AAE 635

Applied Microeconomic Theory

Fall 2009

Last class

- Unconstrained optimization
  - Theory of the Firm: profit maximization problem
  - More applications

Today & Next class (and more)

- Homework 2 out, due Oct. 6th
- Constrained optimization (lecture note 6)
- Cost minimization (lecture note 7, ch. 4&5)

Constrained optimization

- Motivation: cost minimization s.t. PPF

- Mathematical tools
  - Convex set
  - Quasi-concavity (quasi-convexity)
    - Strict quasi-concavity (…)
    - Relationship with concavity, figure 6.2

Unconstrained approach

- General maximization problem with *m* constraints

$$\underset{\mathbf{x}}{Max} \ \{ f(\mathbf{x}) : h_1(\mathbf{x}) = 0, ..h_m(\mathbf{x}) = 0; \mathbf{x} \geq 0, \mathbf{x} \in \mathbf{R}^n \}$$

- Note that $m < n$ (why?)
- Partition $\mathbf{x}=(\mathbf{x}_a, \mathbf{x}_b)$, $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{x}_a \in \mathbf{R}^m$, $\mathbf{x}_b \in \mathbf{R}^{n-m}$
- Constraint Qualification (CQ)
- FONC: $\frac{\partial f(\mathbf{s}(\mathbf{x}_b^*), \mathbf{x}_b^*)}{\partial \mathbf{x}_b} = 0$
- SONC: $\frac{\partial^2 f(\mathbf{s}(\mathbf{x}_b^*), \mathbf{x}_b^*)}{\partial \mathbf{x}_b^2}$ symmetric, negative semi-definite matrix
- SOSC: $\frac{\partial^2 f(\mathbf{s}(\mathbf{x}_b^*), \mathbf{x}_b^*)}{\partial \mathbf{x}_b^2}$ symmetric, negative definite matrix
- Optimization under concavity

Lagrange approach

- The Lagrangean $L(\mathbf{x},\boldsymbol{\lambda}) \equiv f(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{h}(\mathbf{x}) \equiv f(\mathbf{x}) + \sum_j \lambda_j h_j(\mathbf{x})$
- FONC: $$\frac{\partial L}{\partial \mathbf{x}} \equiv L_{\mathbf{x}}(\mathbf{x}^*,\boldsymbol{\lambda}^*) \equiv f_{\mathbf{x}}(\mathbf{x}^*) + (\boldsymbol{\lambda}^*)^T \mathbf{h}_{\mathbf{x}}(\mathbf{x}^*) = 0$$ $$\frac{\partial L}{\partial \boldsymbol{\lambda}} \equiv L_{\boldsymbol{\lambda}}(\mathbf{x}^*,\boldsymbol{\lambda}^*) \equiv \mathbf{h}_{\mathbf{x}}(\mathbf{x}^*)^T = 0$$
- SONC: $[\mathbf{s}_b(\mathbf{x}_b^*)^T, \mathbf{I}_{n-m}] L_{\mathbf{xx}}(\mathbf{x}^*,\boldsymbol{\lambda}^*) \begin{bmatrix} \mathbf{s}_b(\mathbf{x}_b^*) \\ \mathbf{I}_{n-m} \end{bmatrix}$ symmetric, negative semi-definite
- SOSC: $[\mathbf{s}_b(\mathbf{x}_b^*)^T, \mathbf{I}_{n-m}] L_{\mathbf{xx}}(\mathbf{x}^*,\boldsymbol{\lambda}^*) \begin{bmatrix} \mathbf{s}_b(\mathbf{x}_b^*) \\ \mathbf{I}_{n-m} \end{bmatrix}$ symmetric, negative definite
- Optimization under quasi-concavity