

AAE 635

# Applied Microeconomic Theory

Fall 2009

9/7/2009

Lecture 2

## **Last class**

- Theory of the Firm (lecture note 1, HV chapter 2)
  - Profit maximization firms' problem

## **Today**

- Theory of the Firm (lecture note 2, chapter 4)
  - One input profit maximization problem
  - Analytical solutions
- Homework 1 out, due on Sept. 17

## **Next Class**

- Theory of the Firm (lecture note 3, chapter 4)
  - Unconstrained maximization
  - Multiple inputs profit maximization problem

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## Summary

- For an interior maxima,
  - FONC  $f'(x^*) = 0$
  - SONC  $f''(x^*) \leq 0$
  - SOSC  $f''(x^*) < 0$
- For an interior minima,
  - FONC  $f'(x^*) = 0$
  - SONCmin  $f''(x^*) \geq 0$
  - SOSCmin  $f''(x^*) > 0$
- Note that in practice, always consider both FOC and SOC
  - Sufficient condition: FONC + SOSC
    - Useful in deriving solutions
  - Necessary condition: FONC + SONC
    - Useful in obtaining property of equilibrium

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## Analytical solution to firm decision problem

- Recall the problem:  $\text{Max}_x \pi = p \cdot g(x) - w \cdot x$
- FONC:  
$$\frac{\partial \pi}{\partial x} = p \cdot g'(x) - w = 0, \text{ what is the sign of } g'(x)?$$
- SOSC:  
$$\frac{\partial^2 \pi}{\partial x^2} = p \cdot g''(x) < 0 \implies g''(x) < 0 \text{ if } p > 0$$
  - Diminishing marginal productivities

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## An example

$$\text{Max}_x \pi = p \cdot (ax + bx^2) - w \cdot x$$

- FONC:

$$\frac{\partial \pi}{\partial x} = p \cdot (a + 2bx) - w = 0$$

- SOSOC:

$$\frac{\partial^2 \pi}{\partial x^2} = p \cdot 2b < 0$$

- The input demand is  $x^* = \frac{w}{2pb} - \frac{a}{2b}$
- The slope of input demand curve is negative, but how general?

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## General case

$$\text{Max}_x \pi = p \cdot g(x) - w \cdot x$$

- FONC identity:

$$p \cdot g'(x^*(w)) - w \equiv 0$$

- Given an interior solution, the slope of input demand curve,

$$\frac{\partial x}{\partial w} = \frac{1}{p \cdot g''(x^*)} < 0!$$

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Function	Numerical	Example	Analytical
<b>Objective function</b>	$200x - 4x^2 - wx$	$p[ax + bx^2] - wx$	$pg(x) - wx$
<b>Production function</b>	$(200x - 4x^2)/p$	$ax + bx^2$	$g(x)$
<b>FONC</b>	$(200 - 8x) - w = 0$	$p[a + 2bx] - w = 0$	$pg'(x) - w = 0$
<b>SOSC</b>	$-8 < 0$	$2pb < 0$	$pg''(x) < 0$
<b>Input demand</b>	$x^* = 25 - w/8$	$x^* = -a/(2b) + w/(2pb)$	$x^*(p, w)$
<b>Slope of input demand function</b>	$\partial x^*/\partial w = -1/8$	$\partial x^*/\partial w = 1/(2pb)$	$\partial x^*/\partial w < 0$

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## More analytical results (comparative statics)

- Output price on input demand

$$\frac{\partial x^*}{\partial p} = ?$$

- Output price and input price on output supply

$$\frac{\partial y^*}{\partial p} = ? \quad \frac{\partial y^*}{\partial w} = ?$$

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## Refresh your memory: some useful mathematical concepts

- Continuous function, differentiable function, continuously differentiable function...
- Young theorem
- Semi-definite vs. definite
- Concavity (convexity)
  - Strict concavity (convexity)
- Taylor series
- Implicit function theorem
- Homogeneous functions

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