Final Exam

1. Consider a representative consumer making consumption decision between gas (x_1) and other goods (x_2), and deriving utility according to $U(\mathbf{x}) = 10 + \sqrt{(x_1 - 1)x_2}$.
   The market price for these consumptions goods are $\mathbf{p} = (p_1, p_2) > 0$.
   a) Using Lagrangean approach, derive the Marshallian demand functions for x_1, x_2. (10 points)
   b) Find the associated expenditure function $E(\mathbf{p}, U)$, where U is the reference utility level (being maximized in part a) (Hint: obtain the expression for indirectly utility function, then use duality to find $E(\mathbf{p}, U)$). (10 points)
   c) Obtain the Hicksian demand functions and verify the Slutsky equation. (10 points)
   d) Suppose an oil crisis occurs, the government needs to consider some short term policy to restrict the gas consumption. There are two options: price control or quantity control. The government also considers offering a lump sum income tax credit to consumers so that they will vote yes to the policy intervention. Now you are hired by the government to provide advice regarding which policy intervention the government should choose and how much should be offered to consumers as income tax credit. Illustrate how you will conduct this consulting assignment. State your decision criteria for your recommendations. (10 points)

2. You are evaluating a firm that chooses two inputs (x_1, x_2) to minimize cost
   $$C(y, \mathbf{w}) = \underset{\mathbf{x}}{Min}\ \{\mathbf{wx} : y = g(\mathbf{x}), \mathbf{x} \in R^2, \mathbf{x} \geq 0\},$$
   where $\mathbf{w} = (w_1, w_2)$ is the price vector for $\mathbf{x}$, $y = g(\mathbf{x})$ is the production function.
   a. Suppose that the firm adopts a new technology that improves the productivity of x_2 such that each unit of x_2 now is equivalent to tx_2 before, $t > 1$, i.e. the production function is now $y = g(x_1, tx_2)$. Using Lagrangean approach, illustrate how to obtain the cost minimizing input demand functions $x_i^c(y, \mathbf{w}, t)$, $i = 1, 2$. Make necessary assumptions when needed. (10 points)
   b. Use the envelop theorem to evaluate $\frac{\partial C}{\partial t}$, interpret the results. (10 points)
   c. You find that the output elasticities $\frac{\partial \ln(x_i^c)}{\partial \ln(y)}$ are the same for both inputs. You also find that the input price elasticities $\frac{\partial \ln(x_i^c)}{\partial \ln(\mathbf{w})}$ are independent of output y. What does this imply for the production technology? Justify your answer and discuss the implications. (10 points)
   d. Suppose you find that the indirect cost function has the form
   $$C(y, \mathbf{w}, t) = 3w_1^2 + w_2 + (4y^2 + 5)(2w_1 + \tfrac{10w_2}{t^2}).$$
   The firm currently produces 100 units of output ($y = 100$) facing prices $w_1 = 2$ and $w_2 = 1$ and using technology $t = 10$. Find the scale elasticity for current production.
   Under free entry and exit, do you have any recommendation to this firm? (10 points)

3. In Adam Smith's world, the "invisible hands" work, thus free market ideology is promoted. This is consistent with the first and second welfare theorems.
   a. What are the first and second welfare theorems (you can use your own words)? Under what conditions will these theorems hold and why? (12 points)
   b. Do you expect invisible hands work in today's economy? Explain and illustrate examples where they likely work vs. not. (8 points)

Have a nice winter break!