AAE 635

Applied Microeconomic Theory

Fall 2011

Last class

- Consumer Theory I (lecture note 9)
  - Marshallian demand and Hicksian demand

Today and next

- Homework #5 out, due Nov. 22nd
- Consumer Theory I (lecture note 9)
  - Hicksian demand
- Consumer Theory II (lecture note 10)
  - Duality
  - Welfare analysis
  - Household production

The "dual" problem

- *Min* expenditure s.t. utility constraint (and other non-negativity assumptions)
  - Solutions: Hicksian demand functions $\mathbf{x}^c(\mathbf{p}, U)$
- Objective function:

$$E(\mathbf{p},U) = \underset{\mathbf{x}}{Min} \ \{\mathbf{p}^T\mathbf{x} : U(\mathbf{x}) \geq U, \mathbf{x} \in \mathbf{R}^n, \mathbf{x} \geq 0\}$$

- Indirect expenditure function: $E(\mathbf{p},U) = \mathbf{p}^T\mathbf{x}^c(\mathbf{p},U)$
  - *Shephard's Lemma*
- Properties of *E*:
  - Non-decreasing in U and **p**
  - Homogeneous of degree 1 in **p**
  - Concave in **p**
- Properties of $\mathbf{x}_\mathbf{p}^c(\mathbf{p}, U)$
  - symmetric restrictions; sign restrictions…

The "dual" problems

- *Max* Utility st. budget constraint
  - Mashallian demand function $\mathbf{x}^*(\mathbf{p}, y)$
- *Min* expenditure st. utility constraint
  - Hicksian demand functions $\mathbf{x}^c(\mathbf{p}, U)$
- Indirect objective functions
  - *Roy's identity for* $\mathbf{x}^*(\mathbf{p}, y)$
  - *Shephard's Lemma for* $\mathbf{x}^c(\mathbf{p}, U)$

Slutsky Equation

- Duality between $V(\mathbf{p}, y)$ and $E(\mathbf{p}, U)$:

$$E(\mathbf{p}, V(\mathbf{p}, y) = y$$
$$V(\mathbf{p}, E(\mathbf{p}, U)) = U$$

- Slutsky equation: $$\frac{\partial \mathbf{x}^*}{\partial \mathbf{p}} = \frac{\partial \mathbf{x}^c}{\partial \mathbf{p}} - \frac{\partial \mathbf{x}^*}{\partial y} \cdot (\mathbf{x}^*)^T$$
  - Graphical decomposition
  - Relation with Slutsky matrix
  - Implications
    - No income effects case
    - Normal goods case
    - Inferior goods case
- Example…

Application: AIDS

- Given expenditure function

$$\ln(E(\mathbf{p}, U)) = a(\mathbf{p}) + U \cdot b(\mathbf{p})$$

where

$$a(\mathbf{p}) = \alpha_0 + \sum_j \alpha_j \ln(p_j) + 0.5 \sum_i \sum_j \alpha_{ij} \ln(p_i) \ln(p_j)$$
$$\ln(b(\mathbf{p})) = \ln(\beta_0) + \sum_j \beta_j \ln(p_j)$$

- Popular in empirical demand analysis
  - From unobserved $E(\mathbf{p}, U)$ to convenient observable Marshallian behavior

Benefit function, assuming $\mathbf{p}^{T}g = 1$

$$B(\mathbf{x},U) = \underset{\beta}{Max}\ \{\beta : U(\mathbf{x}-\beta\mathbf{g}) \geq U, (\mathbf{x}-\beta\mathbf{g}) \geq \mathbf{0}\}$$

- "Willingness to pay" to achieve consumption **x**, starting from utility level *U*
- Properties:
  - Monetary interpretation
  - Non-increasing in utility level *U*
  - Concave in **x** if *U*(**x**) is quasi-concave

Duality between *E* and *B*, assuming $\mathbf{p}^{T}g = 1$

$$E(\mathbf{p},U) = -\underset{\mathbf{x}}{Max}\ \{B(\mathbf{x},U) - \mathbf{p}^{T}\mathbf{x} : \mathbf{x} \geq \mathbf{0}\}$$

- Choose consumption to maximize the net benefits, B – cost of purchasing **x**
- At the optimum, marginal benefit = marginal cost
- FONC is also sufficient as if *U*(**x**) is quasi-concave (why?)

Summary on Duality

- Additional duality relationships

$$U(\mathbf{x}) = \underset{\mathbf{p}}{Min} \quad \{V(\tfrac{\mathbf{p}}{1}) : \mathbf{p}^T\mathbf{x} \le 1, \mathbf{p} \ge 0\}$$

$$B(\mathbf{x}, U) = \underset{\mathbf{p}}{Min} \quad \{\mathbf{p}^T\mathbf{x} - E(\mathbf{p}, U) : \mathbf{p}^T\mathbf{g} = 1, \mathbf{p} \ge 0\}$$

- Figure 10.1
  - $U(\mathbf{x})$: direct utility function
  - $V(\mathbf{p}, y)$: indirect utility function
  - $E(\mathbf{p}, U)$: expenditure function
  - $B(\mathbf{x}, U)$: benefits function