

(Due Tuesday, September 20, 2011)

Homogeneity and Euler's Theorem

1.

a) Is the function $f(x, y) = \frac{x^3 - y^3}{\sqrt{x} + \sqrt{y}}$ homogeneous of any degree? If so, which degree? If not, prove that it isn't.

b) Is the function $f(x_1, x_2) = 3x_1 + 4x_2 - 2$ homogeneous of any degree? If so, which degree? If not, prove that it isn't.

c) The function $g(x, y)$ is homogeneous of degree r . Is the function f defined by $f(x, y) = g(x, y)/(xy)$ homogeneous of any degree?

d) For the following function, show that it is homogeneous and verify that Euler's theorem holds.

$$f(x_1, x_2) = x_1^{1/2}x_2 + x_1^{3/2}$$

Total Derivative:

2.

a) Find the Total Derivative $\frac{dz}{dx}$ for the following function: Define the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ of two variables by $z = f(x, y) = 8x - 12y$ s.t. $y = \frac{x+1}{x^2}$.

b) Find the Total Derivative $\frac{dz}{dw}$ for the following function: Define the function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ of two variables by $z = f(x, y, w) = x^2 - 8xy - y^3$ s.t. $x = 3w$ $y = 1 - w$.

Implicit Function:

Find $\frac{dy}{dx}$ using the implicit function rule for the following equation:

$$f: \mathbb{R}^2 \rightarrow \mathbb{R} \text{ s.t. } f(x, y) = 7x^2 + 2xy^2 + 9y^4$$

Linear Algebra

3. a) it sometimes happens that $AB = BA$

check this for $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix}$;

show that if B is a scalar multiple of the 2×2 identity matrix, then $AB = BA$ for all 2×2 matrices A .

b) Let $A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. Verify that $(DA)^T = A^T D^T$.

c) invert the coefficient matrix to solve the following systems of equations:

$$\begin{cases} 2x_1 + x_2 = 5 \\ x_1 + x_2 = 3 \end{cases};$$

$$\begin{cases} 2x_1 + x_2 = 4 \\ 6x_1 + 2x_2 + 6x_3 = 20 \\ -4x_1 - 3x_2 + 9x_3 = 3 \end{cases}$$

d) compute the eigenvalues and eigenvectors of the 3×3 matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 5 & 0 \\ 3 & 0 & 2 \end{pmatrix}$$

Function and Optimization

4. a) The economy of Northern Saskatchewan is in equilibrium when the system of equations

$$\begin{cases} 2xz + xy + z - 2\sqrt{z} = 11 \\ xyz = 6 \end{cases}$$

is satisfied. One solution of this set of equations is $x = 3, y = 2, z = 1$, and Northern Saskatchewan is in equilibrium at this point. Suppose that the prime minister discovers that the variable z (output of beaver pelts) can be controlled by simple decree.

If the prime minister raises z to 1.1, use calculus to estimate the change in x and y .

b) For each of the following functions defined on R^2 , find the critical points (the points where the first order necessary condition holds) and classify these as local max, local min, saddle point, or "cannot tell"; which of the maxima and minima you have found are global maxima or global minima? :

$$F = x^3 - y^3 + 9xy$$