Answer Key Homogeneity:

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a) Is the function $f(x,y)=\frac{x^3-y^3}{\sqrt{x}+\sqrt{y}}$ homogeneous of any degree? If so, which degree? If not, prove that it isn't.

2 points
(1 Point correct answer, 1 point effort)

In math camp, I used lambda and "r" when explaining the homogeneity formula. Here, for ease of notation and typing, the letter k will be used instead of lambda. Yes: $((kx)^3 - (ky)^3)/((kx)^{1/2} + (ky)^{1/2}) = k^{5/2}(x^3 - y^3)/(x^{1/2} + y^{1/2})$, so that the function is homogeneous of degree 5/2.

b) Is the function $f(x_1,x_2)=3x_1+4x_2-2$ homogeneous of any degree? If so, which degree? If not, **prove** that it isn't. I wanted everyone to **prove** that the function was not homogenous; the best way to do this was by using a proof of contradiction. However, for people who missed math camp or the first discussion this may not have been very clear. Thus, I only took off a half a point if you simply showed that the theorem did not work to show homogeneity.

Not homogeneous: Suppose, to the contrary, that the function is homogeneous and that there exists some value of r such that

$$3(kx_1)+4(kx_2)-2=k^r(3x_1+4x_2-2)\ \ \forall k \text{ and } (x_1,x_2).$$

Allow k=2. Then $(6x_1)+(8x_2)-2=2^r(3x_1+4x_2-2)\ \ k=2 \text{ and } \forall(x_1,x_2)$.

2 points
(1 Point correct answer, 1 point effort)

Take two different cases of (x_1,x_2).

With $(x_1,x_2)=(1,0)$, we get $6 - 2 = 2^r(3 - 2)$, or $2^r = 4$

With $(x_1,x_2)=(0,1)$, we get $8 - 2 = 2^r(4 - 2)$, or $2^r = 3$

These two conditions are inconsistent, so the function is not homogeneous of any degree.

c) The function $g(x,y)$ is homogeneous of degree r. Is the function f defined by $f(x,y) = g(x,y)/(xy)$ homogeneous of any degree?

We have $f(kx,ky) = g(kx,ky)/(kxky) = k^r g(x,y)/(k^2xy) = k^{r-2}g(x,y)/(xy) = k^{r-2} f(x,y)$.

Thus f is homogeneous of degree $r-2$.

d) For the following function, show that it is homogeneous and verify that Euler's theorem holds.

Recall that Euler's Theorem states the following:

The differentiable function f of n variables is homogeneous of degree r if and only if

$\sum_{i=1}^{n} x_i f_i'(x_1, ..., x_n) = r f(x_1, ..., x_n)$ for all $(x_1, ..., x_n)$. (*)

$$f(x_1,x_2) = x_1^{1/2}x_2 + x_1^{3/2}$$

$$\begin{aligned} f(\lambda x_1,\lambda x_2) &= (\lambda x_1)^{1/2}\lambda x_2 + (\lambda x_1)^{3/2} \\ &= \lambda^{3/2}\left(x_1^{1/2}x_2\right) + \lambda^{3/2}x_1^{3/2} \\ &= \lambda^{3/2}\left(x_1^{1/2}x_2 + x_1^{3/2}\right) \\ &= \lambda^{3/2} f(x_1,x_2) \end{aligned}$$

Result: The function is H.O.D. 3/2 (ie. r=3/2)

3 points
(1 HOD, 1 First Derivatives, 1 Point Euler's Theorem)

$$\begin{aligned} f_1 &= \frac{1}{2}x_1^{-.5}x_2 + \frac{3}{2}x_1^{1/2} \\ &= \frac{1}{2}\left(x_1^{-.5}x_2 + 3x_1^{1/2}\right) \\ f_2 &= x_1^{1/2} \end{aligned}$$

$$\begin{aligned} \sum x_i f_i &= x_1\left(\frac{1}{2}\left(x_1^{-.5}x_2 + 3x_1^{1/2}\right)\right) + x_2 x_1^{1/2} \\ &= \frac{1}{2}x_1^{1/2}x_2 + \frac{3}{2}x_1^{3/2} + x_1^{1/2}x_2 \\ &= \frac{3}{2}x_1^{1/2}x_2 + \frac{3}{2}x_1^{3/2} \\ &= \frac{3}{2}\left(x_1^{1/2}x_2 + x_1^{3/2}\right) \\ &= \frac{3}{2} f(x_1,x_2) \end{aligned}$$

Total Derivative Answer Key:

1. Find the Total Derivative $\frac{dz}{dx}$ for the following function: Define the function $f:\mathbb{R}^2 \to \mathbb{R}$ of two variables by $z = f(x,y) = 8x - 12y$ s.t. $y = \frac{x+1}{x^2}$.

$$z = f(x,y) = 8x - 12y \quad s.t. \quad y = \frac{x+1}{x^2}$$

$$\begin{aligned} \frac{dz}{dx} &= f_x + f_y\frac{dy}{dx} \\ &= 8 - \frac{12\left(-x^2 - 2x\right)}{x^4} \\ &= 8 + \frac{12(x+2)}{x^3} \end{aligned}$$

2 points
(1 point Problem Set Up/effort, one correct answer)

A number of people did not calculate the derivative of dy/dx correctly. It is important that you review your rules of differentiation. Getting the derivative wrong will make it difficult to solve multi-step problems.

2. Find the Total Derivate $\frac{dz}{dw}$ for the following function: Define the function $f:\mathbb{R}^3 \to \mathbb{R}$ of two variables by $z = f(x,y,w) = x^2 - 8xy - y^3$ $s.t.$ $x = 3w$ $y = 1 - w$.

$$\begin{aligned}
\frac{dz}{dw} &= f_x \frac{dx}{dw} + f_y \frac{dy}{dw} \\
&= 3(2x - 8y) + (8x + 3y^2) \\
&= 6x - 24y + 8x + 3y^2 \\
&= 14x - 24y + 3y^2 \\
&= 14 * 3w - 24(1-w) + 3(1-w)^2 \\
&= 42w - 24 + 24w + 3(1 - 2w + w^2) \\
&= 66w - 24 + 3 - 6w + 3w^2 \\
&= 60w - 21 + 3w^2
\end{aligned}$$

2 points

(1 point for setting up the problem to get dz/dw=14x-27y+3y^2
1 point for substituting back in for x=3w and y= 1-w to get the correct answer)

A number of people forgot to substitute back in for x=3w and y=1-w.

Implicit Function Answer Key:

Find $\frac{dy}{dx}$ using the implicit function rule for the following equation:

$$f:\mathbb{R}^2 \to \mathbb{R} \;\; s.t. \;\; f(x,y) = 7x^2 + 2xy^2 + 9y^4$$
$$f_x = 14x + 2y^2$$
$$f_y = 4xy + 36y^3$$

2 points

(1 point for setting up the problem and 1 point for the correct answer and for specifying the conditions under which the IFT holds).

A number of people forgot to specify the conditions under which the IFT holds. This is an important step, because it does not always make sense to use the implicit function theorem. If you recognized that there were conditions under which the theorem does not hold, you received the second point, even if you did not state it exactly correctly.

1. Note that since f(x,y) is differentiable and is a polynomial, it is at least a C^1 function.

2. We can assume that y=g(x). That is to say, the y can be written as a function of x.

2. Note also that to use the IFT, $f_y \neq 0$. Further note that $f_y=0$ if y=0 or if $x=-9y^2$. In economics we generally assume that x>=0. If that is true, then $x=-9y^2$ only if x=0 and y=0. Thus, it is only really important that we recognize that y cannot equal zero and x must be greater than or equal to zero.

$$\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{14x + 2y^2}{4xy + 36y^3}$$

3. a)

$$AB = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} * \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 2\times 3 - 1\times 4 & -2\times 4 + 3 \\ 1\times 3 - 2\times 4 & -4 + 2\times 3 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -5 & 2 \end{pmatrix}$$
$$BA = \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix} * \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2\times 3 - 1\times 4 & 1\times 3 - 2\times 4 \\ 1\times 3 - 2\times 4 & -4 + 2\times 3 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -5 & 2 \end{pmatrix}$$

$$\Rightarrow AB = BA$$

Let us assume $B = \sigma\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \begin{pmatrix}\sigma & 0\\0 & \sigma\end{pmatrix}, A = \begin{pmatrix}a & b\\c & d\end{pmatrix}$, then

$$AB = \begin{pmatrix}a & b\\c & d\end{pmatrix} * \begin{pmatrix}\sigma & 0\\0 & \sigma\end{pmatrix} = \begin{pmatrix}a \times \sigma + 0 & 0 + b \times \sigma\\c \times \sigma + 0 & 0 + d \times \sigma\end{pmatrix} = \begin{pmatrix}a\sigma & b\sigma\\c\sigma & d\sigma\end{pmatrix}$$
$$BA = \begin{pmatrix}\sigma & 0\\0 & \sigma\end{pmatrix} * \begin{pmatrix}a & b\\c & d\end{pmatrix} = \begin{pmatrix}a \times \sigma + 0 & 0 + b \times \sigma\\c \times \sigma + 0 & 0 + d \times \sigma\end{pmatrix} = \begin{pmatrix}a\sigma & b\sigma\\c\sigma & d\sigma\end{pmatrix}$$

$$\Rightarrow AB = BA$$

b)

$$DA = \begin{pmatrix}2 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}2 & 3 & 1\\0 & -1 & 2\end{pmatrix} = \begin{pmatrix}4 & 5 & 4\\2 & 2 & 3\end{pmatrix}$$
$$(DA)' = \begin{pmatrix}4 & 2\\5 & 2\\4 & 3\end{pmatrix}$$
$$A' = \begin{pmatrix}2 & 0\\3 & -1\\1 & 2\end{pmatrix}$$
$$D' = \begin{pmatrix}2 & 1\\1 & 1\end{pmatrix}$$
$$A'D' = \begin{pmatrix}2 & 0\\3 & -1\\1 & 2\end{pmatrix}\begin{pmatrix}2 & 1\\1 & 1\end{pmatrix} = \begin{pmatrix}4 & 2\\5 & 2\\4 & 3\end{pmatrix}$$

$$\Rightarrow (DA)' = A'D'$$

c)

Firstly, let us solve for the system $\begin{cases}2x_1 + x_2 = 5\\ x_1 + x_2 = 3\end{cases}$;

Rewrite in the form of matrix, we could get

$$\begin{pmatrix}2 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}5\\3\end{pmatrix}$$

Using augmented matrix operations, we could derive the solutions by the following steps:

$$\left(\begin{array}{cc|c}2 & 1 & 5\\1 & 1 & 3\end{array}\right)$$
$$\xrightarrow{row\ 1 - row2} \left(\begin{array}{cc|c}1 & 0 & 2\\1 & 1 & 3\end{array}\right)$$
$$\xrightarrow{row\ 2 - row\ 1} \left(\begin{array}{cc|c}1 & 0 & 2\\0 & 1 & 1\end{array}\right)$$
$$\Rightarrow \begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$$

Method 2: Rule of Thumb for 2x2 matrix

$$A = \begin{pmatrix}a & b\\c & d\end{pmatrix}, A^{-1} = \frac{1}{ad - bc}\begin{pmatrix}d & -b\\-c & a\end{pmatrix}$$

It is only valid for 2x2 matrix.

Comments:
1. I would suggest using the second method to solve this problem. It is more efficient. Row operation is intuitive, but tedious.

Secondly, let us solve for the system

$$\begin{cases} 2x_1 + x_2 = 4 \\ 6x_1 + 2x_2 + 6x_3 = 20 \\ -4x_1 - 3x_2 + 9x_3 = 3 \end{cases}$$

Rewrite in the form of matrix, we could get

$$\begin{pmatrix} 2 & 1 & 0 \\ 6 & 2 & 6 \\ -4 & -3 & 9 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 20 \\ 3 \end{pmatrix}$$

$$\det\begin{pmatrix} 2 & 1 & 0 \\ 6 & 2 & 6 \\ -4 & -3 & 9 \end{pmatrix} = 2 \times \det\begin{pmatrix} 2 & 6 \\ -3 & 9 \end{pmatrix} - 1 \times det\begin{pmatrix} 6 & 6 \\ -4 & 9 \end{pmatrix} + 0$$
$$= 2 \times (18 + 18) - (54 + 24)$$
$$= -6$$

$$\det\begin{pmatrix} 4 & 1 & 0 \\ 20 & 2 & 6 \\ 3 & -3 & 9 \end{pmatrix} = 4 \times \det\begin{pmatrix} 2 & 6 \\ -3 & 9 \end{pmatrix} - 1 \times det\begin{pmatrix} 20 & 6 \\ 3 & 9 \end{pmatrix} + 0$$
$$= 4 \times (18 + 18) - (180 - 18)$$
$$= -18$$

$$\det\begin{pmatrix} 2 & 4 & 0 \\ 6 & 20 & 6 \\ -4 & 3 & 9 \end{pmatrix} = 2 \times \det\begin{pmatrix} 20 & 6 \\ 3 & 9 \end{pmatrix} - 4 \times det\begin{pmatrix} 6 & 6 \\ -4 & 9 \end{pmatrix} + 0$$
$$= 2 \times (180 - 18) - 4 \times (54 + 24)$$
$$= 12$$

$$\det\begin{pmatrix} 2 & 1 & 4 \\ 6 & 2 & 20 \\ -4 & -3 & 3 \end{pmatrix} = 2 \times \det\begin{pmatrix} 2 & 20 \\ -3 & 3 \end{pmatrix} - 1 \times det\begin{pmatrix} 6 & 20 \\ -4 & 3 \end{pmatrix} + 4 \times det\begin{pmatrix} 6 & 2 \\ -4 & -3 \end{pmatrix}$$
$$= 2 \times (6 + 60) - (18 + 80) + 4 \times (8 - 18)$$
$$= -6$$

$$x_1 = \frac{\det\begin{pmatrix} 4 & 1 & 0 \\ 20 & 2 & 6 \\ 3 & -3 & 9 \end{pmatrix}}{\det\begin{pmatrix} 2 & 1 & 0 \\ 6 & 2 & 6 \\ -4 & -3 & 9 \end{pmatrix}} = \frac{-18}{-6} = 3$$

$$x_2 = \frac{\det\begin{pmatrix} 2 & 4 & 0 \\ 6 & 20 & 6 \\ -4 & 3 & 9 \end{pmatrix}}{\det\begin{pmatrix} 2 & 1 & 0 \\ 6 & 2 & 6 \\ -4 & -3 & 9 \end{pmatrix}} = \frac{12}{-6} = -2$$

$$x_3 = \frac{\det\begin{pmatrix} 2 & 1 & 4 \\ 6 & 2 & 20 \\ -4 & -3 & 3 \end{pmatrix}}{\det\begin{pmatrix} 2 & 1 & 0 \\ 6 & 2 & 6 \\ -4 & -3 & 9 \end{pmatrix}} = \frac{-6}{-6} = 1$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

1. Again, for this question, I would recommend using cramer's rule or the formula for matrix inversion to solve it. Personally I think row operations are tedious.
2. If you are using inversion formula, do not forget the sign the adj(A). Otherwise you will get the wrong answers.

d)

its characteristic equation is

$$\det\begin{pmatrix} 1-\lambda & 0 & 2 \\ 0 & 5-\lambda & 0 \\ 3 & 0 & 2-\lambda \end{pmatrix} = (5-\lambda)(\lambda-4)(\lambda+1) = 0$$

$$\xrightarrow{yields} \lambda_1 = 5, \lambda_2 = 4, \lambda_3 = -1$$

To compute an eigenvector corresponding to $\lambda = 5$, we solve the system

$$\begin{pmatrix} \lambda & 0 & 2 \\ 0 & 5-\lambda & 0 \\ 3 & 0 & 2-\lambda \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1-5 & 0 & 2 \\ 0 & 5-5 & 0 \\ 3 & 0 & 2-5 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -4 & 0 & 2 \\ 0 & 0 & 0 \\ 3 & 0 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0$$

Whose solution is $v_1 = v_3 = 0, v_2 =$ anything. So, we will take $v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ as an eigenvector for $\lambda = 5$.

To compute an eigenvector corresponding to $\lambda = 4$, we solve the system

$$\begin{pmatrix} \lambda & 0 & 2 \\ 0 & 5-\lambda & 0 \\ 3 & 0 & 2-\lambda \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1-4 & 0 & 2 \\ 0 & 5-4 & 0 \\ 3 & 0 & 2-4 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -3 & 0 & 2 \\ 0 & 1 & 0 \\ 3 & 0 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0$$

A simple eigenvector for $\lambda = 4$ is $v = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$. The same method yields the eigenvector $v = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ for eigenvector $\lambda = -1$.

Comments:

1. To calculate eigenvalues, starting from the second row is the most efficient way.
2. For eigenvectors, especially for this part, a lot of you made mistakes

   To compute an eigenvector corresponding to $\lambda = 5$, we solve the system

$$\begin{pmatrix} \lambda & 0 & 2 \\ 0 & 5-\lambda & 0 \\ 3 & 0 & 2-\lambda \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1-5 & 0 & 2 \\ 0 & 5-5 & 0 \\ 3 & 0 & 2-5 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -4 & 0 & 2 \\ 0 & 0 & 0 \\ 3 & 0 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0$$

   We can get $\begin{cases} -4v_1 + 2v_3 = 0 \\ 3v_1 - 3v_3 = 0 \end{cases}$ These two equations should hold at the same time.

   It is not equivalent to $-4v_1 + 2v_3 = 3v_1 - 3v_3$

4. a)

   This is just a simple application of implicit function theorem. In this example, x, y are endogenous.

   The Jacobian matrix of this system of equations is

$$\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 2z+y & x \\ yz & xz \end{pmatrix}$$

   Evaluated at the given point, we could get

$$\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 2z+y & x \\ yz & xz \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial F_1}{\partial z} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial z} & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 2x+1-\frac{1}{\sqrt{z}} & x \\ xy & xz \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 6 & 3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial z} \end{pmatrix} = \begin{pmatrix} 2z+y & 2x+1-\frac{1}{\sqrt{z}} \\ yz & xy \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 2 & 6 \end{pmatrix}$$

$$\xrightarrow{yields} \frac{\partial x}{\partial z} = -\frac{det\begin{pmatrix} \frac{\partial F_1}{\partial z} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial z} & \frac{\partial F_2}{\partial y} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix}} = -\frac{det\begin{pmatrix} 6 & 3 \\ 6 & 3 \end{pmatrix}}{det\begin{pmatrix} 2z+y & x \\ yz & xz \end{pmatrix}} = -\frac{0}{6} = 0$$

$$\frac{\partial y}{\partial z} = -\frac{det\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial z} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix}} = -\frac{det\begin{pmatrix} 4 & 6 \\ 2 & 6 \end{pmatrix}}{det\begin{pmatrix} 2z+y & x \\ yz & xz \end{pmatrix}} = -\frac{12}{6} = -2$$

$$\xrightarrow{yields} x' = 3, y' = 2 - 2 \times 0.1 = 1.8$$

Comments:
You can also use total differentiation method to solve for the derivatives. One thing you should bear in mind that, when we try to find the corresponding change in new x and y, the new x and y are not necessarily the optimal points to solve the system of equations. We are just using the derivatives at the initial point to approximate the true derivatives at the unknown new point. The reason for our approximation is the closeness of the points changed before and after. Therefore you should not expect the new points can fit in the system of equations any more.

b)
Part b is about unconstrained optimization.

For the function $F = x^3 - y^3 + 9xy$,

The first-order necessary condition is

$$\begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \end{pmatrix} = (3x^2 + 9y \quad -3y^2 + 9x) = 0$$

$$\xrightarrow{yields} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} / \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$

The second order Hessian matrix are respectively

$\left(\begin{matrix} 6x & 9 \\ 9 & -6y \end{matrix}\right| x = 0, y = 0\Big) = \begin{pmatrix} 0 & 9 \\ 9 & 0 \end{pmatrix}$, with $\Delta 1 = 0, \Delta 2 = -81$. Thus this matrix is indefinite, and $x = 0, y = 0$ is a saddle point;

$\left(\begin{matrix} 6x & 9 \\ 9 & -6y \end{matrix}\right| x = 3, y = -3\Big) = \begin{pmatrix} 18 & 9 \\ 9 & 18 \end{pmatrix}$, with $\Delta 1 = 18, \Delta 2 = 243$. Thus this matrix is positive definite; thus $x = 3, y = -3$ is a local min.

In order to find a global extrema, we need to check the Hessian matrix, not only at the critical points, but also within the whole domain of function F.

The Hessian matrix is $\begin{pmatrix} 6x & 9 \\ 9 & -6y \end{pmatrix}$, with $B1 = 6x, B1 = -6y, B2 = -36xy - 81$. Since these principal minors' signs are dependent on x and y, without further restriction on the domain, we cannot tell the convexity/concavity of this function. Therefore, we do not know whether $x = 3, y = -3$ is a global min or not. Actually it cannot be a global min, because at the point $(0 \quad n)$, $F(0, n) = -n^3$, which goes to $-\infty$ as $n \to \infty$.

Comments:

1. please notice the difference between leading principle minors and principle minors.
2. Please notice that I want you to figure out both local and global maxima/minima.

We are sorry if there are any grading messes on your homework.

Please try to use black pen or pencil on your homework. It is hard to grade if you use colored pens. You can also type your homework if you wish.