

AAE 635
Final Exam - Fall 2008

- 1) You are investigating a firm that chooses two inputs to minimize cost

$$C(y, \mathbf{w}, \beta) = \underset{\mathbf{x}}{\text{Min}} \{ \mathbf{w}\mathbf{x} : y = (1 + \beta)g(\mathbf{x}), \mathbf{x} \in R^2, \mathbf{x} \geq 0 \},$$

where $\mathbf{w} = (w_1, w_2)$ is the price vector for \mathbf{x} , $y = (1 + \beta)g(\mathbf{x})$ is the production function, where $\beta > 0$ is a technology enhancement index.

- a) Using Lagrangean approach, illustrate how to obtain the cost minimizing input demand functions $x_i^c(y, \mathbf{w}, \beta)$, $i = 1, 2$. Make necessary assumptions when needed and explain your assumptions. (10 points)
 - b) The firm faces technological progress represented by an increase in β . Evaluate and discuss the properties of $\mathbf{x}_w^c \equiv \frac{\partial \mathbf{x}^c}{\partial \mathbf{w}}$, $\mathbf{x}_y^c \equiv \frac{\partial \mathbf{x}^c}{\partial y}$, and $\mathbf{x}_\beta^c \equiv \frac{\partial \mathbf{x}^c}{\partial \beta}$. Interpret your results (Hint: find primal-dual results, discuss symmetric restrictions and sign restrictions...) (20 points)
 - c) Show that the Allen elasticity of substitution between input 1 and input 2 is necessarily positive (has a lower bound of zero). (Hint: identify homogeneity of $x_i^c(y, \mathbf{w}, \beta)$, then use the Euler theorem...). What is the economic interpretation? (10 points)
- 2) Consider a consumer making consumption decisions between gas (x_1) and other goods (x_2). He/she derives utility according to $U(\mathbf{x}) = 4\sqrt{x_1 x_2}$. The market price for these consumption goods are $\mathbf{p} = (p_1, p_2) > 0$. Consider the case where the consumption decisions are made in order to achieve a certain utility level U_0 .
- a) Derive the Hicksian demand functions for both gas and other goods. (10 points)
 - b) Find the associated indirect utility function $V(\mathbf{p}, y)$, where y is income (Hint: obtain the expression for the expenditure function, then use duality to find $V(\mathbf{p}, y)$). (10 points)
 - c) Use Roy's identity to obtain the Marshallian demand functions and verify the Slutsky equation. (10 points)
 - d) Given the decreasing price of gas, the government launched a program aiming to reduce the carbon dioxide emissions, i.e., imposing a carbon tax of \$0.56 per gallon gas. If the consumer would like to participate in a group action lobbying the government to give up this policy (it does not mean that consumers do not care about green house gas, they may think there are better ways to deal with it), how much would he/she like to contribute to the lobbying fund? Assume the gas price before the tax policy is \$1.69 per gallon, and the price of other goods are normalized to 1 and the household income is normalized to \$72. (10 points)
 - e) Now instead of tax policy, the government imposes a quantity control policy, i.e. each consumer is rationed to two thirds of his/her historical consumption (at price \$1.69 per gallon), how would your answer change compared to part d)? Illustrate how you derive answers here. (10 points)
 - f) Given the information available so far (from both questions), would you expect the "invisible hand" to work or not in this economy? Discuss and justify your answer. (Hint: define necessary terms first, and then comment...). (10 points)

Happy holidays!