Denote by G the *inverse function* of F, where $y = F(h)$ is equivalent to $h = G(y)$. Then, cost minimization can be written as

$$C(y, \mathbf{w}) = \underset{\mathbf{x}}{Min}\ \{\mathbf{w}\cdot\mathbf{x} : y = F(h(\mathbf{x})), \mathbf{x} \geq \mathbf{0}, \mathbf{x} \in \mathbf{R}^n\}.$$

Let $C(1, \mathbf{w}) = \underset{\mathbf{x}}{Min}\ \{\mathbf{w}\cdot\mathbf{x} : 1 = F(h(\mathbf{x})), \mathbf{x} \geq \mathbf{0}, \mathbf{x} \in \mathbf{R}^n\}$ denote the total cost when production level is 1 unit. It follows that the cost minimizing input demand $\mathbf{x}^c(1, \mathbf{w})$ satisfies $G(1) = h(\mathbf{x}^c)$.

Rewrite $C(y, \mathbf{w}) = \underset{\mathbf{x}}{Min}\ \{\mathbf{w}\cdot\mathbf{x} : G(y) = h(\mathbf{x}), \mathbf{x} \geq \mathbf{0}, \mathbf{x} \in \mathbf{R}^n\}$. Now we use the implication of the homothetic technology that $MRS_{ij}(t \cdot \mathbf{x}) = MRS_{ij}(\mathbf{x})$ for all $t > 0$. It follows that $\mathbf{x}^c(y, \mathbf{w}) = k \cdot \mathbf{x}^c(1, \mathbf{w})$. Thus $C(y, \mathbf{w}) = k \cdot \mathbf{w} \cdot \mathbf{x}^c(1, \mathbf{w}) = k \cdot C(1, \mathbf{w})$, where $G(y) = h(k\mathbf{x}^c(1, \mathbf{w})) = k \cdot h(\mathbf{x}^c(1, \mathbf{w})) = k \cdot G(1)$, since $h(t \cdot \mathbf{X}) = t \cdot h(\mathbf{X})$ and $G(1) = h(\mathbf{x}^c)$.

It follows that $k = \frac{G(y)}{G(1)}$. Therefore, under a *homothetic technology*, the indirect cost function takes the form

$C(y, \mathbf{w}) = \frac{G(y)}{G(1)} \cdot C(1, \mathbf{w})$.

When $G(1) = 1$, it reduces to $C(y, \mathbf{w}) = G(y) \cdot C(1, \mathbf{w})$.