

Question 1

- a) Consider the function  $f(x) = X^{\frac{1}{\alpha+\beta+\gamma}}$ .  
We can denote  $u'(x) = f(x) = (x_1 - b_1)^{\frac{\alpha}{\alpha+\beta+\gamma}} (x_2 - b_2)^{\frac{\beta}{\alpha+\beta+\gamma}} (x_3 - b_3)^{\frac{\gamma}{\alpha+\beta+\gamma}}$   
since  $f(x)$  is a monotone transformation, it will preserve the maximum.  
Thus maximizing  $u'(x)$  is equivalent to maximizing  $u(x)$ , which means  
we can set  $\alpha + \beta + \gamma = 1$

- b) Assume  $\alpha + \beta + \gamma = 1$ , and setup the Lagrangian problem using the log transformation of  $u'(x)$ .

$$\mathcal{L} = \alpha \ln(x_1 - b_1) + \beta \ln(x_2 - b_2) + \gamma \ln(x_3 - b_3) + \lambda (y - p_1 x_1 - p_2 x_2 - p_3 x_3).$$

since  $\lambda \alpha = p_i \neq 0$ , we can have interior solutions.

$$\text{(FONC): } \frac{\alpha}{x_1 - b_1} = \lambda p_1 \quad (1) \quad y = p_1 x_1 + p_2 x_2 + p_3 x_3 \quad (4)$$

$$\frac{\beta}{x_2 - b_2} = \lambda p_2 \quad (2)$$

$$\frac{\gamma}{x_3 - b_3} = \lambda p_3 \quad (3)$$

$$(1)/(2) \Rightarrow \frac{x_1 - b_1}{x_2 - b_2} = \frac{p_2 \alpha}{p_1 \beta} \Rightarrow x_1 - b_1 = \frac{\alpha}{\beta} \frac{p_2}{p_1} (x_2 - b_2) \quad (4)$$

$$(2)/(3) \Rightarrow \frac{x_3 - b_3}{x_2 - b_2} = \frac{p_2 \gamma}{p_3 \beta} \Rightarrow x_3 - b_3 = \frac{\gamma}{\beta} \frac{p_2}{p_3} (x_2 - b_2) \quad (5)$$

plugging (4), (5) into budget constraint, we can get

$$\frac{\alpha}{\beta} p_2 (x_2 - b_2) + p_2 (x_2 - b_2) + \frac{\gamma}{\beta} p_2 (x_2 - b_2) + p_1 b_1 + p_2 b_2 + p_3 b_3 = y$$

$$\Rightarrow \left( \frac{1-\beta}{\beta} + 1 \right) p_2 (x_2 - b_2) = y - m$$

$$\Rightarrow x_2 - b_2 = \frac{\beta}{p_2} (y - m) \quad (6)$$

plugging (6) back into (4), (5), we have

$$x_1 - b_1 = \frac{\alpha}{P_1} (y - m)$$

$$x_3 - b_3 = \frac{\gamma}{P_3} (y - m)$$

Now, since  $h(x)$  is a linear function of  $x_i$ , it's quasi-concave

The objective function is  $\alpha \ln(x_1 - b_1) + \beta \ln(x_2 - b_2) + \gamma \ln(x_3 - b_3)$  is a linear combination of log functions. It's concave. So the FONC is also sufficient

The expenditure function: since  $x_1 = \frac{\alpha}{P_1} (y - m) + b_1$ ,  $x_2 = \frac{\beta}{P_2} (y - m) + b_2$ ,  
 $x_3 = \frac{\gamma}{P_3} (y - m) + b_3$ .

We have

$$\left[ \frac{\alpha}{P_1} (y - m) \right]^\alpha \left[ \frac{\beta}{P_2} (y - m) \right]^\beta \left[ \frac{\gamma}{P_3} (y - m) \right]^\gamma = u$$

$$\Rightarrow \left( \frac{\alpha}{P_1} \right)^\alpha \left( \frac{\beta}{P_2} \right)^\beta \left( \frac{\gamma}{P_3} \right)^\gamma (y - m) = u$$

$$\Rightarrow y = P_1 b_1 + P_2 b_2 + P_3 b_3 + u \left( \frac{P_1}{\alpha} \right)^{-\alpha} \left( \frac{P_2}{\beta} \right)^{-\beta} \left( \frac{P_3}{\gamma} \right)^{-\gamma}$$

The indirect utility function:

$$v(P, y) = (y - P_1 b_1 - P_2 b_2 - P_3 b_3) \left( \frac{\alpha}{P_1} \right)^\alpha \left( \frac{\beta}{P_2} \right)^\beta \left( \frac{\gamma}{P_3} \right)^\gamma$$

c) Roy's identity.

$$\frac{\partial v}{\partial P_1} = \left( \frac{\alpha}{P_1} \right)^\alpha \left( \frac{\beta}{P_2} \right)^\beta \left( \frac{\gamma}{P_3} \right)^\gamma (-b_1) + \alpha \left( -\frac{\alpha}{P_1^2} \right) \left( \frac{\alpha}{P_1} \right)^{\alpha-1} \left( \frac{\beta}{P_2} \right)^\beta \left( \frac{\gamma}{P_3} \right)^\gamma (y - P_1 b_1 - P_2 b_2 - P_3 b_3)$$

$$= \left( \frac{\alpha}{P_1} \right)^\alpha \left( \frac{\beta}{P_2} \right)^\beta \left( \frac{\gamma}{P_3} \right)^\gamma \left[ -b_1 - \frac{\alpha}{P_1} (y - P_1 b_1 - P_2 b_2 - P_3 b_3) \right]$$

$$\frac{\partial v}{\partial y} = \left( \frac{\alpha}{P_1} \right)^\alpha \left( \frac{\beta}{P_2} \right)^\beta \left( \frac{\gamma}{P_3} \right)^\gamma$$

$$\Rightarrow x_1 = - \frac{\partial v / \partial P_1}{\partial v / \partial y} = b_1 + \frac{\alpha}{P_1} (y - P_1 b_1 - P_2 b_2 - P_3 b_3) \quad (Q.E.D.)$$

d) Take  $x_1$  for example.

$$\textcircled{1} \quad x_1 = \frac{\alpha}{p_1} (y - p_1 b_1 - p_2 b_2 - p_3 b_3) + b_1$$

$$x_1(kp_1, kp_2, kp_3, ky) = \frac{\alpha}{kp_1} (ky - kp_1 b_1 - kp_2 b_2 - kp_3 b_3) + b_1$$

$$= \frac{\alpha}{p_1} (y - p_1 b_1 - p_2 b_2 - p_3 b_3) + b_1 = x_1(p_1, p_2, p_3, y)$$

$$\textcircled{2} \quad \sum_{i=1}^3 p_i x_i = \left[ \frac{\alpha}{p_1} (y - p_1 b_1 - p_2 b_2 - p_3 b_3) + b_1 \right] p_1 + \left[ \frac{\beta}{p_2} (y - p_1 b_1 - p_2 b_2 - p_3 b_3) + b_2 \right] p_2 + \left[ \frac{\gamma}{p_3} (y - p_1 b_1 - p_2 b_2 - p_3 b_3) + b_3 \right] p_3$$

$$= y.$$

$\textcircled{3}$  Let assume  $u^*$  to be the optimal utility that the consumers can obtain, and the corresponding demands will be  $x_1^*$  and  $x_2^*$ . ( $x_1^*$  could be equal or unequal to  $x_2^*$ ),  $x_1^*, x_2^* \in X(p, y)$   
By the definition of quasi-concave, for the point of  $\alpha x_1^* + (1-\alpha)x_2^*$ , we have

$$u(\alpha x_1^* + (1-\alpha)x_2^*) \geq \min\{u(x_1^*), u(x_2^*)\} = u^*$$

$$\text{Meanwhile } p[\alpha x_1^* + (1-\alpha)x_2^*] = \alpha p x_1^* + (1-\alpha)p x_2^* = \alpha y + (1-\alpha)y = y.$$

since we have a point within the same budget constraint and yield at least the same utility,  $\alpha x_1^* + (1-\alpha)x_2^* \in X(p, y)$

$\Rightarrow X(p, y)$  is convex.

If we assume  $x_1^* \neq x_2^*$ , then under strict quasi-concavity, we'll have

$$u(\alpha x_1^* + (1-\alpha)x_2^*) > \min\{u(x_1^*), u(x_2^*)\} = u^*$$

$$\text{and } p(\alpha x_1^* + (1-\alpha)x_2^*) = y.$$

So here we've a point within the same budget constraint, but generate a

strictly higher utility. In order to maximize utility, consumers should choose  $\alpha x_1^* + (1-\alpha)x_2^*$ , instead of  $x_1^*$  and  $x_2^*$ .

$\Rightarrow x_1^*, x_2^* \notin X(p, y)$ , Contradiction!

$\Rightarrow x_1^* = x_2^*$  under strict quasiconcavity

$\Rightarrow X(p, y)$  has one single element

$$\begin{aligned} \textcircled{4} \quad V(\lambda p_1, \lambda p_2, \lambda p_3, \lambda y) &= (\lambda y - \lambda p_1 b_1 - \lambda p_2 b_2 - \lambda p_3 b_3) \left(\frac{\alpha}{\lambda p_1}\right)^\alpha \left(\frac{\beta}{\lambda p_2}\right)^\beta \left(\frac{\gamma}{\lambda p_3}\right)^\gamma \\ &= (y - p_1 b_1 - p_2 b_2 - p_3 b_3) \left(\frac{\alpha}{p_1}\right)^\alpha \left(\frac{\beta}{p_2}\right)^\beta \left(\frac{\gamma}{p_3}\right)^\gamma \\ &= \tilde{V}(p_1, p_2, p_3, y) \quad \text{where } \alpha + \beta + \gamma = 1. \end{aligned}$$

$$\textcircled{5} \quad \frac{\partial V}{\partial y} = \left(\frac{\alpha}{p_1}\right)^\alpha \left(\frac{\beta}{p_2}\right)^\beta \left(\frac{\gamma}{p_3}\right)^\gamma > 0$$

$$\frac{\partial V}{\partial p_1} = \left(\frac{\alpha}{p_1}\right)^\alpha \left(\frac{\beta}{p_2}\right)^\beta \left(\frac{\gamma}{p_3}\right)^\gamma \left[-b_1 - \frac{\alpha}{p_1} (y - p_1 b_1 - p_2 b_2 - p_3 b_3)\right] \leq 0$$

By symmetry,  $\frac{\partial V}{\partial p_i} \leq 0, \forall i \neq 1$ .

$\textcircled{6}$  remember that a monotone transformation can retain the property of quasiconvexity / quasiconcavity.

$$\text{Let } g(p) = -p, \quad \begin{cases} f_1(p) = \ln(y + b_1 p_1 + p_2 b_2 + p_3 b_3) \\ f_2(p) = \alpha \ln p_1 \\ f_3(p) = \beta \ln p_2 \\ f_4(p) = \gamma \ln p_3 \end{cases} \quad \left. \begin{array}{l} \text{they're quasiconcave} \\ \text{in } p \end{array} \right\} \downarrow$$

$\Rightarrow f_1(g(p)), -f_2(p), -f_3(p), -f_4(p)$  are quasiconvex in  $p$

$\Rightarrow \alpha \ln \alpha + \beta \ln \beta + \gamma \ln \gamma + f_1(g(p)) - f_2(p) - f_3(p) - f_4(p) \Rightarrow$  quasiconvex in  $p$ , and we denote it as  $w$

since  $w = \log V(p, y)$ , we have  $V(p, y) \Rightarrow$  quasiconvex in  $p$ .

Question 2)

$$e(p, u) = a(p) + u b(p)$$

$$v(p, y) = \frac{y - a(p)}{b(p)}$$

$$\frac{\partial v}{\partial y} = \frac{1}{b(p)}$$

$$\frac{\partial v}{\partial p_i} = \frac{b(p)(-a_i(p)) + (y - a(p)) b_i(p)}{[b(p)]^2}$$

$$\Rightarrow x_i = - \frac{\frac{\partial v / \partial p_i}{\partial v / \partial y}}{\frac{\partial v / \partial y}} = \frac{b(p)(-a_i(p)) + (y - a(p)) b_i(p)}{[b(p)]^2} \cdot \frac{1}{\frac{1}{b(p)}} = a_i(p) + (y - a(p)) \frac{b_i(p)}{b(p)}$$

$$\eta = \frac{\partial x_i}{\partial y} \frac{y}{x_i} = \frac{b_i(p)}{b(p)} \cdot \frac{y}{\underbrace{a_i(p) + (y - a(p)) \frac{b_i(p)}{b(p)}}_A} = \frac{b_i(p)}{b(p)} \cdot \frac{1}{\underbrace{\frac{a_i(p)}{y} + \left(1 - \frac{a(p)}{y}\right) \frac{b_i(p)}{b(p)}}_B}$$

When  $y \rightarrow 0$ ,  $\eta \rightarrow 0$

When  $y \rightarrow \infty$ ,  $\eta \rightarrow 1$

$$\lim_{y \rightarrow 0} A = \frac{0}{\infty} = 0$$

$$\lim_{y \rightarrow \infty} B = \frac{b_i(p)}{b(p)} \cdot \frac{1}{\frac{b_i(p)}{b(p)}} = \frac{b_i(p)}{b(p)} \cdot \frac{b(p)}{b_i(p)} = 1$$

Question 3.

a) The budget constraint is  $x_1 + \frac{1}{1+r} x_2 = y_1 + \frac{1}{1+r} y_2$

$$\Rightarrow (1+r)(y_1 - x_1) + y_2 - x_2 = 0$$

Then we need to setup the problem as

$$J = U(x_1, x_2) + \lambda [(1+r)(y_1 - x_1) + y_2 - x_2]$$

$$(cc) \quad \begin{aligned} h_{x_1} &= -(1+r) \neq 0 \\ h_{x_2} &= -1 \neq 0 \end{aligned}$$

According to the primal-dual results,

$$[L_{\alpha x}, L_{\alpha \lambda}] \begin{bmatrix} x_2 \\ \lambda_2 \end{bmatrix} \text{ is symmetric, positive semi-definite.}$$

$$h_{\alpha} = [h_{y_1}, h_{y_2}] = [y_1 - x_1, 1+r, 1] \quad \alpha = [r, y_1, y_2]$$

$$\dots \quad h_{\alpha} u = [y_1 - x_1, 1+r, 1] \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = 0 \Rightarrow u_3 = u_1(x_1 - y_1) - u_2(1+r)$$

$$L_y = \lambda (y_1 - x_1)$$

$$L_{y_1} = \lambda (1+r)$$

$$L_{y_2} = \lambda$$

$$L_{rx_1} = -\lambda, \quad L_{rx_2} = 0, \quad L_{r\lambda} = y_1 - x_1$$

$$L_{y_1 x_1} = 0, \quad L_{y_1 x_2} = 0, \quad L_{y_1 \lambda} = 1+r$$

$$L_{y_2 x_1} = 0, \quad L_{y_2 x_2} = 0, \quad L_{y_2 \lambda} = 1$$

$$\Rightarrow \begin{pmatrix} u_1 \\ u_2 \\ u_1(x_1 - y_1) - u_2(1+r) \end{pmatrix}' \begin{pmatrix} -\lambda & 0 & y_1 - x_1 \\ 0 & 0 & 1+r \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial x_1}{\partial \lambda} & \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial \lambda} & \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \\ \frac{\partial \lambda}{\partial \lambda} & \frac{\partial \lambda}{\partial y_1} & \frac{\partial \lambda}{\partial y_2} \end{pmatrix} \begin{matrix} \\ \\ \\ \end{matrix} \geq 0.$$

$$\text{since } (u_1 \ u_2 \ u_1(x_1 - y_1) - u_2(1+r)) = (u_1 \ u_2) \begin{pmatrix} 1 & 0 & x_1 - y_1 \\ 0 & 1 & -(1+r) \end{pmatrix}$$

$$\Rightarrow (u_1 \ u_2) \begin{pmatrix} 1 & 0 & x_1 - y_1 \\ 0 & 1 & -(1+r) \end{pmatrix} \begin{pmatrix} -\lambda & 0 & y_1 - x_1 \\ 0 & 0 & 1+r \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ x_1 - y_1 & -(1+r) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0.$$

$$\Rightarrow (u_1 \ u_2) \begin{pmatrix} -\lambda & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial x_1}{\partial \lambda} & \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial \lambda} & \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \\ \frac{\partial \lambda}{\partial \lambda} & \frac{\partial \lambda}{\partial y_1} & \frac{\partial \lambda}{\partial y_2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ x_1 - y_1 & -(1+r) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0.$$

$$= -\lambda \begin{pmatrix} \frac{\partial x_1}{\partial \lambda} (1+r) - \frac{\partial x_1}{\partial y_2} \\ 0 \\ 0 \end{pmatrix}$$

symmetric, positive semi-definite.

since  $\lambda \geq 0$

$$\Rightarrow \begin{pmatrix} \frac{\partial x_1}{\partial y_1} + \frac{\partial x_1}{\partial y_2} (x_1 - y_1) & \frac{\partial x_1}{\partial y_1} - (1+r) \frac{\partial x_1}{\partial y_2} \\ 0 & 0 \end{pmatrix} \text{ is symmetric, negative semi-definite.}$$

$$\Rightarrow \begin{cases} \frac{\partial x_1}{\partial y_1} - (1+r) \frac{\partial x_1}{\partial y_2} = 0 & \textcircled{1} \\ \frac{\partial x_1}{\partial y_1} + \frac{\partial x_1}{\partial y_2} (x_1 - y_1) \leq 0 & \textcircled{2} \end{cases}$$

substitute ① into ②, we can get

$$\frac{\partial x_1}{\partial y_1} + \frac{x_1 - y_1}{1+r} \frac{\partial x_1}{\partial y_1} \leq 0.$$

since we don't know the sign of  $(x_1 - y_1) \frac{\partial x_1}{\partial y_1}$ , we cannot sign  $\frac{\partial x_1}{\partial y_1}$  either.

b). if we know the utility function, we can actually know the form of  $\frac{\partial x_1}{\partial y_1}$ ,  $\frac{\partial x_1}{\partial y_2}$

set up the primal problem  $\max u(x_1) + \beta u(x_2)$   
 s.t.  $(1+r)(y_1 - x_1) + y_2 - x_2 = 0$

$$\Rightarrow L = u(x_1) + \beta u(x_2) + \lambda [(1+r)(y_1 - x_1) + y_2 - x_2]$$

$$L_{x_1} = u'(x_1) - \lambda(1+r)$$

$$L_{x_2} = \beta u'(x_2) - \lambda$$

$$L_{\lambda} = (1+r)(y_1 - x_1) + y_2 - x_2$$

$$L_{x_1 x_1} = u''(x_1), \quad L_{x_1 x_2} = 0, \quad L_{x_1 \lambda} = -(1+r).$$

$$L_{x_2 x_1} = 0, \quad L_{x_2 x_2} = \beta u''(x_2), \quad L_{x_2 \lambda} = -1$$

$$L_{\lambda x_1} = -(1+r), \quad L_{\lambda x_2} = -1, \quad L_{\lambda \lambda} = 0$$

$$h_{x_1} = -(1+r) \neq 0, \quad h_{x_2} = -1 \neq 0.$$

$$\Rightarrow \begin{pmatrix} x_{1r} & x_{1y_1} & x_{1y_2} \\ x_{2r} & x_{2y_1} & x_{2y_2} \\ \lambda r & \lambda y_1 & \lambda y_2 \end{pmatrix} = -L_{xx}^{-1} L_{xd} = - \begin{pmatrix} u''(x_1) & 0 & -(1+r) \\ 0 & \beta u''(x_1) & -1 \\ -(1+r) & -1 & 0 \end{pmatrix} \begin{pmatrix} -\lambda & 0 & 0 \\ 0 & 0 & 0 \\ y_1 - x_1 & 1+r & 1 \end{pmatrix}$$

$$= -\frac{1}{|H|} \begin{pmatrix} - & 1+r & \beta u''(x_1)(1+r) \\ + & -(1+r)^2 & u''(x_1) \\ \beta u''(x_1)(1+r) & u''(x_1) & \beta u''(x_1) u''(x_1) \end{pmatrix} \begin{pmatrix} -\lambda & 0 & 0 \\ 0 & 0 & 0 \\ y_1 - x_1 & 1+r & 1 \end{pmatrix}$$

$$\Rightarrow x_{1y_1} = -\frac{1}{|H|} \beta u''(x_1) (1+r)^2$$

$$x_{1r} = -\frac{1}{|H|} (\lambda + \beta u''(x_1) (1+r) (y_1 - x_1))$$

since  $u''(x_2) < 0$  by concavity,  $|H| > 0$  by (SOSC),  $\beta > 0$ .

We can get  $\frac{\partial x_1}{\partial y_1} > 0$ .

$$c) \text{ since } \frac{\partial u^*}{\partial r} = \frac{\partial L^*(r, y_1, y_2)}{\partial r} = \lambda^*(r, y_1, y_2) (y_1 - x_1^*(r, y_1, y_2))$$

The consumer will be better off ( $\frac{\partial u^*}{\partial r} > 0$ ) if he is a net saver ( $y_1 - x_1^*(r, y_1, y_2) > 0$ .)

Question 4.

a). By using budget constraint, we can solve for  $x_3$  easily.

$$b) \text{ since } x_1(p_1, p_2, p_3, w) = 100 - \frac{5\lambda p_1}{\lambda p_3} + \frac{\beta \lambda p_2}{\lambda p_3} + \frac{\delta \lambda w}{\lambda p_3}$$

$$= 100 - \frac{5p_1}{p_3} + \beta \frac{p_2}{p_3} + \delta \frac{w}{p_3}$$

$$= x_1(p_1, p_2, p_3, w)$$

$$\begin{aligned}
 x_2(\lambda p_1, \lambda p_2, \lambda p_3, \lambda w) &= \alpha + \frac{\beta \lambda p_1}{\lambda p_3} + \frac{\gamma \lambda p_2}{\lambda p_3} + \delta \frac{\lambda w}{\lambda p_3} \\
 &= \alpha + \frac{\beta p_1}{p_3} + \frac{\gamma p_2}{p_3} + \frac{\delta w}{p_3} \\
 &= x_2(p_1, p_2, p_3, w)
 \end{aligned}$$

(Q.E.D.)

c) since the Slutsky matrix should be symmetric and negative-semidefinite, we should have  $S_{12} = S_{21}$ ,  $S_{11} \leq 0$ .

$$S_{12} = \frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial w} x_2 = \frac{\beta}{p_3} + \frac{\delta}{p_3} \left( \alpha + \frac{\beta p_1}{p_3} + \frac{\gamma p_2}{p_3} + \frac{\delta w}{p_3} \right)$$

$$S_{21} = \frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial w} x_1 = \frac{\beta}{p_3} + \frac{\delta}{p_3} \left( 100 - \frac{5p_1}{p_3} + \beta \frac{p_2}{p_3} + \delta \frac{w}{p_3} \right)$$

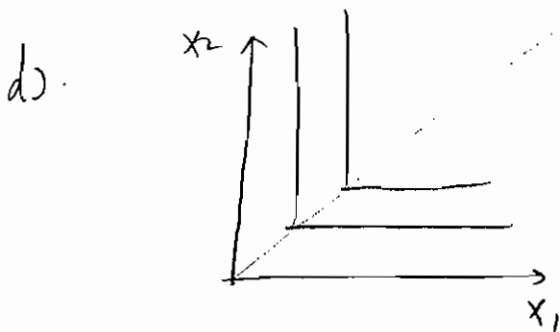
By  $S_{12} = S_{21}$ , we have

$$\begin{cases}
 \beta = -5 \\
 \gamma = \beta = -5 \\
 \alpha = 100
 \end{cases}$$

$$S_{11} = \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial w} x_1 = \frac{-5}{p_3} + \frac{\delta}{p_3} \left( 100 - \frac{5p_1}{p_3} - 5 \frac{p_2}{p_3} + \delta \frac{w}{p_3} \right) \leq 0$$

for  $\forall \frac{p_1}{p_3}, \frac{p_2}{p_3}$ ,

$$\Rightarrow \delta = 0$$



e) since there's no income effect on  $x_3$ , one guess could be

$$u(x_1, x_2, x_3) = \min\{x_1, x_2\} + x_3$$

## NOTES ON GRADING:

### Question 1: Part C

Part 3: Please look at AK carefully. Everyone who was "close" to the correct answer received 4.5 or 5 pts. However, make sure you can answer it fully.

### Part 1-2, 4-6:

You needed to show these properties for this particular functional form.

Otherwise, you are simply copying the generalized proofs from the text or notes.

If you showed it in general, I took off partial points (1/2 - 1 pt) for each sub question.

### Question 2

If you demonstrated how this particular  $E(p, u)$  could be written in the Gorman Form, you received 1 pt. e.c.