

**AAE 635**  
**Problem Set Number 3**  
**Due: Tuesday, October 18<sup>th</sup>**

**Question 1: Economic Application and Verification of LeChatelier's Principle**  
(Silberberg: Page 89 Question 9)

Consider a profit-maximizing firm with the production function  $y = f(x_1, x_2)$ , facing output price  $p$  and factor prices  $w_1$  and  $w_2$ . Suppose this firm is taxed according to the total cost of factor 2 (ie.  $\text{Tax} = tw_2x_2$ ).

Part A: Derive the factor demand functions. Are these choice function homogeneous of any degree in any of the parameters?

Part B: Show that if the tax rate rises, the firm will use less of factor 2.

Part C: Show that  $\frac{\partial x_1^*}{\partial t} = w_2 \frac{\partial x_2^*}{\partial w_1}$

Part D: Suppose that factor 1 is held fixed at its profit-maximizing level. Show that the response of factor 2 to a change in the tax rate is less (in absolute value) than before.

For this question, make sure to include any assumptions that you need to make about the production function.

**Question 2: Profit Maximization with Multiple Outputs**

Consider a profit maximizing firm that produces a good from two plants. Plant 1 is located in the U.S. and produces output  $y_1$  at a cost  $C_1(y_1)$ . Plant 2 is located in Canada and produces output  $y_2$  at a cost of  $C_2(y_2)$ . Total production is  $y = y_1 + y_2$ .

Part A: Assume that the firm is competitive firm facing output price  $p$ . You observe an increase in output price  $p$ . What are the implications of an increase in  $p$  for the following:

- The choice of  $y_1$
- The choice of  $y_2$
- The choice of  $y$

Part B: Plant 2 is subject to a tax in the amount of  $t$  per unit produced. Under competition, what are the implications of the tax "t" for each of the following:

- The choice of  $y_1$
- The choice of  $y_2$
- The choice of  $y$

For this question, make sure to include any assumptions that you need to make about the cost functions.

Question 3:

- (1) Suppose  $f(z)$  is a concave production function with  $J-1$  inputs  $(z_1, \dots, z_{J-1})$ . Suppose also that  $\frac{\partial f(z)}{\partial z_j} \geq 0$  for all  $j$  and  $z \geq 0$  and that the matrix  $D^2f(z)$  is negative definite for all  $z$ . Use the firm's first order conditions and the implicit function theorem to prove that:  
"an increase in the price of an input leads to a reduction in the demand for that input"
- (2) A factor of production is called inferior if the conditional demand for that factor falls as output is increased while factor prices are held constant. Please show that :  
"if marginal cost decreases as the price of some factor increases, then the factor must be inferior".
- (3) Suppose we have the following production function:

$$X = (a + b)^\alpha - \beta b^\vartheta$$

where  $0 < \alpha, \beta < 1$  and  $\vartheta > 1$ . we set  $\vartheta = 2$ . Let input prices be  $P_a, P_b$  respectively. And  $P_a > P_b$ .

$a$  and  $b$  are inputs.  $X$  is output.

Please use either profit maximization or cost minimization to figure out which input is the inferior input by the "inferior" definition given in part (2). (if you use cost minimization approach, you may have trouble checking SOSOC conditions. The calculation is too much. You only need to write down the expression for SOSOC. The related determinants are not necessary.)

Question 4:

Consider the following optimization problem:

$$\underset{x}{\text{Max}} f(x_1, x_2) = x_1^3 x_2 \quad \text{s.t.} \quad 12 - 3x_1 - 2x_2 = 0.$$

- (a) Use the unconstrained method to find the solution candidates.
- (b) Use the Lagrangean approach to find the solution candidates.
- (c) Write the equation that implicitly defines the level set (contour curve) for  $f = 32$ , and then solve it for  $x_2$  to get the function  $x_2 = g(x_1)$  that explicitly defines the level set. Sketch a graph of this level set.
- (d) Is this function quasi-concave? How can you tell? (*Hint: think about how you would show this on the graph you just drew.*)
- (e) What can you say about the solution candidates you find in part a) and b)? How can you validate or invalidate them to be the true solution? Does the special case of quasi-concavity apply here?
- (f) Suppose  $f(x_1, x_2)$  is the production function, and the constrained function is the cost budget. What is the economic interpretation of the Lagrangean multiplier in part b)?