

1) (15 pts.) You hire laborers to weed crops on your vegetable farm. The table below reports the boxes of tomatoes yielded in a week with different numbers of laborers hired to do weeding.

Laborers Hired	Boxes of Tomatoes	Marginal Product	Value of Marginal Product
1	600	--	--
3	1800	600	1800
5	2400	300	900
7	2600	100	300

- a) Using numbers given in this table, show below how to calculate the Marginal Product for one example, and then fill in the Marginal Product column in the table above.

$$MP = \Delta Q / \Delta X = (1800 - 600) / (3 - 1) = 600, \text{ etc.}$$

- b) Tomatoes sell for \$3.00/box. Using numbers from this table, show below how to calculate the Value of Marginal Product for one example, and then fill in the Value of Marginal Product column in the table above.

$$VMP = P * MP = 3 \times 60 = 1800, \text{ etc.}$$

- c) What optimality condition defines the profit maximizing amount of the input to use? (Be brief and to the point.)

$$VMP = r \text{ (the input price)}$$

- d) If wages, taxes, liability insurance, etc. cost \$600/week to hire a laborer, what is the profit maximizing number of laborers to hire? (You may need to interpolate between entries.)

Half way between 900 and 300 is 600 for the $vmp =$ the price of the input, so the laborers to hire are half way between 5 and 7, so 6.

2) (15 pts.) Wheat yield as a function of nitrogen fertilizer is $Y = 35 + 2N - 0.02N^2$, where yield Y is bu/ac and the nitrogen rate N is lbs/ac. The price of wheat is \$10.00/bu and the price of nitrogen \$0.60/lb.

- a) What is the economically optimal nitrogen rate? Set up and solve this economic problem using calculus and the given information (Be sure to check the second order condition).

$$\pi = 10(35 + 2N - 0.02N^2) - 0.6N$$

$$\text{FOC: } d\pi/dN = 10(2 - 0.04N) - 0.6 = 0$$

Solve FOC to get $N = 48.5$ lbs/ac

$$\text{Check SOC: } d^2\pi/dN^2 = 10(-0.04) = -0.4 < 0, \text{ satisfied for a maximum}$$

b) At the nitrogen rate you derived in part a, what is yield (bu/ac)?

$$Y = 35 + 2N - 0.02N^2 = 35 + 2(48.5) - 0.02(48.5^2) = 84.96$$

c) Besides the cost of nitrogen, other fixed costs are \$400/ac. What are net returns (\$/ac)?

$$\pi = pY - 0.6N - 400 = 10(84.96) - 0.6(48.5) - 400 = \$420.45/\text{ac}$$

3) (15 pts.) Laying hens fed the following corn and soybean meal rations produce 1 egg per day.

Soybean Meal (oz)	Corn (oz)	Marginal Rate of Technical Substitution	
5.0	10.0	---	
6.0	9.0	1.0	1.0
7.0	7.5	0.67	1.5
8.0	5.0	0.4	2.5

a) Using numbers from this table, show below how to calculate the Marginal Rate of Technical Substitution between soybean meal and corn for the second row in the table and then fill in the missing entries in the table above.

The answer depends on which input you define as X and Y.

$$\text{MRTS} = - (7.0 - 6.0)/(7.5 - 9.0) = 0.667, \text{ or}$$

or

$$\text{MRTS} = - (7.5 - 9.0)/(7.0 - 6.0) = 1.5$$

b) What optimality condition defines the profit maximizing amount of both inputs to use? (Be brief and to the point.)

$$\text{MRTS} = - \Delta Y/\Delta X = r_x/r_y$$

c) If soybean meal costs \$470/ton and corn costs \$210/ton, what is the profit maximizing level of each to feed? (Note: 1 ton = 2,000 lbs and you may need to interpolate between entries.)

$$\$470/\text{ton} \times 1 \text{ ton}/2000 \text{ lbs} = \$0.235/\text{lb}$$

$$\$210/\text{ton} \times 1 \text{ ton}/2000 \text{ lbs} = \$0.105/\text{lb}$$

Depending on which is X and which is Y,

$$r_x/r_y = 0.105/0.235 = 0.447 \quad \text{or} \quad r_x/r_y = 0.235/0.105 = 2.238$$

These are both between entries, so you need to interpolate. I get about 7.8 for soybean meal and about 5.5 for corn.

4) (20 pts.) Corn production is $Y = 9S - 0.1S^2 + 2N - 0.01N^2 - 0.01SN$, where Y is corn yield as bushels per acre, S is the seeding rate as 1,000 seeds per acre and N is nitrogen fertilizer as pounds per acre. The corn price is \$5/bu, the price of corn seed is \$2 for 1,000 seeds, and the price of nitrogen is \$0.60 per pound.

What is the profit maximizing amount of seeds (S) and nitrogen (N) to use per acre to grow corn? (Note: you will not need to convert prices to set up the profit function.) Be sure to check the second order conditions.

$$\pi = 5(9S - 0.1S^2 + 2N - 0.01N^2 - 0.01SN) - 2S - 0.6N$$

$$\text{FOC: } d\pi/dS = 5(9 - 0.2S - 0.01N) - 2 = 0$$

$$d\pi/dN = 5(2 - 0.02N - 0.01S) - 0.6 = 0$$

Solve 1st FOC for S gives: $S = 43 - 0.05N$

Substitute this into the 2nd FOC and solve for N gives: **N = 74.36** lbs/ac

Substitute this into $S = 43 - 0.05N$ gives **S = 39.28** thousand seeds/ac.

$$\text{SOC: } d^2\pi/dS^2 = 5(-0.2) = \underline{-1.0 < 0} \quad \text{Satisfied for a maximum}$$

$$d^2\pi/dN^2 = 5(-0.02) = \underline{-0.1 < 0} \quad \text{Satisfied for a maximum}$$

$$d^2\pi/dSdN = 5(-0.01) = -0.05$$

$$(d^2\pi/dG^2)(d^2\pi/dH^2) - (d^2\pi/dGdH)^2 =$$

$$(-1.0)(-0.1) - (-0.05)^2 = \underline{0.9975 > 0} \quad \text{Satisfied for a maximum}$$

5) (10 pts.) Your friend is a farmer and asks you for advice when he gets a job offer. From tax records, his typical annual farm revenue is \$730,000 and all annual costs are \$660,000. He says the farm's market value is \$800,000, but he still owes \$600,000 on the mortgage. The new job offer is for \$60,000 per year. You think your friend could earn 10% annually if he sold the farm and invested his money in a money market mutual fund.

a) What is your friend's economic profit for owning and operating the farm?

\$730,000	Revenue
<u>-\$660,000</u>	Operating Costs
\$70,000	Farm Income

Opportunity Cost of Time = \$60,000 (given in question information)

Opportunity Cost of Money or Capital

\$800,000	Market Value
<u>-\$600,000</u>	Mortgage Owed
\$200,000	Equity or money held in the farm

Assumed rate of return is 10% (given in question information)

Opportunity Cost of Money = \$200,000 x 10% = \$20,000

Put it all together

\$730,000	Revenue
-\$660,000	Operating Costs
-\$60,000	Opportunity Cost of Time
-\$20,000	Opportunity Cost of Money
-\$10,000	Economic Profit

- b) Your friend says his goal is to make as much money as possible and, since he is making \$70,000 per year as a farmer, he does not think he should take the \$60,000 per year job offer. Does this make sense? Explain what is your friend forgetting in his analysis?

He's forgetting about the equity he has in the farm and the return he could be making with that money in the mutual fund market.

6) (15 pts.) The table below reports the cost of producing eggs (dozens per year) for a farm.

Eggs (dozens/yr)	Fixed Cost	Variable Cost	Total Cost	Marginal Cost	Average Variable Cost	Average Total Cost
18,000	10,000	15,000	25,000	---	---	---
24,000	10,000	21,000	31,000	1.00	0.88	1.29
28,000	10,000	27,000	37,000	1.50	0.96	1.32
31,000	10,000	33,000	43,000	2.00	1.06	1.39

- a) Using numbers from this table, show below how to calculate Total Cost, Marginal Cost, Average Variable Cost, and Average Total Cost for the second row and then fill in the missing values in the table.

$$TC = FC + VC = 10,000 + 15,000 = 25,000$$

$$MC = \Delta TC / \Delta Q = (31,000 - 25,000) / (24,000 - 18,000) = 1.0$$

or $MC = \Delta VC / \Delta Q = (21,000 - 15,000) / (24,000 - 18,000) = 1.0$

$$AVC = VC / Q = 21,000 / 24,000 = 0.88$$

$$ATC = TC / Q = 31,000 / 24,000 = 1.29$$

- b) What optimality condition defines the profit maximizing amount to produce? (Be brief and to the point.)

$$P = MC$$

- c) At an egg price of \$1.75/dozen, what is the profit maximizing amount of eggs to produce?

$P = MC$ halfway between 1.50 and 2.00, so optimal eggs to produce is halfway between 28,000 and 31,000, or 29,500 eggs.

I'll let you think about the answers to question 7 yourself.