

1) (15 pts.) You manage a vegetable farm and hire labor. This table reports how many carrot bunches are dug, cleaned, and ready for sale in one hour with different numbers of laborers.

Laborers Hired	Carrot Bunches/Hour	Marginal Product	Value of Marginal Product
2	23	--	--
4	66	21.5	64.50
6	85	9.5	28.50
8	97	6.0	18.00

a) Using numbers given in this table, show below how to calculate the Marginal Product for one example, and then fill in the Marginal Product column in the table above.

$$MP = (66 - 23)/(4 - 2) = 21.5$$

b) Carrots sell for \$3.00/bunch. Using numbers from this table, show below how to calculate the Value of Marginal Product for one example, and then fill in the Value of Marginal Product column in the table above.

$$VMP = P \times MP = 3 \times 21.5 = 64.50$$

c) What optimality condition defines the profit maximizing amount of the input to use? (Be brief and to the point.)

$$VMP = r \text{ (input price)}$$

d) If wages, taxes, liability insurance, etc. cost you \$18/hour to hire a laborer, what is the profit maximizing number of laborers to hire? (You may need to interpolate between entries.)

8 laborers

2) (15 pts.) Soybean yield as a function of potassium fertilizer is $Y = 5 + 3K - 0.04K^2$, where yield Y is bu/ac and the potassium rate K is lbs/ac. The price of soybeans is \$15.00/bu and the price of potassium \$0.60/lb.

a) What is the economically optimal potassium rate? Set up and solve this economic problem using calculus and the given information (Be sure to check the second order condition).

$$\pi = 15(5 + 3K - 0.04K^2) - 0.6K$$

$$FOC: 15(3 - 0.08K) - 0.6 = 0$$

$$3 - 0.08K = 0.6/15 = 0.04 \quad 2.96 = 0.08K$$

$$K = 2.96/0.08 = 37$$

$$SOC: 15(-0.08) < 0, \text{ satisfied for a maximum}$$

b) At the potassium rate you derived in part a, what is yield (bu/ac)?

$$Y = 5 + 3K - 0.04K^2 = 5 + 3(37) - 0.04(37)^2 = 61.24$$

c) Besides the cost of potassium, all other costs are \$700/ac. What are net returns (\$/ac)?

$$\pi = P \times Y(K) - rK - 700 = 15(61.24) - 0.6(37) - 700 = \$196.40$$

3) (15 pts.) Feeder pigs starting at 25 lbs fed the following oats and corn rations gain 50 lbs and are ready to transfer to the finishing operation.

Oats (bu)	Corn (bu)	Marginal Rate of Technical Substitution
6.0	11.0	---
7.1	9.4	0.6875 1.4545
8.6	8.4	1.5000 0.6667
10.1	7.8	2.5000 0.4000

a) Using numbers from this table, show below how to calculate the Marginal Rate of Technical Substitution between oats and corn for the second row in the table and then fill in the missing entries in the table above.

$$MRTS = - \Delta Y / \Delta X = - (9.4 - 11.0) / (7.1 - 6.0) = 1.4545, \text{ or } - (7.1 - 6.0) / (9.4 - 11.0) = 0.6875$$

b) What optimality condition defines the profit maximizing amount of both inputs to use? (Be brief and to the point.)

$$MRTS = - \Delta Y / \Delta X = r_x / r_y \text{ (the input price ratio)}$$

c) If oats cost \$2.50/bu and corn costs \$5/bu, what is the profit maximizing level of each to feed? (Note: you may need to interpolate between entries.)

$$r_o / r_c = 0.5 \text{ or } r_o / r_o = 2.0$$

If you use 2.0, happens at oats = 9.4 and corn = 8.1, but if you use 0.5, then closer to 10.1 for oats and 7.8 for corn or about 9.5 for oats and 8.0 for corn.

4) (20 pts.) Corn production is $Y = 28 + 8S - 0.1S^2 + 2W - 0.05W^2 + 0.01SW$, where Y is corn yield as bushels per acre, S is the seeding rate as 1,000 seeds per acre and W is inches of irrigation water applied per acre. The corn price is \$6/bu, the price of corn seed is \$2 for 1,000 seeds, and the cost to pump water (its price) is \$4.00 for an inch spread over an acre.

What is the profit maximizing amount of seeds (S) and water (W) to use per acre to grow corn?

(Note: you will not need to convert prices to set up the profit function.)

Be sure to check the second order conditions.

$$\pi = 6(28 + 8S - 0.1S^2 + 2W - 0.05W^2 + 0.01SW) - 2S - 4W$$

$$FOCs: 6(8 - 0.2S + 0.01W) - 2 = 0$$

$$FOC_w: 6(2 - 0.1W + 0.01S) - 4 = 0$$

Solve FOCs for s: $S = 38.35 + 0.05W$, then substitute this into FOC_w and solve for W to get $W = 17.253$, then put this into $S = 38.35 + 0.05W$ to get $S = 39.196$.

SOC: $d^2 \pi / dS^2 = 6(-0.2) = -1.2 < 0$ (satisfied for max), $d^2 \pi / dW^2 = 6(-0.1) = -0.6 < 0$ (satisfied for max), $d^2 \pi / dSdW = 6(0.1) = 0.06$, then $(-1.2)(-0.6) - 0.6^2 = 0.7164 > 0$ (satisfied for max)

5) (10 pts.) Your parents are thinking of quitting farming and ask your advice. Their typical annual farm revenue is \$200,000 and all annual costs are \$165,000. They tell you the farm's market value is \$260,000, but they owe \$110,000 for the mortgage. Also, they say they could get jobs and earn \$45,000 total for the both of them. You think they could invest the farm equity they have and earn 3% annually in the bond market.

a) Given these numbers, what is their economic profit for owning and operating the farm?

Revenue	200,000	given
Operating costs	- 165,000	given
Opportunity cost of equity	- 4,500	$260,000 - 110,000 = 150,000 \times 3\% = 4,500$
Opportunity cost of time	<u>- 45,000</u>	given
	- 14,500	

b) They want to make the most money they can so they have more to retire on and to give to their kids. Given these numbers and their goal, what is your recommendation and why?

Given their stated goal and the negative economic profit, they should sell the farm, take the jobs and invest the equity and make \$14,500 more per year.

6) (15 pts.) The table below reports the cost of producing turkeys for a farm.

Turkeys (number/year)	Fixed Cost	Variable Cost	Total Cost	Marginal Cost	Average Variable Cost	Average Total Cost
2,000	4,000	28,000	32,000	---	---	---
2,900	4,000	41,000	45,000	14.44	14.14	15.52
3,700	4,000	54,000	58,000	16.25	14.59	15.68
4,200	4,000	67,000	71,000	26.00	15.95	16.90

a) Using numbers from this table, show below how to calculate Total Cost, Marginal Cost, Average Variable Cost, and Average Total Cost for the second row and then fill in the missing values in the table.

$$TC = FC + VC = 4,000 + 28,000 = 32,000$$

$$MC = \Delta TC / \Delta Q = (41,000 - 28,000) / (2900 - 2000) = 14.44$$

$$AVC = VC / Q = 41,000 / 2900 = 14.14$$

$$ATC = TC / Q = 45,000 / 2900 = 15.52$$

b) What optimality condition defines the profit maximizing amount to produce? (Be brief and to the point.)

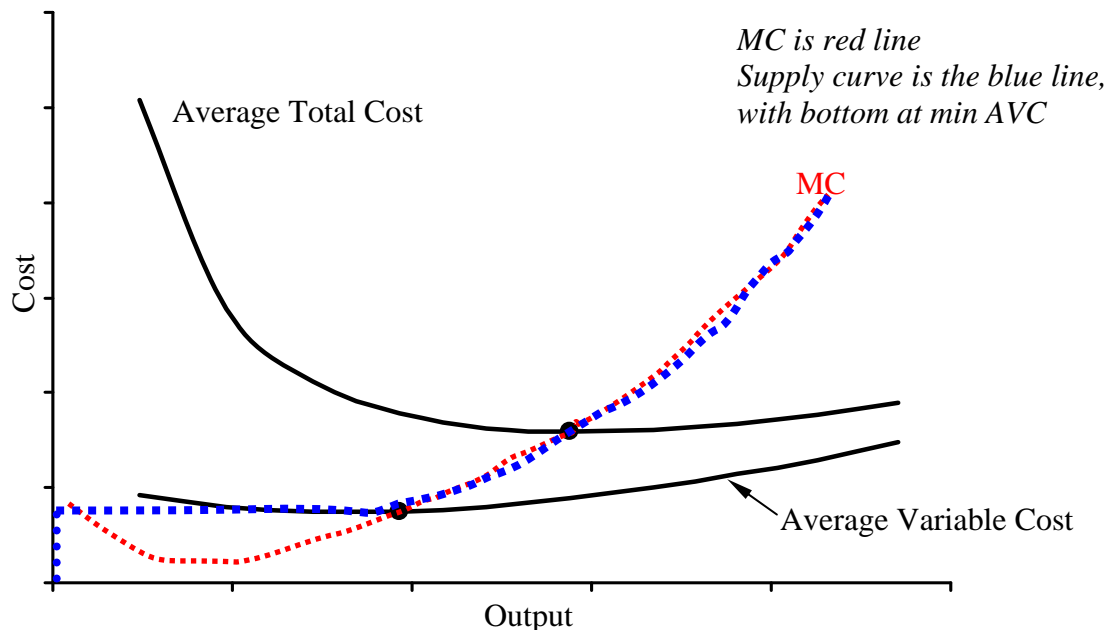
$$P = MC$$

c) At a turkey price of \$20, what is the profit maximizing amount of turkeys to produce? (Note: you may need to interpolate between entries.)

About midway between 3700 and 4200 or 3950 or 4000.

7) (10 pts.) The figure below illustrates Average Total Cost and Average Variable Cost for a hypothetical farm, where the two dots represent the minimum of each curve.

- a) On the figure below, draw in a hypothetical marginal cost curve and clearly mark the profit maximizing supply curve—the quantity the profit maximizing farm will produce at any given price. Clearly indicate the bottom end of the supply curve—the lowest price a profit maximizing farm will operate at and the quantity it will supply.



- b) Why would a profit maximizing farm choose to operate in the short run at a price below Average Total Cost and above Average Variable Cost? (Be brief and to the point.)

Because the firm covers some, but not all of the fixed costs at $\min ATC > p > \min AVC$. The firm still losses money (has negative economic profit) for a p in this range, but would lose more money if stopped producing, so better to lose less money.

- c) Briefly explain what I call the “Flat Objective Problem” and what it means for input use in crop production.

Output and profit/returns often become very “flat” (non-responsive) to changes in input use at or near optimal levels, so that there is lots of room for mistakes by the farmer and not really change the profit much, and the small differences in profit for these large input changes are swamped by all the other random noise in the production system, so cannot tell if you are at the optimum or just close to it.