

AAE635 Homework #2
(Due 10/6/2009 before class)

1. Suppose the production function of a competitive firm with two inputs, labor (L) and capital (K), and a single output Y is given as:

$$Y(L, K) = AL^\alpha K^\beta, \text{ where } A > 0, \alpha > 0, \beta > 0,$$

- a) Under what conditions (i.e. parameter values of α and β) the production function is strictly concave?
(From now on, assume that conditions in part a hold.)
- b) Given output price p and factor prices w for labor and r for capital, derive the profit maximizing output supply and input demand functions;
- c) Show that $\frac{\partial L^*}{\partial r} = \frac{\partial K^*}{\partial w}$, and is negative;
- d) Now the government is imposing an output tax in the amount of t per unit produced. How would the optimal production level Y^* respond to the tax imposed?
- e) Suppose that the firm is currently experiencing some financing problem, i.e. it cannot adjust capital through new loan or liquidation. How would your answer to d change? (Show the detail that supports your answer)
2. Assume corn farmers have production technology in the quadratic form:
- $$Y(\mathbf{x}) = a_1x_1^2 + a_2x_2^2 + b_1x_1 + b_2x_2, \text{ where inputs } \mathbf{x} = \{x_1, x_2\}, Y \text{ is the total corn yield, and } a\text{'s and } b\text{'s are parameters.}$$
- a) Based on your understanding of the economics of production, what do you expect the signs of a 's and b 's are? Justify your answer.
- b) Assume positive output price p and input price \mathbf{w} , what are the optimal input levels under profit maximization?
- c) What are the optimal input levels under yield maximization? How would your answers compare to those in part b? Discuss the source of differences.
- d) (Bonus) Can you draw some general conclusions about the linkages between the profit function and the production function?