

# AAE635 Homework #1

(Due 09/17/2009)

Please refer to your math review material for help.

1. Let  $y = f(x_1, x_2) \equiv g(h(x_1, x_2))$ , where  $h(x_1, x_2) = \ln(x_1 x_2)$ . Using chain rule to show that

a. 
$$\frac{\partial y}{\partial h} \equiv x_1 \cdot \frac{\partial y}{\partial x_1} \equiv x_2 \cdot \frac{\partial y}{\partial x_2}$$

b. 
$$\frac{\partial^2 y}{\partial h^2} \equiv x_1 \cdot \frac{\partial y}{\partial x_1} + x_1^2 \cdot \frac{\partial^2 y}{\partial x_1^2} \equiv x_2 \cdot \frac{\partial y}{\partial x_2} + x_2^2 \cdot \frac{\partial^2 y}{\partial x_2^2}$$

2. Show that the following functions are homogeneous and verify that Euler's theorem holds.

a.  $f(x_1, x_2) = Ax_1^{\alpha_1} x_2^{\alpha_2}$

b.  $f(x_1, x_2) = A(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho})^{\frac{1}{\rho}}$

c.  $f(x_1, x_2) = (x_1 x_2) / (x_1^2 - x_2^2)$

3. Let  $f : [-2, 2] \rightarrow \mathbb{R}$  be given as  $f(x) = 2x^3 - 3x^2$ .
- a. Use first and second order conditions to identify local minimum and maximum points;
- b. What are the global minimum and maximum points?

4. Evaluate the determinants of the following matrices.

a. 
$$\begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$$

b. 
$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

c. 
$$\begin{bmatrix} 4 & 2 & 7 \\ 9 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

d. 
$$\begin{bmatrix} \frac{1}{a} & b \\ c & 2d \end{bmatrix}$$

Obtain the inverse of each matrix (if invertible).