AAE 635 Fall 2011
Problem Set #5
Due: November 22nd, 2011

1. Consider the 3-good setting in which the consumer has utility function $u(x) = (x_1 - b_1)^{\alpha}(x_2 - b_2)^{\beta}(x_3 - b_3)^{\gamma}$:

(1) Can you assume that $\gamma + \alpha + \beta = 1$ without loss of generality? Justify your answer.

(2) Assume that $\gamma + \alpha + \beta = 1$. Write down the First-order conditions for the utility maximization problem, and derive the consumer's Marshallian demand and the associated indirect utility functions. Note that the latter is proportional to the amount of "discretionary income" $y - \sum_{i=1}^{n} p_i b_i$. Such a system of demands is known as the linear expenditure system and is due to Stone (1954).

(3) Verify Roy's identity.

(4) Verify that these demand function and indirect utility function satisfy the properties listed as below:

a. The demand function $x(p.w)$ is homogeneous of degree zero in $(p.w)$;

b. Walras' law holds: $px = w$ for all $x \in x(p,w)$;

c. If $u()$ is quasiconcave, then $x(p.w)$ is a convex set. Moreover, if $u()$ is strictly quasiconcave, then $x(p.w)$ consists of a single element;

d. The indirect utility function is homogeneous of degree zero;

e. The indirect utility function is strictly increasing in w and nonincreasing in p_i for any I;

f. The indirect utility function is quasiconvex.

2. You may derive the expenditure function in Question1, and note that it is linear in utility. It is a special case of the Gorman polar form: $e(p,u) = a(p) + ub(p)$, where $a(p)$ and $b(p)$ are both linear homogeneous and concave. Show that for a consumer with this expenditure function, the income elasticity of demand for every good approaches zero as $y \to 0$, and approaches unity as $y \to \infty$.

3. Suppose a consumer has income y1 this year and y2 next year. He or she consumes x1 $\in$ R this year and x2 $\in$ R next year, being able to borrow and lend at interest rate r. Assume that the consumer maximizes the utility of consumption over these two years.

a) Derive the comparative statics for this problem. Will an increase in this year's income necessarily lead to an increase in consumption this year? Please setup a primal dual approach to answer this question.
b) Will your answer change if we know his utility function is $U(x_1, x_2) = u(x_1) + \beta u(x_2)$, where beta is the discount rate, and $u(x)$ is concave.
c) Prove that the consumer will be better off (worse off) if the interest rate rises when he or she is a net saver (dissaver) this year.

4. A consumer in a three-good economy (goods denoted x_1, x_2, x_3; prices denoted p_1, p_2, p_3) with wealth level w>0 has demand functions for commodities 1 and 2 are given by

$$x_1 = 100 - 5p_1 / p_3 + \beta p_2 / p_3 + \delta w / p_3$$
$$x_2 = \alpha + \beta p_1 / p_3 + \gamma p_2 / p_3 + \delta w / p_3$$

where Greek letters are nonzero constants.

(1) Indicate how to calculate the demand for good3 (but do not actually do it).

(2) Are the demand functions for x_1, x_2 appropriately homogeneous?

(3) Calculate the restrictions on the numerical values of $\alpha, \beta, \gamma, \delta$ implied by utility maximization and related properties.

(4) Given your results in (3), for a fixed level of x_3 draw the consumer's indifference curve in the x_1, x_2 plane.

(5) What does your answer to (4) imply about the form of the consumer's utility function $u(x_1, x_2, x_3)$?