_N)$ is not feasible.
- If the largest feasible aggregate benefit W(**U**) is positive, this means that reaching utility level **U** can be attained using a positive aggregate quantity of the bundle **g**. This implies that reaching utility level $\mathbf{U} = (U_1, \ldots, U_N)$ is feasible. In this context, W(**U**) can be interpreted as a measure of aggregate surplus (expressed in terms of quantity of the bundle **g**). When $W(\mathbf{U}) > 0$, this surplus can in general be redistributed among the N households.
- If the largest feasible aggregate benefit W(**U**) is equal to zero, this corresponds to a feasible allocation where aggregate benefit have been maximized but there is no aggregate surplus to redistribute.

This suggests considering the following allocations.

Definition: An allocation **z** is zero-maximal if it is maximal and if **U** is chosen such that

$$W(\mathbf{U}) = 0. \quad (5)$$

Following our discussion, a zero-maximal allocation is feasible, maximizes aggregate benefit, and corresponds to a situation where there is no aggregate surplus to redistribute. We show next that, under some regularity conditions, a zero-maximal allocation identifies a Pareto efficient allocation.

Proposition 1: Assume that there is at least one household that is non-satiated in **g** (i.e., with $U_i(\mathbf{x}_i + \alpha\mathbf{g})$ being strictly increasing in α for some i). If the allocation **z*** is Pareto efficient, then it is zero maximal.

Proof: The allocation **z*** is feasible. It follows from (2) that $B_i(\mathbf{x}_i, U_i) \geq 0$ for all $i = 1, \ldots, n$. Assume that $B_{i'} > 0$ for some household i'. This implies that $B > 0$. But the aggregate surplus B can be redistributed to the household that is non-satiated in **g**. This would make this household better off without making any other worse off, thus contradicting Pareto efficiency. It follows that Pareto efficiency implies zero maximality.

Proposition 2: If **z*** is zero maximal, then it is Pareto efficient compared to all feasible allocations satisfying $\mathbf{x}_i > 0$ for all i.

Proof: Assume that there is a feasible allocation **z** satisfying $\mathbf{x}_i > 0$, where $U_i(\mathbf{x}_i) \geq U_i(\mathbf{x}_i^*)$ for all *i*, but with $U_{i'}(\mathbf{x}_{i'}) > U_{i'}(\mathbf{x}_{i'}^*)$ for some household *i*'. This means that **z*** is not Pareto efficient. It follows from (2) that $B_i(\mathbf{x}_i, U_i(\mathbf{x}_i^*)) \geq 0$ for all *i*, and $B_{i'}(\mathbf{x}_{i'}, U_{i'}(\mathbf{x}_{i'}^*)) > 0$. This implies that **z** cannot be zero-maximal. Thus, zero maximality implies Pareto efficiency.

Propositions 1 and 2 establish close relationships between Pareto efficiency and zero-maximality.

They state the equivalence of two concepts when the following two conditions hold

1. Non-satiation in **g** for at least one household.
2. Any allocation that does not satisfy $\mathbf{x}_i > 0$ for all *i* is not Pareto efficient.

These two conditions appear reasonable. Condition 1 is intuitive. Condition 2 states that, under Pareto efficiency, the consumption of any commodity must be positive (e.g., this can be expected to hold if household preferences are non-satiated). In the discussion presented below, we will assume that both conditions are satisfied. Thus, we proceed with our analysis assuming that Pareto efficiency and zero-maximality are equivalent. Note that such equivalence holds without imposing any restriction on the production technology (represented by the feasible set **Y**).

Under zero-maximality, the distributable aggregate surplus $W(\mathbf{U})$ is zero. This provides a simple and intuitive interpretation of Pareto efficiency. An allocation is Pareto efficient when:

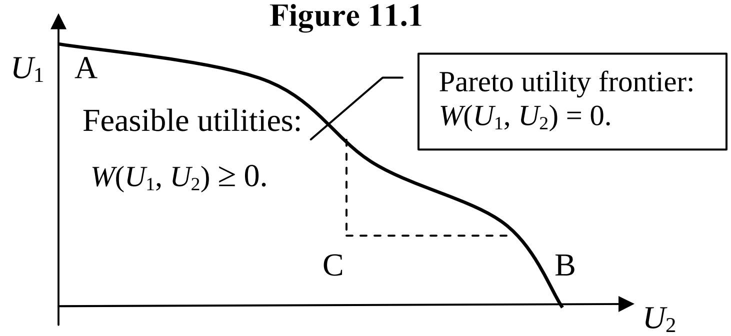
- First, resource allocation $\mathbf{z} = (\mathbf{x}, \mathbf{y})$ is chosen such as to maximize aggregate benefit $B(\mathbf{x}, \mathbf{U})$, conditional on $\mathbf{U} = (U_1, \ldots, U_N)$ (as stated in (4)).
- Second, the level of utilities $\mathbf{U} = (U_1, \ldots, U_N)$ is chosen such that the associated distributable surplus $W(\mathbf{U})$ is entirely redistributed to the *N* households.

We have seen above that a positive distributable surplus $W(\mathbf{U}) \geq 0$ corresponds to a feasible allocation. In other words, the set of feasible utilities $\mathbf{U} = (U_1, \ldots, U_N)$ is given by $\{\mathbf{U}: W(\mathbf{U}) \geq 0\}$. The boundary of this feasible set is of special interest.

Definition: The Pareto utility frontier is given by the set of utilities $\mathbf{U} = (U_1, \ldots, U_N)$ satisfying $W(\mathbf{U}) = 0$.

Note that $W(\mathbf{U}) = 0$ is an equation involving *N* variables: $U_1, \ldots, U_N$. This equation typically has an infinite number of solutions. The set of its solutions constitutes the Pareto utility frontier. It means that an allocation is efficient if and only if it is associated with a point on the Pareto utility frontier. Alternatively, an allocation is inefficient if it generates utilities **U** that are below the Pareto utility frontier. This is illustrated in figure 11.1 in the context of a two-household economy ($N = 2$).

Figure 11.1



This allows the identification of feasible points that are inefficient. The feasible point C in figure 11.1 illustrates this. Point C is Pareto inefficient since there exist feasible points to the north-east of C that can make both households better off (i.e., with higher utilities U_1 and U_2). As such, the allocation associated with point C is not socially desirable.

Figure 11.1 also shows that any point along the line between A and B is Pareto efficient. This illustrates that the Pareto efficiency criterion says little about distribution issues. Indeed, there exist efficient allocations that are not equitable. For example, point A corresponds to an efficient point that benefits greatly household 1 at the expense of household 2. Alternatively, point B is an efficient point that benefits greatly household 2 at the expense of household 1. The Pareto efficiency criterion provides no information about whether point A is better (or worse) than point B.

Again, it is worth stressing that all these results are obtained without imposing any restriction on the production technology, as represented by the feasible set **Y**.

11.2.2 Welfare measurements

The aggregate benefit $B(\mathbf{x}, \mathbf{U})$ can provide a convenient way to measure aggregate welfare. In general, $B(\mathbf{x}, \mathbf{U})$ measures the aggregate quantity of the bundle **g** that the N households facing consumption **x** must give up to reach utility levels **U**. And in the case where the price of the bundle g is one unit, then $B(\mathbf{x}, \mathbf{U})$ is the aggregate amount of money the N households facing **x** are willing to pay to reach utility levels **U**.

Given the maximal allocations defined in (4), aggregate benefit $B(\mathbf{x}, \mathbf{U})$ can be compared with $W(\mathbf{U})$. From (4), we have

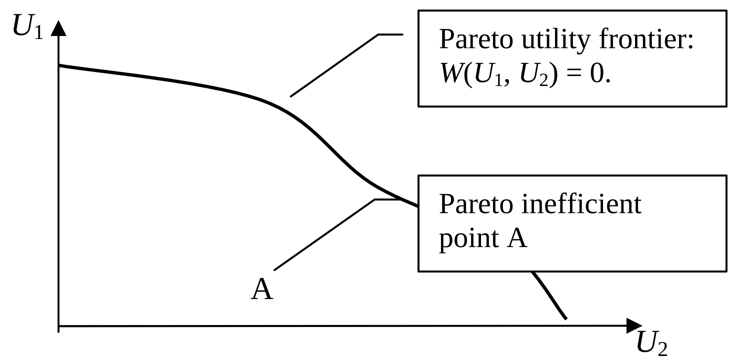
$$W(\mathbf{U}) \geq B(\mathbf{x}, \mathbf{U}) \text{ for all feasible } \mathbf{z} = (\mathbf{x}, \mathbf{y}). \quad (6)$$

And from (5), a zero maximal/Pareto efficient allocation satisfies $W(\mathbf{U}) = 0$. This suggests measuring welfare loss by the amount $W(\mathbf{U}) - B(\mathbf{x}, \mathbf{U}) \geq 0$

When **U** is chosen such that $W(\mathbf{U}) = 0$, this gives $- B(\mathbf{x}, \mathbf{U}) \geq 0$.

This suggests that, with **U** chosen such that $W(\mathbf{U}) = 0$, $- B(\mathbf{x}, \mathbf{U}) \geq 0$ is a measure of the welfare loss associated with inefficient resource allocation. This is illustrated in figure 11.2, where point A is Pareto inefficient (being below the Pareto utility frontier), and where $- B(\mathbf{x}, \mathbf{U})$ provides a welfare measurement of the distance between point A and the Pareto utility frontier. And in the case where the price of the bundle **g** is one unit, then $- B(\mathbf{x}, \mathbf{U})$ is an aggregate monetary amount of the welfare loss associated with Pareto inefficiency.

Figure 11.2



Again, these results are very general in the sense that they apply without imposing any restriction on the production technology, as represented by the feasible set **Y**.

11.3 Shadow Prices

The zero-maximal allocations just identified provide useful insights in to the valuation of resources. To see that, consider the Lagrangean associated with the maximization problem in (4)

$$L(\mathbf{z}, \boldsymbol{\lambda}) = B(\mathbf{x}, \mathbf{U}) + \boldsymbol{\lambda}^T (\sum_j \mathbf{y}_j - \sum_i \mathbf{x}_i), \quad (7)$$

where $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$ is a $(n\times1)$ vector of Lagrange multipliers associated with constraint (1c).

Definition: Define a saddle point of the Lagrangean $L(x, \lambda) = f(x) + \lambda^T h(x)$ as a point $(x^* \in X, \lambda^* \geq 0)$, satisfying $L(x, \lambda^*) \leq L(x^*, \lambda^*) \leq L(x^*, \lambda)$, for all $x \in X$ and all $\lambda \geq 0$.

The maximization problem (4) can be equivalently written in terms of the saddle point of the Lagrangean (7) under some regularity conditions. These regularity conditions are

1. There exists a feasible point $\mathbf{z}^0$ such that $\sum_j \mathbf{y}_j^0 - \sum_i \mathbf{x}_i^0 > 0$ (Slater's condition, a form of constraint qualification (CQ) that applies to constraint functions $\mathbf{h}(\mathbf{x})$ that are not necessarily differentiable.)
2. The objective function and constraint functions are concave
3. The feasible set is convex.

Here, condition 1 (Slater's condition) corresponds to assumption A2. Condition 2 is satisfied since the objective function $\mathbf{B}(\mathbf{x}, \cdot)$ is concave, and the constraint (1c) is linear hence concave. Finally, since the feasible set for $\mathbf{x} \geq 0$ is convex, condition 3 is satisfied if the following assumption holds.

Assumption A3: The feasible set **Y** is convex.

Assumption 3 imposes convexity restrictions on the production technology. Intuitively, it imposes diminishing marginal productivity in production activities. This is important. While the results presented above applied for any production technology, the assumption of diminishing marginal productivity is needed to obtain the results presented below.

Using the saddle point characterization under assumptions A2 and A3, a maximal allocation (defined in (4)) can be equivalently expressed in terms of the saddle point of the Lagrangean (7):

$$W(\mathbf{U}) = \underset{\boldsymbol{\lambda}\geq 0}{Min} \ \underset{\mathbf{z}}{Max} \ \{L(\mathbf{z},\boldsymbol{\lambda}) : \mathbf{x} \geq 0, \mathbf{y} \in \mathbf{Y}\}, \tag{8a}$$

which has for solution **z***(**U**), λ*(**U**). Note that (8a) can be written as

$$\begin{aligned} W(\mathbf{U}) &= \underset{\boldsymbol{\lambda}\geq 0}{Min} \ \underset{\mathbf{z}}{Max} \ \{B(\mathbf{x},\mathbf{U}) + \boldsymbol{\lambda}^T(\textstyle\sum_j \mathbf{y}_j - \sum_i \mathbf{x}_i) : \mathbf{x} \geq 0, \mathbf{y} \in \mathbf{Y}\}, \\ &= \underset{\boldsymbol{\lambda}\geq 0}{Min} \ \{\underset{\mathbf{x}\geq \mathbf{0}}{Max} \ \{B(\mathbf{x},\mathbf{U}) - \boldsymbol{\lambda}^T \textstyle\sum_i \mathbf{x}_i\} + \underset{\mathbf{y}}{Max} \ \{\boldsymbol{\lambda}^T \sum_j \mathbf{y}_j : \mathbf{y} \in \mathbf{Y}\}\}, \\ &= \underset{\boldsymbol{\lambda}\geq 0}{Min} \ \{\underset{\mathbf{x}\geq \mathbf{0}}{Max} \ \{\textstyle\sum_i B_i(\mathbf{x}_i, U_i) - \boldsymbol{\lambda}^T \sum_i \mathbf{x}_i\} + \underset{\mathbf{y}}{Max} \ \{\boldsymbol{\lambda}^T \sum_j \mathbf{y}_j : \mathbf{y} \in \mathbf{Y}\}\}, \\ &= \underset{\boldsymbol{\lambda}\geq 0}{Min} \ \{\textstyle\sum_i [\underset{\mathbf{x}_i\geq \mathbf{0}}{Max} \ \{B_i(\mathbf{x}_i, U_i) - \boldsymbol{\lambda}^T \mathbf{x}_i\}] + \underset{\mathbf{y}}{Max} \ \{\boldsymbol{\lambda}^T \sum_j \mathbf{y}_j : \mathbf{y} \in \mathbf{Y}\}\}, \end{aligned} \tag{8b}$$

The Lagrange multipliers λ*(**U**) have the standard interpretation: they measure the shadow value of the constraint (1c). In the case where the bundle **g** has a unit price (i.e., where λ^T **g** = 1), they are the **shadow prices** of the *n* commodities.

In addition, note that (8b) can be decomposed into the following sub-problems

1. $\pi(\boldsymbol{\lambda}) = \underset{\mathbf{y}}{Max} \ \{\boldsymbol{\lambda}^T \sum_j \mathbf{y}_j : \mathbf{y} \in \mathbf{Y}\}$, (8c1)
2. $-E_i(\lambda, U_i) = \underset{\mathbf{x}_i\geq \mathbf{0}}{Max} \ \{B_i(\mathbf{x}_i, U_i) - \boldsymbol{\lambda}^T \mathbf{x}_i\}$, (assuming that λ^T **g** = 1), (8c2)
3. $E(\lambda, \mathbf{U}) = \Sigma_i E_i(\lambda, U_i)$, (8c3)
4. $W(\mathbf{U}) = \underset{\boldsymbol{\lambda}\geq 0}{Min} \ \{\pi(\boldsymbol{\lambda}) - E(\boldsymbol{\lambda}, \mathbf{U})\}$. (8c4)

Expression (8c1) implies profit maximization for production activities, using the Lagrange multipliers λ as prices for the *n* commodities. Note that it applies even in the presence of externalities across firms (as the set **Y** represents the joint production technology across all firms). The fact that aggregate profit maximization is consistent with Pareto efficiency is called the Coase theorem. This identifies decision rules that lead to the efficient management of externalities. Expression (8c1) also defines the aggregate indirect profit function π(λ).

Expression (8c2) defines the expenditure function for the *i*-th household, again using the Lagrange multipliers λ as prices.

Expression (8c3) defines the aggregate expenditure function as the sum of the individual expenditure functions across all households.

Finally, expression (8c4) defines the Lagrange multipliers/ shadow prices λ* as the solution of a minimization problem involving aggregate profit π net of aggregate expenditures *E*. Equation (8c4) also defines the distributable surplus *W*(**U**). It provides an alternative intuitive interpretation of the distributable surplus: it is the value of aggregate profit net of consumer expenditures, evaluated at λ*.

It follows that, under assumptions A2 and A3, **z***(**U**) obtained from (8c1)-(8c3) provides a characterization of a maximal allocation. In this context, the feasibility condition $W(\mathbf{U}) \geq 0$ can be interpreted intuitively in terms of the aggregate budget constraint: $\pi(\lambda) - E(\lambda, \mathbf{U}) \geq 0$ (evaluated at λ^*), stating that consumers cannot spend more than aggregate income π.

To obtain a zero-maximal allocation, from (5), it remains to choose U such that the distributable surplus is completely redistributed: $W(\mathbf{U}) = 0$. Then, from (8c4), $W(\mathbf{U}) = 0$ is equivalent to stating that the aggregate budget constraint must be binding: $\pi(\lambda) - E(\lambda, \mathbf{U}) = 0$ (evaluated at λ^*), as the distributable surplus is completely redistributed among the N households.

It should be emphasized that, so far, we have not assumed the existence of markets. Thus, the role of shadow prices λ^* is relevant with or without markets. This stresses that the concept of Pareto efficiency is relevant whether or not resource allocation is supported by a market economy.

11.4 Competitive market allocations

In this section, we explore the role of markets in attaining Pareto efficiency.

Assumption A4: The production technology is $\mathbf{Y} = (\mathbf{Y}_1 \times \mathbf{Y}_2 \times \ldots \times \mathbf{Y}_M)$, where $\mathbf{y}_i \in \mathbf{Y}_i$, $i = 1, \ldots, M$.

Assumption A4 means that the production technology for $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_M)$ is **non-joint** across firms in the sense that the feasible set for the j-th firm, $\mathbf{Y}_j$, is independent of the other firms. It follows that the production goods $\mathbf{y}_j$ are private goods for the j-th firm, implying the absence of external effect of the decisions of each firm on any other firm. In this context, assumption A4 implies a situation of **well-defined property rights** and **no externalities**.

Note: All the results presented above applied without assumption A4. It means that all our earlier results allowed for **external effects** across firms, as represented by the **joint** technology **Y**, where $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_M) \in \mathbf{Y}$ allows for the decision of each firm to affect other firms. See below.

Assumption A4 implies some modifications to the analysis presented in the previous section. From (8a), under assumptions A2 and A3, a maximal allocation (defined in (4)) can be equivalently expressed in terms of the saddle point of the Lagrangean (7)

$$W(\mathbf{U}) = \underset{\lambda \geq 0}{Min} \ \underset{\mathbf{z}}{Max} \ \{L(\mathbf{z}, \lambda) : \mathbf{x} \geq 0, \mathbf{y}_j \in \mathbf{Y}_j, j = 1, \ldots, M\}, \qquad (9a)$$

which has for solution $\mathbf{z}^*(\mathbf{U})$, $\lambda^*(\mathbf{U})$. Note that (9a) can be written as

$$W(\mathbf{U}) = \underset{\lambda \geq 0}{Min} \ \underset{\mathbf{z}}{Max} \ \{B(\mathbf{x}, \mathbf{U}) + \lambda^T (\textstyle\sum_j \mathbf{y}_j - \sum_i \mathbf{x}_i) : \mathbf{x} \geq 0, \mathbf{y}_j \in \mathbf{Y}_j, j = 1, \ldots, M\},$$

$$= \underset{\lambda \geq 0}{Min} \ \{\underset{\mathbf{x} \geq \mathbf{0}}{Max} \ \{\textstyle\sum_i B_i(\mathbf{x}_i, U_i) - \lambda^T \sum_i \mathbf{x}_i\} + \underset{\mathbf{y}}{Max} \ \{\lambda^T \sum_j \mathbf{y}_j : \mathbf{y}_j \in \mathbf{Y}_j, j = 1, \ldots, M\}\},$$

$$= \underset{\lambda \geq 0}{Min} \ \{\textstyle\sum_i [\underset{\mathbf{x}_i \geq \mathbf{0}}{Max} \ \{B_i(\mathbf{x}_i, U_i) - \lambda^T \mathbf{x}_i\}] + \sum_j \underset{\mathbf{y}_j}{Max} \ \{\lambda^T \mathbf{y}_j : \mathbf{y}_j \in \mathbf{Y}_j\}\}. \qquad (9b)$$

Now, let λ be **actual market prices** for the n commodities, and assume that the bundle $\mathbf{g}$ has a unit price where $\lambda^T \mathbf{g} = 1$. Then, in a way similar to (8b), note that (9b) can be decomposed into the following sub-problems

1. $\pi_i(\lambda) = \underset{\mathbf{y}_j}{Max} \; \{\lambda^T \mathbf{y}_j : \mathbf{y}_j \in \mathbf{Y}_j\}$, (9c1)
2. $\pi(\lambda) = \sum_j \pi_j(\lambda)$, (9c2)
3. $- E_i(\lambda, U_i) = \underset{\mathbf{x}_i \geq \mathbf{0}}{Max} \; \{B_i(\mathbf{x}_i, U_i) - \lambda^T \mathbf{x}_i\}$, (assuming that $\lambda^T \mathbf{g} = 1$), (9c3)
4. $E(\lambda, \mathbf{U}) = \Sigma_i E_i(\lambda, U_i)$, (9c4)
5. $W(\mathbf{U}) = \underset{\lambda \geq 0}{Min} \; \{\pi(\lambda) - E(\lambda, \mathbf{U})\}$. (9c5)

Expression (9c1) implies profit maximization for the j-th firm. Since this optimization problem takes prices as given, this corresponds to the behavior of a **competitive firm** that takes market prices λ as being exogenous.

Equation (9c2) defines the aggregate indirect profit function $\pi(\lambda)$ as the sum of the firm profit functions across all **competitive firms**.

Expression (9c3) defines the expenditure function for the i-th household, given market prices λ.

Expression (9c4) defines the aggregate expenditure function as the sum of the individual expenditure functions across all households.

Finally, expression (9c5) defines the competitive market prices λ^* as the solution of a minimization problem involving aggregate profit π net of aggregate expenditures E. Equation (9c5) also defines the distributable surplus $W(\mathbf{U})$. Again, in the context of competitive markets, it provides another intuitive interpretation of the distributable surplus: it is the value of aggregate profit net of consumer expenditures, evaluated at λ^*.

Finally, to obtain a zero-maximal allocation, from (5), it remains to choose $\mathbf{U}$ such that the distributable surplus is completely redistributed: $W(\mathbf{U}) = 0$. Then, from (9c5), $W(\mathbf{U}) = 0$ is equivalent to stating that the aggregate budget constraint must be binding: $\pi(\lambda) - E(\lambda, \mathbf{U}) = 0$ (evaluated at λ^*), as the distributable surplus is completely redistributed among the N households.

Definition: A competitive equilibrium is an allocation satisfying

- market prices $\lambda^* \geq 0$, normalized such that $\lambda^{*T} \mathbf{g} = 1$,
- $\mathbf{z}^* = (\mathbf{x}^*, \mathbf{y}^*)$ satisfying (9c1)-(9c5),
- and where $\pi(\lambda^*) = E(\lambda^*, \mathbf{U})$.

This means that a competitive equilibrium $(\mathbf{z}^*, \lambda^*)$ is characterized by

- Allocations satisfying the feasibility conditions $\mathbf{x} \geq 0$, $\mathbf{y}_j \in \mathbf{Y}_j$, $j = 1, \ldots, N$.
- M competitive firms maximizing profit (as stated in (9c1).

- N households minimizing expenditures (as expressed through the dual relationship (9c3)
- Allocations satisfying the aggregate budget constraint $\pi(\lambda^*) = E(\lambda^*, \mathbf{U})$.

We obtain the following important results.

First welfare theorem: Assume that assumption A4 holds, and that household preferences are non-satiated. Then, a competitive equilibrium ($\mathbf{z}^*$, λ^*) is Pareto efficient.

Proof: By definition, a competitive equilibrium is equivalent to (9c1)-(9c5). But this is equivalent to the saddle point of the Lagrangean in (9a), which implies that $\mathbf{z}^*$ is a maximal equilibrium (saddle point theorem). In addition, $\pi(\lambda^*) = E(\lambda^*, \mathbf{U})$ implies that $\mathbf{z}^*$ is a zero maximal equilibrium. Thus, under non-satiation, $\mathbf{z}^* = (\mathbf{x}^*, \mathbf{y}^*)$ is Pareto efficient.

Second welfare theorem: Assume that assumptions A2, A3 and A4 hold, and that household preferences are non-satiated. Assume that $\mathbf{z}^*$ is a Pareto efficient allocation. Then, there exists prices $\lambda^* \geq 0$ such that ($\mathbf{x}^*$, λ^*) is a competitive equilibrium.

Proof: The Pareto efficient $\mathbf{z}^*$ allocation is zero maximal. Under assumptions A2, A3 and A4, a maximal equilibrium $\mathbf{z}^*$ implies that ($\mathbf{z}^*$, λ^*) is a saddle point of the Lagrangean in (9a). But this implies (9c). And zero maximality implies that $W(\mathbf{U}) = \pi(\lambda^*) - E(\lambda^*, \mathbf{U}) = 0$. It follows that ($\mathbf{z}^*$, λ^*) is a competitive equilibrium.

The two welfare theorems state that, under some regularity conditions, competitive markets can support a Pareto efficient allocation. This is a remarkable result in that it requires no central planning. It shows that competitive market prices can reflect the social value of commodities and provide appropriate incentives for efficient decentralized decision making by firms and households. This is sometimes called Adam Smith's invisible hand. However, this result holds only under some regularity conditions. They are:

- Assumption A2, stating the existence of a feasible allocation such that $\sum_j \mathbf{y}_j > 0$.
- Assumption A3: the feasible set for production activities, $\mathbf{Y}$, is convex.
- Assumption A4: The production technology is non-joint: $\mathbf{Y} = (\mathbf{Y}_1 \times \mathbf{Y}_2 \times \ldots \times \mathbf{Y}_M)$.

While assumption A2 appears non-controversial, assumptions A3 and A4 can be more problematic.

- There are situations where the feasible set $\mathbf{Y}$ is non-convex (thus violating A3). For example, this includes cases where the production technology exhibits increasing returns to scale (e.g., national defense, public utilities).
- Also, there are situations where the production technology $\mathbf{Y}$ is joint. This includes the case where external effects exist (violating assumption A4). This includes the case of positive externalities (e.g., bees producing honey while they help fertilize flowers) as well as negative externalities (e.g., pollution). In such

situations, the Coase theorem states that aggregate profit maximization applies. But externalities mean that profit maximization does not apply at the firm level (since it would neglect the external effects across firms). In this context, decentralized profit-maximizing decision making would typically be inefficient. This means that efficiency requires the implementation of coordination schemes (e.g., using mergers, contracts or government policy) across firms affected by externalities, schemes that would implement aggregate profit maximizing outcomes.

11.5 Gains from Trade

Assume that the economy involves *N* locations (e.g., *N* countries), where each location is associated with a representative consumer and a representative firm. Then, $M = N$, and each location exhibits production as well as consumption activities and has the option to trade the *n* goods with any other location. This raises the question: what are the benefits from trade?

To answer that question, consider the case where there are trade restrictions. Write these trade restrictions as

$$\mathbf{x}_i = \mathbf{y}_i, \text{ for some locations } i = 1, \dots, N. \tag{10}$$

The constraint (10) implies that the restricted locations do not trade: they do not export and do not import. As a result, they are in autarky: they consume all of their domestic production. Given (10), define the autarky-constrained maximal allocation

$$W^a(\mathbf{U}) = \underset{\mathbf{z}}{Max}\ \{B(\mathbf{x}, \mathbf{U}): \text{ equ. (1a) - (1c), } \mathbf{x}_i = \mathbf{y}_i\}. \tag{11}$$

It is clear that (11) is a restricted version of (4). Since the restrictions (10) can only reduce the feasible set in (4), it follows that the maximum value attained $W^a(\mathbf{U})$ cannot increase, implying that

$$W^a(\mathbf{U}) \leq W(\mathbf{U}),$$

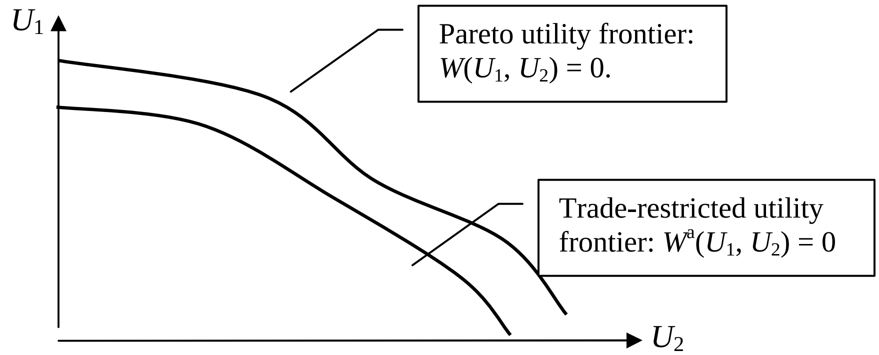
or

$$W(\mathbf{U}) - W^a(\mathbf{U}) \geq 0. \tag{12}$$

This suggests that $W(\mathbf{U}) - W^a(\mathbf{U}) \geq 0$ is a welfare measure of the gains from trade.

We have seen that the sets of $\mathbf{U} = (U_1, \dots, U_N)$ satisfying $W(\mathbf{U}) = 0$ identifies the (unconstrained) Pareto utility frontier. This suggests that the set of utilities $\mathbf{U} = (U_1, \dots, U_N)$ satisfying $W^a(\mathbf{U}) = 0$ can be interpreted as the trade-constrained utility frontier. From (12) and using the fact that that *W* and W^a are non-increasing in $\mathbf{U}$, it follows that the trade-constrained Pareto utility frontier must be below the unconstrained Pareto utility frontier. This is illustrated in Figure 11.3, showing that trade restrictions imply an inward shift of the utility frontier.

Figure 11.3



In this context, $[W(\mathbf{U}) - W^a(\mathbf{U})] \geq 0$ measures the distance between the Pareto utility frontier and the trade restricted frontier. And in the case where **U** is chosen such that $W(\mathbf{U}) = 0$ and g is worth \$1, it follows that $-W^a(\mathbf{U}) \geq 0$ provides a monetary measure of the gains from trade. Alternatively stated, $-W^a(\mathbf{U}) \geq 0$ is measure of the economic losses from the trade restrictions (10).

11.6 Externalities

Come back to the case of N households and M firms. Externalities exist when some economic agent (a household or a firm) makes decisions that also affect directly the welfare of other economic agents. We have seen that in the absence of externalities, competitive market allocations can be Pareto efficient. Alternatively, in the presence of externalities, market allocations are in general Pareto inefficient.

We have just seen the implications of externalities in production activities. To see that, contrast the decentralized decisions (9c1) with the centralized decision (8c1) when the technology **Y** is joint. It suggests that decentralized decision-making will typically be Pareto inefficient whenever it fails to consider the external effects across firms.

Next, we address the issue of externalities in consumption activities. The analysis presented so far assumed that all consumption goods were private goods, without external effects across households. We now investigate the effects of externalities in consumption goods, where a good $\mathbf{x}_i$ can affect the welfare of possibly all N households. Thus, we consider the case where the utility of the i-th household is given by $U_i(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N)$, $i = 1, \ldots, N$.

Note: This can include as special case public goods, i.e. goods that are nonexclusive and are consumed equally by all households (e.g., clean air). For example, if $\mathbf{x}_0$ denote a public good, and private goods are denoted by $\mathbf{x}_i^p$, then the i-th household utility function is $U_i(\mathbf{x}_0, \mathbf{x}_i^p)$, $i = 1, \ldots, N$.

In the presence of externalities, the aggregate benefit function $B(\mathbf{x}, \mathbf{U})$ defined in (3) is no longer an appropriate measurement of social benefit. However, this does not imply that an aggregate benefit measure cannot be obtained. In the case where the reference bundle g includes only private goods, consider replacing the aggregate benefit measure $B(\mathbf{x}, \mathbf{U})$ in (3) by the following measure

$$B(\mathbf{x}, \mathbf{U}) = \underset{\beta}{Max} \ \{\textstyle\sum_i \beta_i : U_i(\mathbf{x}_1 - \beta_1\mathbf{g}, ..., \mathbf{x}_N - \beta_N\mathbf{g}) \geq U_i, (\mathbf{x}_i - \beta_i\mathbf{g}) \geq 0, i = 1, ..., N\}, \quad (13)$$

where $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$ and $\mathbf{U} = (U_1, \ldots, U_N)$. Then, $B(\mathbf{x}, \mathbf{U})$ in (13) measures the aggregate number of units of the reference bundle **g** the group of N households would be willing to give up starting from utilities **U** to obtain **x**.

Note: In the case of private consumption goods with utility functions $U_i(\mathbf{x}_i)$, note that (13) reduces to the aggregate benefit measure given in (3). Thus, expression (13) is a generalization of (3) in the presence of external effects among households.

In the case where the bundle g involves private goods, it should be emphasized that, after replacing (3) by (13), the zero maximal allocation given by (4)-(5) would still identify a Pareto efficient allocation in the presence of externalities. This means that external effects do not invalidate the benefit measurements discussed above. It just stresses that such measurements must reflect the possible external effects among agents (e.g., as illustrated in (13)). To the extent that (13) differs from (3), this suggests that decentralized decision-making will typically be Pareto inefficient whenever it fails to consider the external effects across households.

It is clear that the aggregate benefit (13) can involve complex interactions among the N households. As such, it cannot be easily decentralized. Similarly, in the presence of joint technology **Y**, (8c1) can involve complex interactions among the M firms. Again, this cannot be easily decentralized. Indeed, using (6), decentralized competitive markets will typically generate allocations $\mathbf{z} = (\mathbf{x}, \mathbf{y})$ satisfying $B(\mathbf{x}, \mathbf{U}) \leq W(\mathbf{U})$. This implies that, in general, decentralized competitive markets will fail to yield Pareto efficient allocations in the presence of externalities. In this case, from (6), $W(\mathbf{U}) - B(\mathbf{x}, \mathbf{U}) \geq 0$ provides a measurement of the welfare loss (or social cost) of externalities under markets allocations.

There are several possible solutions to this problem.

- Internalization: The externalities can be "internalized" through institutional change (e.g., the merging of firms having external effects on each other).
- The market solution: Create new markets such that the external effects become subject of exchange at a given market price, thus making the markets "more complete."
- The contract solution: If (9c1) is inappropriate, then all firms affected by the externalities can get together and sign a contract that would refine property rights and generate an allocation that is consistent with the solution of the aggregate optimization problem (8c1). Such contracts can implement a Pareto efficient allocation (e.g., as given by the zero maximal allocation (4)-(5)). This is the Coase theorem. Similar arguments would apply to households affected by externalities.
- The government solution: If decentralized decision-making is inefficient, government could step in and affect resource allocation. There are two types of instruments for government policy:

- Price instruments: Taxes/subsidies that would affect market prices and private incentives, with the objective that market prices should reflect the social value of the commodities (as represented by λ^* in (8b) or (8c4)).
- Quantity instruments: Government regulations constraining the choices made by private agents in an attempt to reduce the social cost of the externalities.

11.7 Uncertainty

So far, we have not addressed issues of uncertainty or imperfect information. In this section, we discuss how uncertainty can be introduced in the analysis.

Consider the case where the uncertainty facing the *N* households and the *M* firms can be represented by mutually exclusive states. The states are events that are not known by at least one agent at the time of the decisions. For example, it is known for sure whether it will snow tomorrow in Madison: this generates two mutually exclusive states: e_1 = it will snow, and e_2 = it will not snow. Consider the case where there are a total of *S* states. Let the list of all relevant states be $\mathbf{e} = (e_1, e_2, \ldots, e_S)$.

How does information about the states affect decisions? In the context where $\mathbf{z} = (\mathbf{x}, \mathbf{y})$, let $\mathbf{z}(\mathbf{e}) = (\mathbf{z}(e_1), \ldots, \mathbf{z}(e_S))$, where $\mathbf{z}(e_s)$ is the decision rule indicating which decisions z are made under state e_s. The vector $\mathbf{z}(\mathbf{e})$ represents state-contingent decisions. Then, the analysis presented above can still apply provided that the vector $\mathbf{z} = (\mathbf{x}, \mathbf{y})$ is replaced by the state contingent decisions $\mathbf{z}(\mathbf{e})$. This means that $\mathbf{x}(\mathbf{e}) = (\mathbf{x}_1(\mathbf{e}), \ldots, \mathbf{x}_N(\mathbf{e}))$, and $\mathbf{y}(\mathbf{e}) = (\mathbf{y}_1(\mathbf{e}), \ldots, \mathbf{y}_M(\mathbf{e}))$, where $\mathbf{x}_i(e_s)$ is the consumption decision of the *i*-th household under the *s*-th state of nature, and $\mathbf{y}_j(\mathbf{e}_s)$ is the production decision of the *j*-th firm under the *s*-th state of nature, $s = 1, \ldots, S$.

In the context of state-contingent decisions, the *i*-th household *ex ante* utility function becomes

$$U_i(\mathbf{x}_i(\mathbf{e})) = U_i(\mathbf{x}_i(e_1), \ldots, \mathbf{x}_i(e_S)), \tag{14a}$$

$i = 1, \ldots, N$. The specification (14a) is very general. For example, it does not require *a priori* assessment of the relative likelihood of the *S* events. However, it is often convenient to impose some structure on household preferences and to rely on probability assessments of the uncertain states. This includes as a special case the utility function

$$U_i(v_{i1}(\mathbf{x}_i(e_1)), \ldots, v_{iS}(\mathbf{x}_i(e_S));\ \text{Prob}_i(e_1), \ldots,\ \text{Prob}_i(e_S)), \tag{14b}$$

where $\text{Prob}_i(e_s)$ is the (subjective) probability that the *i*-th household will face the state *s*, $s = 1, \ldots, S$. Equation (14b) allows for preferences to be a non-linear function of the probabilities $\text{Prob}_i(\mathbf{e})$.

11.8 Imperfect and Asymmetric Information

Typically, many decisions are made without perfect information. This means that agents often decide without knowing which state will occur. Then, each decision must depend on the information available at the time of the decision. This information can be represented by information partitions. Let $\mathbf{\Omega}$ be the set of all possible states: $\mathbf{\Omega} = \{e_1, \ldots.,$

e_S}. Consider partitioning the set $\mathbf{\Omega}$ such that the *k*-th partition $\mathbf{P}_k$ satisfies $\mathbf{P}_k = \{\mathbf{P}_{k1}, \mathbf{P}_{k2}, \ldots\}$, $\mathbf{P}_{kj} \cap \mathbf{P}_{kj'} = \varnothing$ for all $j \neq j'$, and $\cup_j \mathbf{P}_{kj} = \mathbf{\Omega}$.

Definition: The partition $\mathbf{P}_k$ defines an information structure as a collection of subsets of $\mathbf{\Omega}$ that are mutually disjoint but whose union is $\mathbf{\Omega}$.

The *k*-th information structure has the following interpretation: the decision maker can distinguish between situations where the true state falls into different subsets $\mathbf{P}_{kj}$ and $\mathbf{P}_{kj'}$, $j \neq j'$, but he/she cannot distinguish between states found within a given subset $\mathbf{P}_{kj}$. At one extreme, there is the case of "no information" corresponding to $\mathbf{P}_k = \{\mathbf{\Omega}\}$, where $\mathbf{P}_{k1} = \mathbf{\Omega}$ and the decision maker cannot distinguish between any of the states. At the other extreme, there is the case of "perfect information" corresponding to $\mathbf{P}_k = \{e_1, \ldots, e_S\}$, where $\mathbf{P}_{ks} = e_s$ and the decision maker can distinguish between any two states. In intermediate situations, the decision maker can distinguish between some states but not others. A comparison of the quality of information between information structures can be established a follows.

Definition: A information structure $\mathbf{P}_k^a$ is at least as fine as $\mathbf{P}_k^b$ if, for every subsets $\mathbf{P}_{kj}$ of $\mathbf{P}_k^a$ and every subset $\mathbf{P}_{k'j'}$ of $\mathbf{P}_k^b$, either $\mathbf{P}_{kj} \subset \mathbf{P}_{k',j'}$ or $\mathbf{P}_{kj} \cap \mathbf{P}_{k'j'} = \varnothing$.

In addition, if $\mathbf{P}_k^a \neq \mathbf{P}_k^b$, then $\mathbf{P}_k^a$ is more informative than $\mathbf{P}_k^b$, as $\mathbf{P}_k^a$ gives a more refined partition of the state space $\mathbf{\Omega}$.

Let $\mathbf{z}(\mathbf{e}) = (\mathbf{z}_1(\mathbf{e}), \mathbf{z}_2(\mathbf{e}), \ldots)$. Assume that the *k*-th decision $\mathbf{z}_k(\mathbf{e})$ is made under the information structure $\mathbf{P}_k$. Since decisions can only depend on the information available, this imposes the following information restrictions on the k-th state-contingent decisions

$$\text{if } \mathbf{e}_s \text{ and } \mathbf{e}_{s'} \text{ belong to the same subset } \mathbf{P}_{kj},\text{, then } \mathbf{z}_k(\mathbf{e}_s) = \mathbf{z}_k(\mathbf{e}_{s'}), \tag{15}$$

$k = 1, 2, \ldots$ Equation (15) simply restricts the state-contingent decisions to be the same when the decision maker cannot distinguish between states. In the case of "no information", this restricts the *k*-th decision to be made *ex ante*, i.e., the same no matter which state eventually occurs. In the case of perfect information, (15) imposes no restriction at all, as the *k*-th decision is made *ex post* and can be different under each state. In intermediate situations, this simply restricts the *k*-th decision rule $\mathbf{z}_k(\mathbf{e})$ to depend only on the information available to the decision maker.

Information typically varies across decisions. This can happen when a decision maker learns over time and not all decisions are made at the same time. This can also happen when there are different decisions makers, with some decision makers being better informed than others. This is the case of asymmetric information. In general, this means that each decision $\mathbf{z}_k(\mathbf{e})$ can be associated with a different information structure $\mathbf{P}_k$, $k = 1, 2, \ldots$ Then, let $\mathbf{P} = (\mathbf{P}_1, \mathbf{P}_2, \ldots)$ denote the information structure relevant for making all decisions in the economy.

Given imperfect and asymmetric information, the analysis presented above remains valid after making two adjustments:

- Replace the decisions $\mathbf{z} = (\mathbf{y}, \mathbf{x})$ by the state-contingent decision rules $\mathbf{z}(\mathbf{e})$.
- Given an information structure $\mathbf{P} = (\mathbf{P}_1, \mathbf{P}_2, \ldots)$, impose the information restrictions (15) in the definition of the maximal allocation (4).

Then the maximal allocation (4) becomes

$$W(\mathbf{U}, \mathbf{P}) = \underset{\mathbf{z(e)}}{Max} \ \{B(\mathbf{x}(\mathbf{e}), \mathbf{U}): \text{ equ. (1a) - (1c), and (15)}\}, \qquad (4')$$

Let $\mathbf{P}^+ = ((\mathbf{e}_1, \ldots, \mathbf{e}_S), (\mathbf{e}_1, \ldots, \mathbf{e}_S), \ldots)$ be the information structure under perfect information, where all decisions are made *ex post*. And let $\mathbf{P}^- = (\mathbf{\Omega}, \mathbf{\Omega}, \ldots)$ be the information structure under "no information", where all decisions are made *ex ante*. We know that, under $\mathbf{P}^+$, (15) imposes no information constraints in (4'). And under $\mathbf{P}^-$, (15) imposes the most informational restrictions in (4'). In addition, consider two information structures $\mathbf{P}^a = (\mathbf{P}_1^a, \mathbf{P}_2^a, \ldots)$ and $\mathbf{P}^b = (\mathbf{P}_1^b, \mathbf{P}_2^b, \ldots)$ where $\mathbf{P}^a$ is more informative than $\mathbf{P}^b$, i.e. where $\mathbf{P}^a \neq \mathbf{P}^b$ and $\mathbf{P}_k^a$ is at least as fine as $\mathbf{P}_k^b$ for $k = 1, 2\ldots$ Then, (15) imposes at least as many restrictions in (4') under $\mathbf{P}^b$ than under $\mathbf{P}^a$. It follows that, in general,

$$W(\mathbf{U}, \mathbf{P}^-) \leq W(\mathbf{U}, \mathbf{P}^b) \leq W(\mathbf{U}, \mathbf{P}^a) \leq W(\mathbf{U}, \mathbf{P}^+). \qquad (16)$$

This shows that good information contributes to increasing the distributable surplus $W(\mathbf{U}, \mathbf{P})$ and increasing economic efficiency as it shifts up the Pareto utility frontier. Alternatively, poor information contributes to decreasing the distributable surplus $W(\mathbf{U}, \mathbf{P})$ and lowering economic efficiency as it generates an inward shift in the Pareto utility frontier. The negative effects of poor information have two sources:

- Under **imperfect information**, the quality of production and consumption decisions deteriorates, generating adverse welfare effects for firms and households.
- Exchange between two agents can only be based on common information. Thus, the presence of significant **asymmetric information** decreases the possibilities for exchange. This reduces the gains from trade and contributes to market failures. This also reduces the ability of contracts to improve the efficiency of resource allocation.