

AAE 635

Applied Microeconomic Theory

Fall 2009

10/26/2009

Classes 14-16

Last class

- Duality (lecture note 8, chapters 3,5&6)
- Midterm

Today & Next class (and more)

- Cost minimization (continued) (lecture note 9, chapter 3,5&6)
- Homework 4 out next class

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Review: Duality

- Primal approach: FONC's
- Dual approach: via indirect objective function
- Envelop theorem: $\frac{\partial V}{\partial \mathbf{a}} \equiv V_{\mathbf{a}} = L_{\mathbf{a}}(\mathbf{x}^*(\mathbf{a}), \lambda^*(\mathbf{a}), \mathbf{a})$
 - Hotelling Lemma (profit max.)
 - Shephard Lemma (cost min.)

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Profit maximization: competitive case

- Profit maximization always implies cost minimization
 - Cost minimizing behavior consistent with profit max.
- Implications
 - Marginal cost pricing: $P = MC =$ supply curve
 - Slope of MC (SONC and SOSOC)
 - Concave production in $\mathbf{x} \rightarrow$ convex cost function in y
 - Response to input price change is greater under profit maximization than under cost minimization
- What about the monopoly case?
 - $MR = MC$

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Return to scale (RTS): production function

- IRTS, CRTS, DRTS, and VRTS
- RTS and homogeneous production function (degree r)
 - Scale elasticity $SE(\mathbf{x}) = r = \sum_i \frac{\partial \ln(g(\mathbf{x}))}{\partial \ln(x_i)}$
- What about the cost function?
 - $SE = AC/MC$
 - RTS vs. shape of AC, Figure 9.3

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Input substitution

- Substitutes vs. complements
 - Symmetric restrictions
- Allen elasticity of substitution (AES)
$$\sigma_{ij} = \frac{\partial x_i^c}{\partial w_j} \cdot \frac{C}{x_i^c \cdot x_j^c} = \frac{\partial \ln(x_i^c)}{\partial \ln(w_j)} \cdot \frac{1}{S_j}, \quad i \neq j$$
 - AES is symmetric and unit free
- Implication of the homogeneity restriction
 - Inputs cannot be all complements, but can be all substitutes
 - $n=2$: inputs must be substitutes
- CES technology
 - AES constant (same) for any pair of inputs
 - AES does not depend on any other parameters

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CES technology

- General case

$$y = g(\mathbf{x}) = A \cdot (\alpha_1 \cdot x_1^\rho + \alpha_2 \cdot x_2^\rho + \dots + \alpha_n \cdot x_n^\rho)^{1/\rho},$$

- Homogeneous of degree v
- $AES = 1/(1 - \rho)$
- Cost minimizing input demand and indirect cost function

- Special cases

- Leontief technology: $AES = 0$
- Cobb-Douglas technology: $AES = 1$
- Linear technology: $AES = \infty$

- What about flexible functional forms?

- CES when $n=2$

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Output effects

- On average output elasticity is positive
- Homothetic technology: $y = g(\mathbf{x}) = F(h(\mathbf{x}))$
 - MRS homogeneous of degree zero in \mathbf{x} (same slope along a ray through the origin), figure 9.5
 - Output elasticity equals to $(1/SE)$, same for all inputs
 - Price elasticity of input demand are independent of y
- What about homogeneous technology?
 - Special case of homothetic technology, $SE = r$

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Long run equilibrium: free entry and exit

- Optimization problem: minimizing AC
- Entry/exit condition: zero profits
 - If profits are positive, entry
 - If profits are negative, exit
 - If profits are zero, equilibrium
- Properties of the equilibrium
 - $p^e(\mathbf{w})$ concave in \mathbf{w} (why?)
 - The Hessian matrix is symmetric, n.s.d
 - Price effects on relative demand (\mathbf{x}^e/y^e) are symmetric
 - Relative demand is downward sloping w.r.t. own input price

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Relationships with profit max.

- Input demand identity: $\mathbf{x}^e(\mathbf{w}) = \mathbf{x}^*(p^e(\mathbf{w}), \mathbf{w})$
- Output supply identity: $y^e(\mathbf{w}) = y^*(p^e(\mathbf{w}), \mathbf{w})$
- Linkage between short run profit max. behavior and long run equilibrium under free entry/exit

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