AAE 635 Fall 2011

Problem Set Number 2

Due: Tuesday, October 4th

Question1:

Please derive the profit function $\pi(p)$ and supply function (or correspondence) $y(p)$ for the single-output technologies whose production functions $f(z)$ are given by

(1) Given the production function $f(x_1, x_2) = \alpha_1 lnx_1 + \alpha_2 lnx_2$, calculate the profit-maximizing supply functions, and the profit function. Assume that $\alpha_i > 0$.

(2) Consider the technology described by $y = 0$ for $x \leq 1$ and $y = lnx$ for $x > 1$. Calculate the profit function for this technology.

(3) Given the production function $f(x_1, x_2) = \min\{x_1, x_2\}^{\alpha}$, calculate the profit-maximizing supply functions, and the profit function. What restriction must α satisty? (You need to check the second-order conditions here.)

(4) (optional) Given the production function $(x_1, x_2) = \frac{1}{2}{x_1}^2 + lnx_2$, could you find the optimal inputs to achieve the maximum profits? Justify your intuition by relate it to marginal revenue per dollar input, and the function's shape.

Question 2:

Consider the following production function:

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}\ such\ that\ y = f(x_1, x_2) = 24x_1 + 14x_2 - x_1^2 + x_1x_2 - x_2^2$$

1) Assume (for this part only) that x_2 can be written as a function of x_1. That is to say, $x_2=x_2(x_1)$. Find the technical rate of substitution between x_2 and x_1 (That is to say, find $\frac{dx_2}{dx_1}$).

2) Find the input factor demand functions $x_1^*(w_1, w_2, p)$, and $x_2^*(w_1, w_2, p)$ that maximizes profit. Find the optimal output $y^*(w_1, w_2, p)$. Make sure to define the profit function and to check the second order conditions.

3) **Using the First Order Conditions**, find the following comparative statics.

$$\frac{\partial x_1^*}{\partial w_1}, \frac{\partial x_2^*}{\partial w_2}, \frac{\partial x_1^*}{\partial w_2}, \frac{\partial x_2^*}{\partial w_1}$$

You can check your answers using the input factor demand functions. Describe the signs of each and any symmetry that you notice.

4) Find $\frac{\partial x_1^*}{\partial p}$ *and* $\frac{\partial x_2^*}{\partial p}$ using either the first order conditions or the input factor demand functions.

Show that the weighted average of $\frac{\partial x_1^*}{\partial p}$ *and* $\frac{\partial x_2^*}{\partial p}$ is greater than zero. That is to say, show that

$\frac{\partial f}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{x}}{\partial p} > 0$.

5) Are the input factor demand functions $x_1^*(w_1, w_2, p)$, and $x_2^*(w_1, w_2, p)$ homogenous in p and w? if so, to what degree? Interpret.

6) Using the results from part 2, find the comparative static $\frac{\partial y^*}{\partial p}$ and interpret its sign.

7) Find $\frac{\partial y^*}{\partial w_1}$, and $\frac{\partial y^*}{\partial w_2}$. Show that the sum of the price elasticities of supply across all prices (p, w_1, w_2) equals zero. Note that the elasticity of y* with respect to the output price p is $\frac{\partial \ln y^*}{\partial \ln p} = \frac{\partial y^*}{\partial p} \cdot \frac{p}{y^*}$, the elasticity of y* with respect to w_1 is $\frac{\partial \ln y^*}{\partial \ln w_1} = \frac{\partial y^*}{\partial w_1} \cdot \frac{w_1}{y^*}$ and the elasticity of y* with respect to w_2 is $\frac{\partial \ln y^*}{\partial \ln w_2} = \frac{\partial y^*}{\partial w_2} \cdot \frac{w_2}{y^*}$.

When answering the questions above, please **box in** and **label** any equations that you reference in later questions.