

AAE 635

Applied Microeconomic Theory

Fall 2009

9/9/2009

Lecture 3&4

Last class

- Theory of the Firm (lecture note 2, HV ch. 2)
 - Single input profit maximization

Today & Next class (and more)

- Mathematical tools for unconstrained optimization (lecture note 3)
- Theory of the Firm (lecture note 4, HV ch. 2&3)
 - N-input profit maximization problem
 - 2-input case
 - General

9/9/2009

Lecture 3&4

Mathematical tools: Concepts

- Continuous function, differentiable function, continuously differentiable function...
- Young theorem
- Semi-definite vs. definite
- Concavity (convexity)
 - Strict concavity (convexity)
- Taylor series
- Implicit function theorem
- Homogeneous functions
 - Euler theorem

9/9/2009

Lecture 3&4

Unconstrained optimization

- General optimization problem

$$\underset{x}{\text{Max}} \{ f(\mathbf{x}) : \mathbf{x} \geq 0, \mathbf{x} \in \mathbf{R}^n \}$$

- FONC: $f'(\mathbf{x}^*) = 0$
- SONC: $f''(\mathbf{x}^*)$ negative semi-definite matrix
- SOSOC: $f''(\mathbf{x}^*)$ negative definite matrix
- Optimization under concavity

9/9/2009

Lecture 3&4

Summary

- For an interior maximum (minimum),
FONC $f'(\mathbf{x}^*) = 0$
SONC $f''(\mathbf{x}^*)$ negative (positive) semi-definite
SOSC $f''(\mathbf{x}^*)$ negative (positive) definite

- If objective function is concave (convex)

FOC both nec. and suf. for global interior solution for
maxima (minima): $f'(\mathbf{x}^*) = 0$

9/9/2009

Lecture 3&4

Profit maximization, n -input

- The problem:

$$\underset{y, \mathbf{x}}{\text{Max}} \{ p \cdot y - \mathbf{w} \cdot \mathbf{x} : y \leq g(\mathbf{x}), \mathbf{x} \geq 0, y \geq 0 \}$$

- Technical efficiency: $y = g(\mathbf{x})$
- Technical inefficiency: $y < g(\mathbf{x})$
- If $p > 0$, then profit max implies tech. eff. (why?)
- Firm's problem is rewritten as

$$\underset{\mathbf{x}}{\text{Max}} \{ \pi(\mathbf{x}) = R(\mathbf{x}) - C(\mathbf{x}) = p \cdot g(\mathbf{x}) - \mathbf{w} \cdot \mathbf{x} : \mathbf{x} \geq 0 \}$$

9/9/2009

Lecture 3&4

Production Behavior, assuming interior solutions...

$$\underset{\mathbf{x}}{\text{Max}} \{ \pi(\mathbf{x}) = R(\mathbf{x}) - C(\mathbf{x}) = p \cdot g(\mathbf{x}) - \mathbf{w} \cdot \mathbf{x} : \mathbf{x} \geq 0 \}$$

- FONC:

$$\frac{\partial \pi(\mathbf{x}^*)}{\partial \mathbf{x}} = \left[\frac{\partial \pi(\mathbf{x}^*)}{\partial x_1}, \dots, \frac{\partial \pi(\mathbf{x}^*)}{\partial x_n} \right] = 0 \Leftrightarrow p \cdot g'(\mathbf{x}^*)_{1 \times n} = \mathbf{w}_{1 \times n}^T$$

- Economic interpretation: MR = MC for each input x_i
- Implication: MP > 0 in profit maximizing behavior if $p > 0$, $w_i > 0$

9/9/2009

Lecture 3&4

Diminishing marginal productivity

- $g(\mathbf{x})$ being concave, $\Leftrightarrow g''(\mathbf{x})$ being negative semi-definite

Then,

- FONC is also sufficient
- SONC always satisfied
 - If SONC not satisfied, then production cannot be profit maximizing
- Does not imply SOSC
- Figure 4.1

9/9/2009

Lecture 3&4

Input demand functions $\mathbf{x}^*(p, \mathbf{w})$, assuming differentiable...

- Solution of FONC: $\frac{\partial \pi(\mathbf{x}^*)}{\partial \mathbf{x}} = \pi_{\mathbf{x}}(\mathbf{x}^*, p, \mathbf{w}) = 0$

- Strengthen SONC to SOSC,

$\pi_{\mathbf{xx}}(\mathbf{x}^*, p, \mathbf{w})_{n \times n}$ being negative definite

- SONC + $\det[\pi_{\mathbf{xx}}(\mathbf{x}^*, p, \mathbf{w})] \neq 0 \iff \text{SOSC}$

- FONC identity

$$\pi_{\mathbf{x}}(\mathbf{x}^*(p, \mathbf{w}), p, \mathbf{w}) = 0$$

9/9/2009

Lecture 3&4

Effects of input prices \mathbf{w}

- FONC identity: $\pi_{\mathbf{x}}(\mathbf{x}^*(p, \mathbf{w}), p, \mathbf{w}) \equiv 0$

- (mathematical tools: implicit function theorem or chain rule)

- $\frac{\partial \mathbf{x}^*}{\partial \mathbf{w}} \equiv \mathbf{x}_{\mathbf{w}}^* = -[\pi_{\mathbf{xx}}]^{-1} \pi_{\mathbf{xw}} = [\pi_{\mathbf{xx}}]^{-1}$,
symmetric and negative definite

9/9/2009

Lecture 3&4

Implications

- Applied to any arbitrary technology
- $\partial x_i^*/\partial w_i < 0$ for all i , downward sloping input demand function
- $\partial x_i^*/\partial w_j = \partial x_j^*/\partial w_i$ for all $i \neq j$, can be negative or positive

9/9/2009

Lecture 3&4

Effects of output price p

- FONC identity: $\pi_{\mathbf{x}}(\mathbf{x}^*(p, \mathbf{w}), p, \mathbf{w}) = 0$
- (mathematical tools: implicit function theorem and chain rule)
- $\partial \mathbf{x}^*/\partial p \equiv \mathbf{x}_p^* = -[\pi_{\mathbf{xx}}]^{-1} \pi_{\mathbf{x}p} = -[\pi_{\mathbf{xx}}]^{-1} (\partial g/\partial \mathbf{x})^T$, either positive or negative
- $(\partial g/\partial \mathbf{x}) (\partial \mathbf{x}^*/\partial p) = -(\partial g/\partial \mathbf{x}) [\pi_{\mathbf{xx}}]^{-1} (\partial g/\partial \mathbf{x})^T \geq 0$, the weighted average of the price slope is non-negative

9/9/2009

Lecture 3&4

Properties of the supply function $y^*(p, \mathbf{w})$

- Given $p > 0$, technical efficiency implies

$$y^*(p, \mathbf{w}) = g(\mathbf{x}^*(p, \mathbf{w}))$$

- $\partial y^*/\partial p = ?$
- $\partial y^*/\partial \mathbf{w} = ?$
- Implications

9/9/2009

Lecture 3&4

Homogeneity Property

- Input demand function $\mathbf{x}^*(p, \mathbf{w})$
 - Homogeneous of degree zero in prices
- Implications
 - For each input demand, sum of all price elasticities must be equal to zero
- Output supply function $y^*(p, \mathbf{w})$
 - Homogeneous of degree zero in prices
 - Sum of all price elasticities must be equal to zero
- $\mathbf{x}^*(p, \mathbf{w}) = \mathbf{x}^*(\mathbf{w}/p)$, and $y^*(p, \mathbf{w}) = y^*(\mathbf{w}/p)$

9/9/2009

Lecture 3&4

Two inputs, $n=2$

- All results obtained earlier apply
- Additional implications
 - Price response is larger (smaller) when the production function is "flatter" ("more curved")
 - Cross price effect is positive (negative) when x_2 has a negative (positive) impact on the marginal product of x_1

9/9/2009

Lecture 3&4

General case: m outputs, n inputs

- Netput vector $\mathbf{y} = (\underbrace{y_1, \dots, y_m}_{m \text{ output (non-negative)}}, \underbrace{y_{m+1}, \dots, y_{m+n}}_{n \text{ input (non-positive)}}) \in \mathbf{R}^{m+n}$

- Firm's profits are

$$\pi(\mathbf{y}, \mathbf{p}) = R - C = \sum_{i \leq m} p_i \cdot y_i - \sum_{i \geq m+1} p_i \cdot (-y_i) = \sum p_i \cdot y_i = \mathbf{p} \cdot \mathbf{y}$$

- Weak axiom of profit maximization (WAPM)
 - Netput functions $\mathbf{y}^*(\mathbf{p})$ tend to be upward sloping
 - Input functions tend to be downward sloping
 - $\partial \mathbf{y}^* / \partial \mathbf{p} = \text{symmetric, positive semi-definite matrix}$

9/9/2009

Lecture 3&4

The LeChartelier Pinciple

- Firms are constrained to choose only part of netputs
 - E.g. short term decisions vs. long term decisions
 - Conditional rule $\mathbf{y}_a^c(\mathbf{p}_a, \mathbf{y}_b)$ vs. unconditional rule $\mathbf{y}_a^*(\mathbf{p}_a, \mathbf{p}_b)$
- *The unconditional price effects $[\partial \mathbf{y}_a^* / \partial \mathbf{p}_a]$ exceed the conditional price effects $[\partial \mathbf{y}_a^c / \partial \mathbf{p}_a]$ by a positive semi-definite matrix*
- Unconditional price effects are larger than conditional price effects
 - Long-term decisions are more price responsive than short-term decisions

9/9/2009

Lecture 3&4