

AAE 635

# Applied Microeconomic Theory

Fall 2011

9/7/2011

Class 2

## **Last class**

- Introduction (lecture note 1)

## **Today**

- Theory of the Firm (lecture note 2)
  - One input profit maximization problem
  - Analytical solutions
- Homework 1 out, due Sept. 20<sup>th</sup>

## **Next Class**

- Theory of the Firm (lecture note 3)
  - Unconstrained maximization
  - Multiple inputs profit maximization problem

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## Summary

- For an interior maxima,
  - FONC  $f'(x^*) = 0$
  - SONC  $f''(x^*) \leq 0$
  - SOSC  $f''(x^*) < 0$
- For an interior minima,
  - FONC  $f'(x^*) = 0$
  - SONCmin  $f''(x^*) \geq 0$
  - SOSCmin  $f''(x^*) > 0$
- Note that in practice, always consider both FOC and SOC
  - Sufficient condition: FONC + SOSC
    - Useful in deriving solutions
  - Necessary condition: FONC + SONC
    - Useful in obtaining property of equilibrium

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## Neoclassical Theory of the Firm: Single input

*I: Set up the objective function*

- Firms' objective: profit maximization

$$\underset{x,y}{\text{Max}} \quad \pi = p \cdot y - w \cdot x$$

$$\text{s.t. } y \leq g(x),$$

where  $x$  is input,  $y$  is output,

$y = g(x)$  is the PPF (production possibility frontier),

$p$  and  $w$  are prices of the output and input (exogenous variables).

Rewrite it if  $p > 0$ :  $\underset{x}{\text{Max}} \quad \pi = p \cdot g(x) - w \cdot x, \quad (\text{Why?})$

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## Firm's decision problem: single input

### *Step II: Solve the optimization problem*

- Given interior solutions, apply the first order condition

$$\frac{\partial \pi}{\partial x} = 0 \Rightarrow \frac{\partial(TR - TC)}{\partial x} = 0 \Rightarrow MR - MC = 0$$

- $MR = MC \rightarrow x^*(w)$ : Input demand function
- Moreover, the slope of the profit maximization input demand curve must be downward sloping, why?

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## Analytical solution to firm decision problem

- Recall the problem:  $\underset{x}{Max} \pi = p \cdot g(x) - w \cdot x$

- FONC:

$$\frac{\partial \pi}{\partial x} = p \cdot g'(x) - w = 0, \text{ what is the sign of } g'(x)?$$

- Positive marginal product

- SOSC:

$$\frac{\partial^2 \pi}{\partial x^2} = p \cdot g''(x) < 0 \Rightarrow g''(x) < 0 \text{ if } p > 0$$

- Diminishing marginal productivities
- How about the SONC? What does the SONC imply?

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*Step III: Comparative statics*

- FONC identity:

$$p \cdot g'(x^*(w)) - w \equiv 0$$

- Taking derivative wrt  $w$  and solve...
- Given an interior solution, the slope of input demand curve,

$$\frac{\partial x}{\partial w} = \frac{1}{p \cdot g''(x^*)} < 0!$$

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More comparative statics...

- Output price on input demand

$$\frac{\partial x^*}{\partial p} = ?$$

- Output price and input price on output supply

$$\frac{\partial y^*}{\partial p} = ? \quad \frac{\partial y^*}{\partial w} = ?$$

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## Refresh your memory: some useful mathematical concepts

- Continuous function, differentiable function, continuously differentiable function...
- Young theorem
- Semi-definite vs. definite
- Concavity (convexity)
  - Strict concavity (convexity)
- Taylor series
- Implicit function theorem
- Homogeneous functions

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