AAE 635
Problem Set 6
Due: December 13th, 2011

1. Assume your utility function is given by $u(x_1, x_2) = \ln x_1 + x_2$. For the following questions, assume that interior solutions exist.

a) Find the Marshallian Demands and the Indirect Utility Function.
b) Find the Hicksian Demands and the Indirect Expenditure Function using the duality identities.
c) Verify the Slutsky equation for a change in good 2 as good 2's price changes.
d) The income elasticity of demand is given by $\varepsilon_{yi} = \frac{\partial x_i(y,p)}{\partial y}\frac{y}{x_i(y,p)}$, where $x_i(y,p)$ is the Marshallian demand for good *i*. You are told that $\frac{\partial x_2(y,p)}{\partial p_1} < \frac{\partial x_1(y,p)}{\partial p_2}$. Without solving for ε_{yi}, what can you say about the income elasticities of good 1 and good 2? (Hint: You will need to use the Slutsky matrix and symmetry results to show the relationship between the income elasticity of good 1 and the income elastic of good 2.) Verify your answer by solving for ε_{y1} and ε_{y2}.
e) Suppose that the initial prices and income level are $(y, p_1, p_2) = (200, 1, 2)$. The governor is proposing a new budget bill which will lead to an increase in the price of good one, healthcare, increases to $p_1^1 = 4$. In order to get approval, the governor is also offering a proposal of lump sum subsidy to you. How much do you think the governor should offer?
f) You have a strong feeling that the bill will go through. You believe that the only way to stop it to recall the governor. How much will you be willing to give to the recall campaign?
g) What do you notice about your answers to e) and f)? This utility function is an example of a "quasilinear utility" function. Quasilinear utility functions can always be rewritten in the following functional form: $U(x_1,...x_n) = x_1 + \eta(x_2,...,x_n)$. Carefully explain the relationship between the CV and the EV for this type of utility function.

2. You want to investigate the welfare of an individual facing 2 goods, **x**=(x_1, x_2). The individual's preferences are given by the utility function:

$$U(x_1, x_2) = \left(x_1^{1/2} + x_2^{1/2}\right)^2$$

a) Let **g** = (1,0). Find the benefit function B(**x**,U) associated with this utility function.

Under the expenditure minimization problem, $E(p,U) = \min_x \left\{ p_1x_1 + p_2x_2 \;\; st. \left(x_1^{1/2} + x_2^{1/2}\right)^2 \geq U \right\}$, we find that the indirect expenditure function equals $E(p,U) = \frac{Up_1p_2}{(p_1+p_2)}$.

b) Show that the below duality relationship holds:

$$B(\mathbf{x}, U) = Min_p\{p_1x_1 + p_2x_2 - E(\mathbf{p}, U) : \mathbf{p}^T\mathbf{g} = 1, \mathbf{p} \geq 0\}$$

Hint: Create an unconstrained minimization problem by substitution $\mathbf{p}^T\mathbf{g} = 1$ into the objective function.