

**AAE 635 Homework #3**  
(Due 10/20/2009 before class)

1. Consider a profit maximizing firm with production function  $y = f(x_1, x_2)$  that sells its output competitively at price  $p$ . The firm obtains input  $x_1$  at a competitively determined unit wage  $w_1$ , but the firm faces an upward-sloping supply function for  $x_2$  given by  $w_2 = w_2^0 + kx_2$ , where  $w_1$ ,  $w_2$ ,  $p$  and  $k$  are positive parameters.
  - a. Set up the problem and derive the first- and (sufficient) second-order conditions. Is the “law of diminishing marginal product” hold for each factor?
  - b. Derive the comparative statics results of  $w_1$ ,  $w_2^0$  and  $k$  on optimal inputs.  
Show that  $\frac{\partial x_1}{\partial w_2^0} = -\frac{\partial x_2}{\partial w_1}$ .
  - c. Suppose the government put a tax  $t$  on the output price  $p$ . Drive the comparative statics results of the tax rate  $t$  on optimal inputs.
  - d. Suppose now that the firm is a monopolist in the output market, facing a demand curve  $p = ay + b$ , where  $a < 0, b > 0$ . Set up the problem and derive the first- and (sufficient) second-order conditions. Drive the comparative statics results of input price  $w_1$  on optimal inputs. Compare the results with those from the previous case and discuss.
  - e. Returning to the competitive market output market model, suppose  $x_2$  is held fixed. Drive the “short-run” input demand function for  $x_1$ , and prove that it is downward-sloping in  $w_1$ .

2. Find the maximum or minimum values of the following functions  $f(x_1, x_2)$  subject to the constraints by the method of direct substitution (i.e. unconstrained approach) and by Lagrange multipliers. Be sure to check the second-order conditions or the quasi-concavity to see if a maximum or minimum (if either) is satisfied.

a. Min  $f(x_1, x_2) = 2x_1 + x_2$  s.t.  $2 - x_1x_2 = 0$ .

b. Max  $f(x_1, x_2) = x_1x_2$  s.t.  $4 - (2x_1 + x_2) = 0$ ;

Verify that the optimal solution from a) and b) are the same. Suppose that  $x_1x_2$  is production function and  $2x_1 + x_2$  is its cost, explain the economic meaning of the Lagrange multipliers from a) and b).

3. Consider the general maximization problem

$$\text{Max}_{x_1, x_2} f(x_1, x_2, \alpha), \text{ subject to } g(x_1, x_2) = k,$$

where  $x_1$  and  $x_2$  are choice variables and  $\alpha$  and  $k$  are parameters. Using the Lagrangian

$$L = f(x_1, x_2, \alpha) + \lambda(k - g(x_1, x_2)),$$

a) Prove that  $\frac{\partial^2 f}{\partial x_1 \partial \alpha} \cdot \frac{\partial x_1^*}{\partial k} + \frac{\partial^2 f}{\partial x_2 \partial \alpha} \cdot \frac{\partial x_2^*}{\partial k} = \frac{\partial \lambda^*}{\partial \alpha}$ .

b) What functional forms of the objective function and constraint would lead to the simple reciprocity result  $\frac{\partial x_1^*}{\partial k} = \frac{\partial \lambda^*}{\partial \alpha}$  ?